Lesson 11-5 Multiplying Polynomials

▶ **BIG IDEA** To multiply a polynomial by a polynomial, multiply each term of one polynomial by each term of the other polynomial and add the products.

Picturing the Multiplication of Polynomials with Area

The Area Model for Multiplication shows how to multiply two polynomials with many terms. For example, to multiply $a + b + c + d$ by $x + y + z$, draw a rectangle with length $a + b + c + d$ and width $x + y + z$.

	a	b	c	d
x	ax	bx	cx	dx
y	ay	by	cy	dy
z	az	bz	cz	dz

The area of the largest rectangle equals the sum of the areas of the twelve smaller rectangles.

$$\text{total area} = ax + ay + az + bx + by + bz + cx + cy + cz + dx + dy + dz$$

But the area of the biggest rectangle also equals the product of its length and width.

$$\text{total area} = (a + b + c + d) \cdot (x + y + z)$$

The Distributive Property can be used to justify why the two expressions must be equal. Distribute $(x + y + z)$ over $(a + b + c + d)$ to get $(a + b + c + d) \cdot (x + y + z) = a(x + y + z) + b(x + y + z) + c(x + y + z) + d(x + y + z) = ax + ay + az + bx + by + bz + cx + cy + cz + dx + dy + dz$.

Because of the multiple use of the Distributive Property, we call this general property the *Extended Distributive Property*.

Extended Distributive Property

To multiply two sums, multiply each term in the first sum by each term in the second sum and then add the products.

Mental Math

Evaluate.

a. $27^{\frac{1}{3}}$

b. $27^{\frac{2}{3}}$

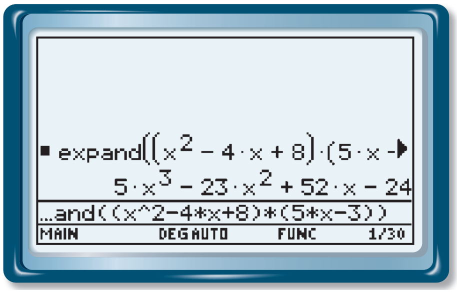
c. $27^{\frac{2}{3}}$

If one polynomial has m terms and the second has n terms, there will be mn terms in their product. This is due to the Multiplication Counting Principle. If some of these are like terms, you can simplify the product by combining like terms.

GUIDED

Example 1

Cassandra used the EXPAND feature on a CAS to multiply the polynomials $x^2 - 4x + 8$ and $5x - 3$. Her result is shown at the right.



The CAS does not display steps that most people would show in order to find the answer. Using the Extended Distributive Property, show the steps that the CAS does not display to expand $(x^2 - 4x + 8)(5x - 3)$.

Solution $(x^2 - 4x + 8)(5x - 3)$

$= x^2 \cdot \underline{\ ?\ } + x^2 \cdot \underline{\ ?\ } + (-4x) \cdot \underline{\ ?\ } + (-4x) \cdot \underline{\ ?\ } + 8 \cdot \underline{\ ?\ } + 8 \cdot \underline{\ ?\ }$

$= \underline{\ ?\ } + \underline{\ ?\ } + \underline{\ ?\ } + \underline{\ ?\ } + \underline{\ ?\ } + \underline{\ ?\ }$

Now combine like terms.

$= \underline{\ ?\ } - \underline{\ ?\ } + \underline{\ ?\ } - \underline{\ ?\ }$

Example 2

Expand $(4x + 3)(x - 6)$.

Solution Think of $x - 6$ as $x + -6$. Multiply each term in the first polynomial by each in the second. There will be four terms in the product.

$$(4x + 3)(x - 6) = 4x \cdot x + 4x \cdot (-6) + 3 \cdot x + 3 \cdot (-6)$$

$$= 4x^2 + (-24)x + 3x + (-18)$$

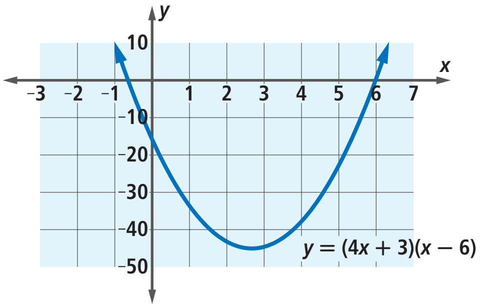
Now simplify by adding or subtracting like terms.

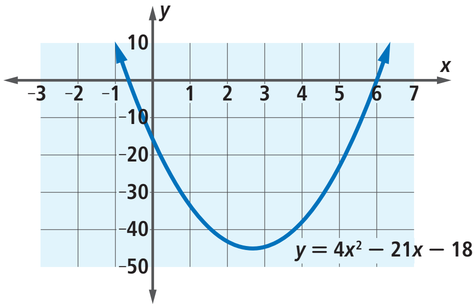
$$= 4x^2 - 21x - 18$$

Check 1 Let $x = 10$. (Ten is a nice value to use in checks because powers of 10 are so easily calculated.) Then $(4x + 3)(x - 6) = (4 \cdot 10 + 3)(10 - 6) = 43 \cdot 4 = 172$.
When $x = 10$, $4x^2 - 21x - 18 = 4 \cdot 10^2 - 21 \cdot 10 - 18 = 400 - 210 - 18 = 172$; so it checks.

(continued on next page)

Check 2 It is possible that when x = 10, the expression $(4x + 3)(x - 6)$ just happened to have the same value as $4x^2 - 21x - 18$. A better check is to set each expression equal to y and graph the resulting equation.





The two graphs are identical, so it checks.

STOP **QY**

Because multiplication is associative and commutative, to multiply three polynomials you can start by multiplying any two of them, and then multiply their product by the third polynomial.

Example 3

Expand $n(n - 1)(n - 2)$.

Solution 1 Multiply n by $n - 1$ first.

$$n(n - 1)(n - 2) = n(n - 1) \cdot (n - 2)$$
$$= (n^2 - n)(n - 2)$$
$$= n^2 \cdot n + n^2(-2) - n \cdot n - n(-2)$$
$$= n^3 - 2n^2 - n^2 + 2n$$
$$= n^3 - 3n^2 + 2n$$

Solution 2 Multiply $n - 1$ by $n - 2$ first.

$$n(n - 1)(n - 2) = n \cdot (n - 1)(n - 2)$$
$$= n \cdot (n \cdot n + n \cdot (-2) - 1 \cdot n - 1 \cdot (-2))$$
$$= n \cdot (n^2 - 2n - n + 2)$$
$$= n \cdot (n^2 - 3n + 2)$$
$$= n^3 - 3n^2 + 2n$$

▶ QY

Austin used a CAS to multiply $a^2 + 5n - 14$ by $4n + 1$. The screen shows $4a^2n + a^2 + 20n^2 - 51n - 14$.

a. How many terms did the CAS combine to get the product shown?

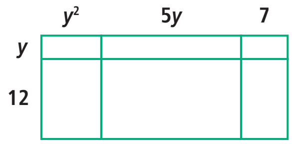
b. Which like terms produced a^2?

c. Which like terms produced $20n^2$?

d. Which like terms produced $-51n$?

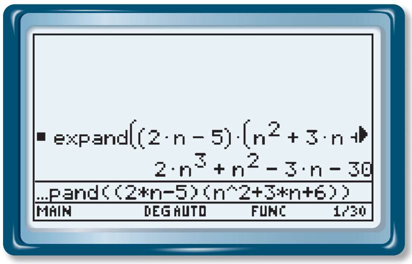
Questions

COVERING THE IDEAS



1. **a.** What multiplication is shown at the right?
   b. Do the multiplication.

2. Simone used a CAS to multiply $2n - 5$ by $n^2 + 3n + 6$. The screen at the right shows her result. Explain which multiplications were done and combined to get each term in the product.

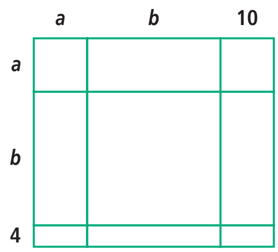


3. **a.** Multiply $2x - 5$ by $6x + 4$.
   b. Check your answer by letting $x = 10$.
   c. Check your answer by graphing $y = (2x - 5)(6x + 4)$ and graphing your answer.

In 4–7, expand and simplify the expression.

4. $(3x^2 + 7x + 4)(x + 6)$
5. $(n + 1)(2n^2 + 3n - 1)$
6. $(m^2 - 10m - 3)(2m^2 - 5m - 4)$
7. $4(x^2 + 2x + 2)(x^2 - 2x + 2)$

8. Find the area of the rectangle at the right, and simplify the result.



9. Expand $(5c - 4d + 1)(c - 7d)$.

10. **a.** Expand $(n - 3)(n + 4)(2n + 5)$ by first multiplying $n - 3$ by $n + 4$, then multiplying that product by $2n + 5$.
    b. Do the same expansion starting with a different multiplication.

APPLYING THE MATHEMATICS

11. **a.** Expand $(3x + 5)(4x + 2)$.
    b. Expand $(3x - 5)(4x - 2)$.
    c. Make a generalization from the pattern of answers in Parts a and b.

12. **a.** Expand $(4p - 1)(2p + 3)$.
    b. Expand $(4p + 1)(2p - 3)$.
    c. Make a generalization from the pattern of answers in Parts a and b.

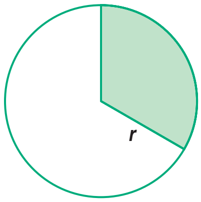
13. Expand $\left(\frac{1}{5}x - 2.7\right)^2$ by writing the power as $\left(\frac{1}{5}x - 2.7\right)\left(\frac{1}{5}x - 2.7\right)$.

14. How much larger is the volume of a cube with edges of length $n + 1$ than the volume of a cube with edges of length n?

15. Solve the equation $(2n - 7)(n + 14) = 57$ by multiplying the binomials and then using the Quadratic Formula.

REVIEW

16. The sector at the right is one-third of a circle with radius r. Write a formula for the perimeter p of this sector in factored form. **(Lesson 11-4)**



17. If $p(x) = 2x^2 + 5$ and $q(x) = 3x^2 + 6$, simplify $2 \cdot p(x) - 3 \cdot q(x)$. **(Lesson 11-3)**

18. Twice the larger of two numbers is six more than four times the smaller. If the sum of eight times the smaller and three times the larger is 93, what are the two numbers? **(Lessons 10-5, 10-4, 10-2)**

19. Given $f(x) = 4^{-x} + 2$, find each value. **(Lessons 8-4, 7-6)**
   a. $f(2)$ **b.** $f(-1)$ **c.** $f(0) + f(-3)$

20. Find two algebraic fractions whose product is $\frac{14m}{39p}$. **(Lesson 5-1)**

EXPLORATION

21. Multiply each of the polynomials in Parts a–d by $x + 1$.
   a. $x - 1$
   b. $x^2 - x + 1$
   c. $x^3 - x^2 + x - 1$
   d. $x^4 - x^3 + x^2 - x + 1$
   e. Look for a pattern and use it to multiply $(x + 1)(x^8 - x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1)$.
   f. Predict what you think the product of $(x + 1)$ and $(x^{100} - x^{99} + x^{98} - x^{97} + ... + x^2 - x + 1)$ is when simplified. Can you explain why your answer is correct?

22. A multidigit number in base 10 is shorthand for a polynomial in x. When $x = 10$, $436 = 4x^2 + 3x + 6$, and $2{,}187 = 2x^3 + 1x^2 + 8x + 7$.

   When you multiply 2,187 by 436, you are essentially multiplying two polynomials.

$$(4x^2 + 3x + 6)(2x^3 + 1x^2 + 8x + 7)$$

   Multiply these polynomials and show how their product equals the product of 436 and 2,187 when $x = 10$.

QY ANSWERS

a. two like terms were combined

b. a^2 times 1

c. $5n$ times $4n$

d. $5n$ times 1 and –14 times $4n$