Lesson 11-3 Multiplying a Polynomial by a Monomial

BIG IDEA To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial and add the products.

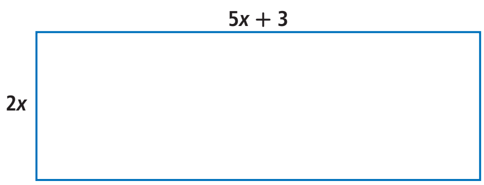
In earlier chapters, you saw several kinds of problems involving multiplication by a monomial. To multiply a monomial by a monomial, you can use properties of powers.

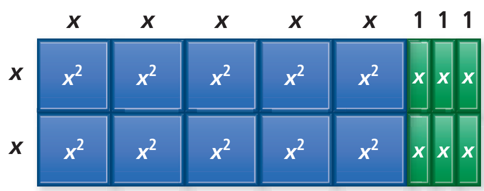
$$(9a^4b^5)(8a^3b) = 9 \cdot 8 \cdot a^{4+3}b^{5+1}$$
$$= 72a^7b^6$$

To multiply a monomial by a binomial, you can use the Distributive Property $a(b + c) = ab + ac$.

$$2x(5x + 3) = 2x \cdot 5x + 2x \cdot 3$$
$$= 10x^2 + 6x$$

Some products of monomials and binomials can be pictured using the Area Model for Multiplication. For example, the product $2x(5x + 3)$ is the area of a rectangle with dimensions $2x$ and $5x + 3$. Such a rectangle is shown below at the left.





This rectangle can be split into tiles as shown above at the right. The total area of the rectangle is $10x^2 + 6x$, which agrees with the result obtained using the Distributive Property.

Mental Math

Find the multiplicative inverse of each number.

a. $-5x$

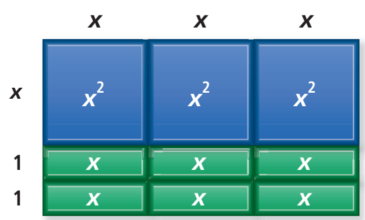
b. $\frac{4q}{-9}$

c. 0

d. $a + b$

Example 1

Find two equivalent expressions for the total area of the rectangle pictured at the right.



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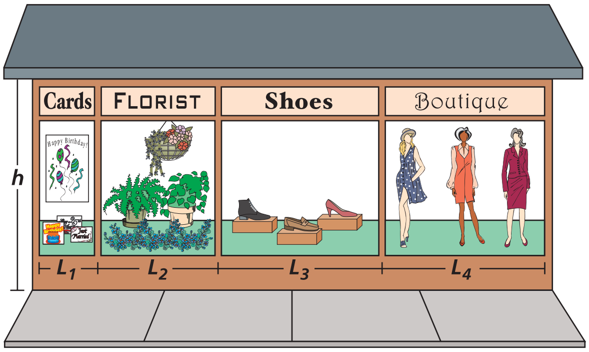
Solution The total area is the same as the sum of the areas of the individual tiles, or $3x^2 + 6x$. Also, the total area is length times width, or $3x(x + 2)$.

So this drawing shows $3x(x + 2) = 3x^2 + 6x$.

The area representation of a polynomial shows how to multiply a monomial by any other polynomial. The picture shows a view of some storefronts at a shopping mall.



The displays in the windows are used to attract shoppers, so store owners and mall managers are interested in the areas of storefronts. Note that the height h of each storefront is a monomial, and the sum of the lengths of the storefronts $(L_1 + L_2 + L_3 + L_4)$ is a polynomial.

The total area of the four windows can be computed in two ways. One way is to consider all the windows together. They form one big rectangle with length $(L_1 + L_2 + L_3 + L_4)$ and height h. Thus, the total area equals $h \cdot (L_1 + L_2 + L_3 + L_4)$.

A second way is to compute the area of each storefront and add the results. Thus, the total area also equals $hL_1 + hL_2 + hL_3 + hL_4$.

These areas are equal, so
$h \cdot (L_1 + L_2 + L_3 + L_4) = hL_1 + hL_2 + hL_3 + hL_4$.

In general, to multiply a monomial by a polynomial, extend the Distributive Property: multiply the monomial by each term in the polynomial and add the results.

Example 2

Expand $6r(x^2 - \sqrt{3}x + 7rx)$.

Solution Multiply each term in the trinomial by the monomial $6r$.

$$6r(x^2 - \sqrt{3}x + 7rx) = 6r \cdot x^2 - 6r \cdot \sqrt{3}x + 6r \cdot 7rx$$
$$= 6rx^2 - 6\sqrt{3}rx + 42r^2x$$

Check 1 Test a special case by substituting for both r and x. We let $r = 5$ and $x = 3$.

Does

$6 \cdot 5(3^2 - \sqrt{3} \cdot 3 + 7 \cdot 5 \cdot 3) = 6 \cdot 5 \cdot 3^2 - 6\sqrt{3} \cdot 5 \cdot 3 + 42 \cdot 5^2 \cdot 3$?

Remember to follow order of operations on each side.

Does $30(114 - 3\sqrt{3}) = 270 - 90\sqrt{3} + 3{,}150$?

A calculator shows that each side has the value 3,264.115... .

Check 2 Use a CAS. Enter

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EXPAND(6*r*(x^2-[√‾](3)*x+7*r*x)).
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You should get an expression equivalent to the answer.

As always, you must be careful with the signs in polynomials.

Activity

A student was given the original expressions below and asked to expand them. The student's answers are shown below.

Original Expressions	Student's Expanded Expressions
1. $2x(3x^2y^3z^7)$	1. $6x^3 + 2xy^3 + 2xz^7$
2. $-3a^2(4a^2b + 7ab - 5a^3b^2)$	2. $-12a^4b - 21a^3b + 15a^5b^2$
3. $7m^3n(4mn^4)$	3. $28m^4n^5$
4. $7xy(2x^3y - 5xy^5 + x^2y)$	4. $14x^4y^2 - 5xy^5 + x^2y$
5. $\frac{1}{4}a^5b(8ab^2 + 2a^2 - 20a^3b)$	5. $2a^6b^3 + \frac{1}{2}a^7b - 5a^8b^2$

Step 1 Identify the expressions you believe the student expanded correctly.

Step 2 Expand the original expressions. Did you accurately find the expanded expressions with mistakes?

Step 3 For each expression the student did not expand correctly, write a sentence explaining what the student did incorrectly.

Explaining a Rule from Arithmetic

Recall the rule for multiplying a decimal by a power of 10: To multiply by 10^n, move the decimal point n places to the right. Multiplication of a monomial by a polynomial can show why this rule works. For example, suppose 81,026 is multiplied by 1,000. Write 81,026 as a polynomial in base 10, and 1,000 as the monomial 10^3.

$$1{,}000 \cdot 81{,}026 = 10^3 \cdot (8 \cdot 10^4 + 1 \cdot 10^3 + 2 \cdot 10 + 6)$$

Now use the Distributive Property.

$= 10^3 \cdot 8 \cdot 10^4 + 10^3 \cdot 1 \cdot 10^3 + 10^3 \cdot 2 \cdot 10 + 10^3 \cdot 6$

The products can be simplified using the Product of Powers Property and the Commutative and Associative Properties of Multiplication.

$= 8 \cdot 10^7 + 1 \cdot 10^6 + 2 \cdot 10^4 + 6 \cdot 10^3$

Now simplify the polynomial.

$= 81{,}026{,}000$

This same procedure can be repeated to explain the product of any decimal and any integer power of 10.

Questions

COVERING THE IDEAS

In 1 and 2, find the product.

1. $(5x)(11x)$
2. $(200xy^3)(3x^2y)$

In 3 and 4,

a. find the product, and

b. draw a rectangle to represent the product.

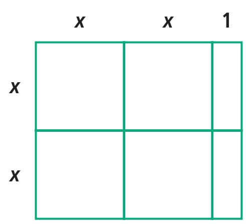
3. $3h(h + 5)$
4. $4n(n + 3)$

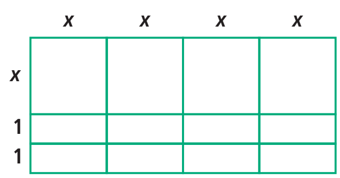
In 5 and 6, a large rectangle is shown.

a. Express its area as the sum of areas of smaller rectangles.

b. Express its area as length times width.

c. Write an equality from Parts a and b.

5. 

6. 

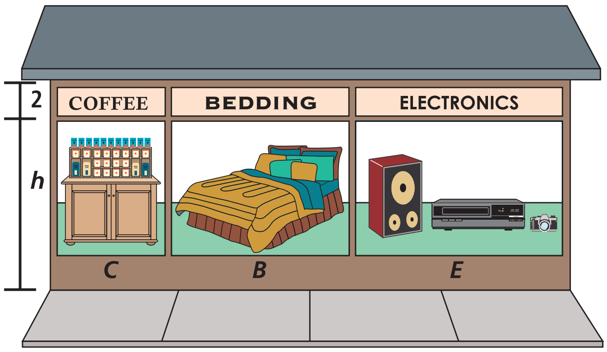
7. **Fill in the Blank** Using the Distributive Property, $a(b - c + d) = \underline{\quad ? \quad}$.

In 8–11, expand the expression.

8. $3x^2(x^2 + 2x - 8)$
9. $5x(-5x^2 - x + 6.2)$
10. $p(2 + p^2 + p^3 + 5p^4)$
11. $-0.5ab(4b - 2a + 10)$
12. Use multiplication of a monomial by a polynomial to explain why the product of 7,531 and 100,000 is 753,100,000.

APPLYING THE MATHEMATICS

13. Suppose the building below had to increase its height by 2 feet.

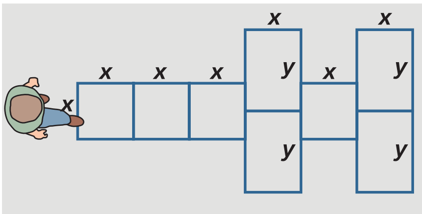


a. Express the entire building's new storefront area as the sum of the three individual stores' storefront areas.

b. Express the entire building's new storefront area as new height times length.

c. Express the new storefront area as the sum of its old area and the additional area.

14. The arrangement of rectangles at the right is used by children in many countries for playing hopscotch. What is the total area?



In 15–17, write the expression as a polynomial in standard form.

15. $(6x)(3x) - (5x)(2x) - (4x)(x)$

16. $2(x^2 + 3x) + 3x^2$

17. $m^3(m^2 - 3m + 2) - m^2(m^3 - 5m^2 - 6)$

In 18 and 19, simplify the expression.

18. $a(2b - c) + b(2c - a) + c(2a - b)$

19. $(x^2 + 2xy + y^2) - (x^2 - 2xy + y^2)$

20. At the right is a circle in a square.

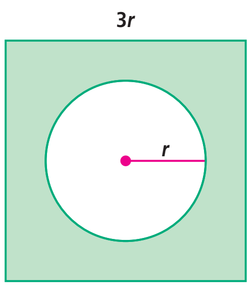
a. What is the area of the square?

b. What is the area of the circle?

c. What is the area of the shaded region?

d. If a person had 3 copies of the shaded region, how much area would be shaded?

e. If a person had c copies of the shaded region, how much area would be shaded?



21. **Fill in the Blank** Find the missing polynomial in the given equation.

$$3n^2 \cdot (\underline{\ \ ?\ \ }) = 60n^4 + 27n^3 - 30n^2$$

22. **a.** What is the rule for dividing a decimal by 1,000?
b. Make up an example like the one on pages 671–672 to explain why the rule works.

REVIEW

In 23 and 24, an expression is given.
a. Tell whether the expression is a polynomial.
b. If it is a polynomial, give its degree. If it is not a polynomial, explain why not. (Lesson 11-2)

23. $4a^4 + 2a^{-2}$

24. $n^3m^5 + n^2m^4 + nm^3$

25. Write a trinomial with degree 4. **(Lesson 11-2)**

26. After five years of birthdays, T.J. has received and saved $50x^4 + 70x^3 + 45x^2 + 100x + 80$ dollars. He put the money in a savings account at a yearly scale factor x. **(Lessons 11-1, 7-1)**
a. How much did T.J. get on his last birthday?
b. How much did T.J. get on the first of these birthdays?
c. If $x = 1$, how much has T.J. saved?
d. What does a value of 1 for x mean?

27. For what value(s) of c does the quadratic equation $3x^2 - 4x + c = 0$ have no solutions? **(Lesson 9-6)**

In 28 and 29, simplify the expression. (Lessons 8-4, 5-2)

28. $\frac{18xy^3}{6xy}$

29. $\frac{4m}{9} \div \frac{6m^3}{15m}$

In 30 and 31, write an equation for the line with the given characteristics.

30. contains the points (8, –2) and (3, 13) **(Lesson 6-6)**

31. slope $\frac{1}{2}$, x-intercept 1 **(Lesson 6-4)**

EXPLORATION

32. **a.** Suppose a monomial of degree 3 is multiplied by a monomial of degree 4. What must be true about the degree of the product? Support your answer with an example.
b. Suppose a monomial of degree m is multiplied by a monomial of degree n. What must be true about the degree of the product? Support your answer with an example.