

Lesson

11-4

Common Monomial Factoring

BIG IDEA Common monomial factoring is the process of writing a polynomial as a product of two polynomials, one of which is a monomial that factors each term of the polynomial.

When two or more numbers are multiplied, the result is a single number. *Factoring* is the reverse process. In **factoring**, we begin with a single number and express it as the product of two or more numbers. For example, the product of 7 and 4 is 28. So, factoring 28, we get $28 = 7 \cdot 4$. In Lesson 11-3, you multiplied monomials by polynomials to obtain polynomials. In this lesson you will learn how to reverse the process.

If factors are not integers, then every number has infinitely many factors. For example, 8 is not only $4 \cdot 2$ and $8 \cdot 1$, but also $24 \cdot \frac{1}{3}$ and $2.5 \cdot 3.2$. For this reason, in this book all factoring is *over the set of integers*.

Factoring Monomials

Every expression has itself and the number 1 as a factor. These are called the **trivial factors**. If a monomial is the product of two or more variables or numbers, then it will have factors other than itself and 1.

Example 1

What are the factors of $49x^3$?

Solution The factors of 49 are 1, 7, and 49. The monomial factors of x^3 are 1, x , x^2 , and x^3 . The factors of $49x^3$ are the 12 products of a factor of 49 with a factor of x :

$$1, 7, 49, x, 7x, 49x, x^2, 7x^2, 49x^2, x^3, 7x^3, 49x^3$$

STOP QY

The **greatest common factor** (GCF) of two or more monomials is the product of the greatest common factor of the coefficients and the greatest common factors of the variables.

Vocabulary

factoring
trivial factors
greatest common factor
factorization
prime polynomials
complete factorization

Mental Math

Consider the parabola
 $y = 3(x - 6)^2 + 4$.

- What is its vertex?
- Does the parabola open up or down?
- True or False** The parabola is congruent to $y = -3x^2$.
- What is an equation for its line of symmetry?

QY

Which of the factors of $49x^3$ are trivial factors?

Example 2

Find the greatest common factor of $6xy^2$ and $18y$.

Solution The GCF of 6 and 18 is 6. The GCF of xy^2 and y is y . Because the factor x does not appear in all terms, it does not appear in the GCF.

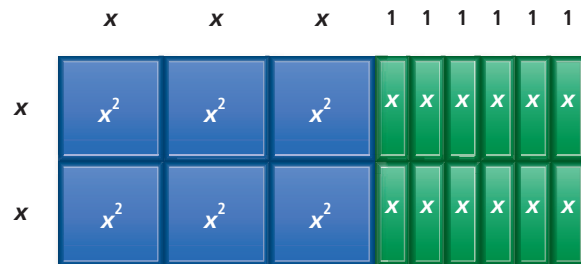
So the GCF of $6xy^2$ and $18y$ is $6 \cdot y$, which is $6y$.

Notice that the GCF of the monomials includes the GCF of the coefficients of the monomials. It also includes any common variables raised to the *least* exponent of that variable found in the terms.

As with integers, the result of factoring a polynomial is called a **factorization**. Here is a factorization of $6x^2 + 12x$.

$$6x^2 + 12x = 2x(3x + 6)$$

Again, as with integers, a factorization with two factors means that a rectangle can be formed with the factors as its dimensions. Here is a picture of the factorization.

**Activity**

Step 1 Build or draw two other rectangles with an area of $6x^2 + 12x$.

Step 2 Write the factorization that is shown by each rectangle.

Step 3 Do any of the rectangles have the greatest common factor of $6x^2$ and $12x$ as a side length? If so, which rectangle?

The Activity points out that there is more than one way to factor $6x^2 + 12x$. When factoring a polynomial, the goal is that the GCF of all the terms is one factor. In $6x^2 + 12x$, $6x$ is the greatest common factor, so $6x^2 + 12x = 6x(x + 2)$.

Monomials such as $6x$, and polynomials such as $x + 2$ that cannot be factored into polynomials of a lower degree, are called **prime polynomials**. To factor a polynomial completely means to factor it into prime polynomials. When there are no common numerical factors in the terms of any of the prime polynomials, the result is called a **complete factorization**. The complete factorization of $6x + 12$ is $6(x + 2)$.

There are many techniques for factoring polynomials, and you will study several in this and the next chapter. No matter which technique you use, you will obtain the same answer. As with integers, the prime factorization of a polynomial is unique except for order.

Unique Factorization Theorem for Polynomials

Every polynomial can be represented as a product of prime polynomials in exactly one way, disregarding order and integer multiples.

GUIDED

Example 3

Factor $20a^3b + 8a - 12a^5b^2$ completely.

Solution The greatest common factor of 20, 8, and -12 is $\underline{\quad? \quad}$. The greatest common factor of a^3 , a , and a^5 is $\underline{\quad? \quad}$. Because the variable b does not appear in all terms, b does not appear in the greatest common factor.

The greatest common factor of $20a^3b$, $8a$, and $-12a^5b^2$ is $\underline{\quad? \quad}$.

Divide each term by the GCF to find the terms in parentheses. With practice you'll be able to do these steps in your head.

$$\frac{20a^3b}{4a} = \underline{\quad? \quad} \quad \frac{8a}{4a} = \underline{\quad? \quad} \quad \frac{-12a^5b^2}{4a} = \underline{\quad? \quad}$$

So $20a^3b + 8a - 12a^5b^2 = 4a(\underline{\quad? \quad} + \underline{\quad? \quad} - \underline{\quad? \quad})$.

Factoring provides a way of simplifying some fractions.

Example 4

Simplify $\frac{22m + 4m^2}{m}$. ($m \neq 0$)

Solution 1 Factor the numerator, simplify the fraction, and multiply.

$$\frac{22m + 4m^2}{m} = \frac{2m(11 + 2m)}{m} = 2(11 + 2m) = 22 + 4m$$

Solution 2 Separate the given expression into the sum of two fractions and then simplify each fraction.

$$\frac{22m + 4m^2}{m} = \frac{22m}{m} + \frac{4m^2}{m} = 22 + 4m$$

Check The solutions give the same answer, so they check.

Questions

COVERING THE IDEAS

1. List all the factors of $33x^4$.

In 2 and 3, find the GCF.

2. $25y^5$ and $40y^2$

3. $17a^2b^2$ and $24ba^2$

4. Represent the factorization $12x^2 + 8x = 4x(3x + 2)$ with rectangles.

5. a. Factor $15c^2 + 5c$ by finding the greatest common factor of the terms.

b. Illustrate the factorization by drawing a rectangle whose sides are the factors.

6. Showing tiles, draw two different rectangles each with area equal to $16x^2 + 4x$.

7. Explain why $x^2 + xy$ is not a prime polynomial.

8. In Parts a–c, complete the products.

a. $36x^3 + 18x^2 = 6(\underline{\quad?} + \underline{\quad?})$

b. $36x^3 + 18x^2 = 18(\underline{\quad?} + \underline{\quad?})$

c. $36x^3 + 18x^2 = 18x^2(\underline{\quad?} + \underline{\quad?})$

d. Which of the products in Parts a–c is a complete factorization of $36x^3 + 10x^2$? Explain your answer.

9. Simplify $\frac{24n^6 + 20n^4}{4n^2}$.

10. Find the greatest common factor of $28x^5y^2$, $-14x^4y^3$, and $49x^3y^4$.

In 11–14, factor the polynomial completely.

11. $33a - 33b + 33ab$

12. $x^{2,100} - x^{2,049}$

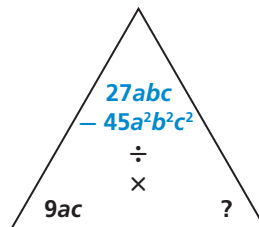
13. $12v^9 + 16v^{10}$

14. $46cd^3 - 69cd^2 + 18c^2d^2$

APPLYING THE MATHEMATICS

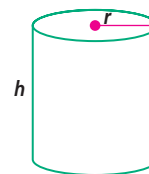
15. The area of a rectangle is $14r^2h$. One dimension is $2r$. What is the other dimension?

16. The top vertex in the fact triangle at the right has the expression $27abc - 45a^2b^2c^2$. What expression belongs in the position of the question mark?



17. a. Graph $y = 2x^2 - 8x$.
 b. Graph $y = 2x(x - 4)$.
 c. What do you notice about the graphs of the equations?
 Explain why this occurs.

In 18 and 19, a circular cylinder with height h and radius r is pictured at the right. Factor the expression giving its surface area.



18. $\pi r^2 + 2\pi r h$, the surface area with an open top
 19. $2\pi r^2 + 2\pi r h$, the surface area with a closed top

In 20 and 21, simplify the expression.

20. $\frac{9x^2y + 54xy - 9xy^2}{9xy}$ 21. $\frac{-100n^{100} - 80n^{80} + 60n^{60}}{2n^2}$

REVIEW

In 22 and 23, simplify the expression. (Lesson 11-3)

22. $-4x^3(3 - 5x^2 + 7x^4)$ 23. $k(k + 2k^2 + n) - 2n(k - 2n) - k^2$

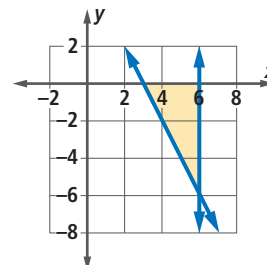
24. Which investment plan is worth more at the end of 10 years if the annual yield is 6%? Justify your answer. (Lesson 11-1)

Plan A: Deposit \$50 each year on January 2, beginning in 2008.

Plan B: Deposit \$100 every other year on January 2, beginning in 2008.

25. **Multiple Choice** Which system of inequalities describes the shaded region in the graph at the right? (Lesson 10-9)

A $\begin{cases} y - x < 6 \\ x \leq 0 \\ y \leq 6 \end{cases}$ B $\begin{cases} y + 2x \geq 6 \\ x \leq 6 \\ y \leq 0 \end{cases}$ C $\begin{cases} y + 2x \leq 6 \\ x \leq 0 \\ y \leq 0 \end{cases}$ D $\begin{cases} y \leq 6 + 4x \\ x \geq 3 \\ y \geq 1 \end{cases}$



26. Simplify $\sqrt{50x^3y^4}$. (Lesson 8-6)
27. There are 4 boys, 7 girls, 6 men, and 5 women on a community youth board. How many different leadership teams consisting of one adult and one child could be formed from these people? (Lesson 8-1)

EXPLORATION

28. The number 6 has four factors: 1, 2, 3, and 6. The number 30 has eight factors: 1, 2, 3, 5, 6, 10, 15, and 30.
- a. Find five numbers that each have an odd number of factors.
 b. Give an algebraic expression that describes all numbers with an odd number of factors. Explain why you think these numbers have an odd number of factors.

QY ANSWER

1 and $49x^3$