# Lesson 11-2 Classifying Polynomials

**Vocabulary**

monomial
polynomial
binomial
trinomial
degree of a monomial
degree of a polynomial
linear polynomial
quadratic polynomial

**BIG IDEA** Polynomials are classified by their number of terms and by their degree.

## Classifying Polynomials by Numbers of Terms

Recall that a *term* can be a single number, variable, or product of numbers and variables. In an expression, addition (or subtraction, which is "adding the opposite") separates terms.

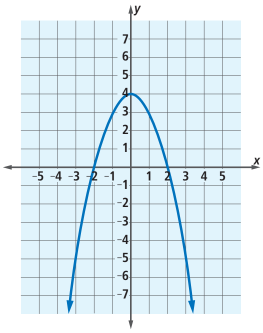
Polynomials are identified by their number of terms. A **monomial** is a single term in which the exponent for every variable is a positive integer. A **polynomial** is an expression that is either a monomial or sum of monomials. Polynomials with two or three terms are used so often they have special names. A **binomial** is a polynomial that has two terms. A **trinomial** is a polynomial that has three terms. Here are some examples.

| Monomials | Not Monomials | |
|---|---|---|
| $6x$ | $6x + y$ | (a binomial) |
| $-16t^2$ | $-16t^{-2}$ | (negative exponent on a variable) |
| $x^2y^4$ | $\frac{x^2}{y^4}$ | (variables divided) |
| **Binomials** | **Not Binomials** | |
| $x + 26\sqrt{2}$ | $26x\sqrt{2}$ | (monomial) |
| $\frac{x}{3} - y^3$ | $-\frac{xy^3}{3}$ | (monomial) |
| $0.44 - 2^{-10}pq^4$ | $0.44 - 2^{-10}p + q^4$ | (trinomial) |
| **Trinomials** | **Not Trinomials** | |
| $18x^2 + 5x + 9$ | $(15x^2)(5x)(9)$ | (monomial) |
| $a^2 + 2ab - b^{20}$ | $a^{-2} + 2ab - b^{-20}$ | (negative exponent on variables) |
| $pq + qr + rp$ | $\frac{1}{pq + qr + rp}$ | (variables divided) |

There are no special names for polynomials with more than three terms.

**Mental Math**

**Refer to the graph of a function.**




**a.** State the domain of the function.

**b.** State the range of the function.

**c.** State the *x*-intercepts.

**d.** State the *y*-intercept.

## Classifying Polynomials by Degree

Every nonconstant term of a polynomial has one or more exponents. For example, $3x^2$ has 2 as its exponent. $10t$ has an unwritten exponent of 1, since $t^1 = t$. $15a^2b^3c^4$ has 2, 3, and 4 as its exponents.

The **degree of a monomial** is the sum of the exponents of the variables in the expression.

$3x^2$ has degree 2.

$10t$ has degree 1.

$15a^2b^3c^4$ has degree $2 + 3 + 4$, or 9.

The degree of a single number, such as 15, is considered to be 0 because $15 = 15x^0$. However, the number 0 is said not to have any degree, because $0 = 0 \cdot x^n$, where $n$ could be any number. The **degree of a polynomial** is the highest degree of any of its monomial terms after the polynomial has been simplified. For example, $6x - 17x^4 + 8 + x^2$ has degree 4. $p + q^2 + pq^2 + p^2q^3$ has degree 5 (because $2 + 3 = 5$).

STOP **QY1**

> **QY1**
>
> Classify each polynomial by the number of its terms and its degree.
>
> **a.** $x^6 + x^7 + x^5$
>
> **b.** $8y^3z^2 - 40yz^6$
>
> **c.** $\frac{4}{3}\pi r^3$

When a polynomial has only one variable, writing it in standard form makes it easy to determine its degree. When the polynomial in $x$ above is written in standard form, the degree is the exponent of the leftmost term.

$-17x^4 + x^2 + 6x + 8$ has degree 4.

Function notation can be used to represent a polynomial in a variable. For example, let $p(x) = -17x^4 + x^2 + 6x + 8$. Then values of the polynomial are easily described. For example, $p(2) = -17 \cdot 2^4 + 2^2 + 6 \cdot 2 + 8 = -248$.

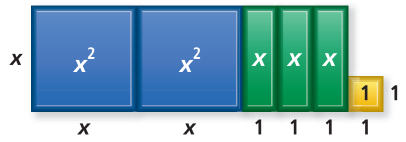
STOP **QY2**

> **QY2**
>
> If $p(x) = -x + 3 + 4x^4$, what is $p(-2)$?

The polynomial $p + q^2 + pq^2 + p^2q^3$ is a polynomial in $p$ and $q$. There is no standard form for writing polynomials that have more than one variable, like this one. However, sometimes one variable is picked and the polynomial is written in decreasing powers of that variable. For example, written in decreasing powers of $q$, this polynomial is $p^2q^3 + pq^2 + q^2 + p$, or, to emphasize the powers of $q$, $p^2q^3 + (p + 1)q^2 + p$.

A polynomial of degree 1, such as $13t - 6$, is called a **linear polynomial.** A polynomial of degree 2, such as $2x^2 + 3x + 1$ or $\ell w$, is called a **quadratic polynomial.** Linear and quadratic polynomials whose coefficients are positive integers can be represented by tiles.

The tiles below represent the polynomial $2x^2 + 3x + 1$ because this polynomial is the area of the figure.




## Using the Degree of a Polynomial to Check Operations with Polynomials

The simplest polynomials in one variable are the monomials $x$, $x^2$, $x^3$, $x^4$, and so on. You know how to add, subtract, multiply, and divide these monomials. For example, $x^2 \cdot x^3 = x^5$, so in this case a polynomial of degree 2 multiplied by a polynomial of degree 3 gives a polynomial of degree 5. In general, the degree of an answer to a polynomial computation is as easy to determine with complicated polynomials as it is with the simplest ones. Consider these examples of polynomial addition and subtraction.

**GUIDED**

### Example

**Collect like terms and determine the degree of the resulting polynomial.**

1. $(17w + 14) - (6 - 5w) =$ ___?___
   degree 1 degree 1 degree 1
2. $(6ab - 22) + (2a + 8b) = 6ab + 2a + 8b - 22$
   degree 2 degree 1 degree ___?___
3. $(4x + x^3 - 7) - (x^2 + 4x + 5) =$ ___?___
   degree 3 degree 2 degree ___?___
4. $(x^7 + 4x - 5) + (3 - 2x - x^7) =$ ___?___
   degree 7 degree 7 degree ___?___

Notice that the degree of the sum or difference of two polynomials is never greater than the highest degree of the polynomial addends. Can you see why this is so?

## Questions

**COVERING THE IDEAS**

1. Explain why $3x^2 + 4$ is a polynomial but $\frac{3}{x^2} + 4$ is not.
2. **Fill in the Blank** A binomial is a polynomial with ___?___ term(s).

**In 3–6, an expression is given.**

**a. Tell whether the expression is a monomial.**

**b. If it is a monomial, state its degree.**

**3.** $17x^{11}$

**4.** $2w^{-4}$

**5.** $\frac{1}{2}bh$

**6.** $2a^4b^5$

**7.** Is $xyz$ a trinomial? Explain your reasoning.

**8.** Classify each polynomial by its degree and number of terms.

**a.** $x^2 + 10$

**b.** $x^2 + 10x + 21$

**c.** $x^2 + 10xy + y^2$

**d.** $x^3 + 10x^2 + 21x$

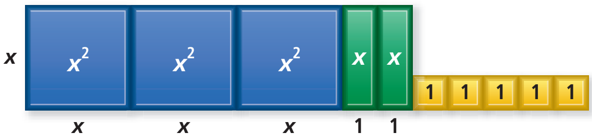
**9.** Write the polynomial $12 - 4x - 3x^5 + 8x^2$ in standard form.

**10.** **a.** Write the polynomial $a^3 - 3ab^2 - b^3 - 3a^2b$ in standard form as a polynomial in $a$.

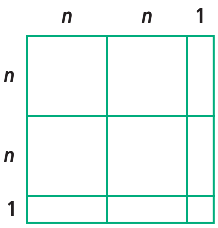
**b.** Write the polynomial $a^3 - 3ab^2 - b^3 - 3a^2b$ in standard form as a polynomial in $b$.

**In 11 and 12, what polynomial is represented by the tiles?**

**11.**




**12.**




**13.** Fill in the blank with *always, sometimes but not always*, or *never*. Explain your answer. The degree of the sum of two polynomials is __?__ greater than the degree of either polynomial addend.

## APPLYING THE MATHEMATICS

**In 14–18, an expression is given.**

**a. Show that the expression can be simplified into a monomial.**

**b. Give the degree of the monomial.**

**14.** $10x - 14x$

**15.** $10x(-14x)$

**16.** $(5n^3)(6n)^2$

**17.** $xy + yx$

**18.** $12x^4 - (3x^4 + 2x^4 + x^4)$

19. Let $p(x) = 50x^3 + 50x^2 + 50x + 50$ and $q(x) = 100x^3 + 120x^2 + 140x + 160$. Give the degree of each polynomial.

**a.** $p(x)$ **b.** $q(x)$ **c.** $p(x) + q(x)$ **d.** $p(x) - q(x)$

20. Repeat Question 19 if $p(x) = x^{200} - x^{100} + 1$ and $q(x) = x^{100} - x^{200} + 1$.

**In 21–24, give the degree of these polynomials used to find length, area, and volume of geometric figures.**

21. perimeter of a triangle $= a + b + c$

22. volume of a circular cone $= \frac{1}{3}\pi r^2 h$

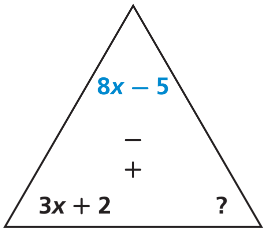
23. area of a trapezoid $= \frac{1}{2}hb_1 + \frac{1}{2}hb_2$

24. surface area of a cylinder $= 2\pi r^2 + 2\pi rh$

25. **a.** Write a monomial with one variable whose degree is 70.

**b.** Write a monomial with two variables whose degree is 70.

26. Complete the fact triangle below and write the related polynomial addition and subtraction facts.




27. **a.** Give an example of two trinomials in $x$ of degree 5 whose sum is of degree 5.

**b.** Give an example of two trinomials in $x$ of degree 5 whose sum is not of degree 5.

28. **a.** Write 318 and 4,670 as polynomials with 10 substituted for the variable.

**b.** Add your polynomials from Part a. Is your sum equal to the sum of 318 and 4,670?

## REVIEW

29. **a.** If you received \$1,000 as a present on the day you were born, and the money was put into an account at an annual scale factor of $x$, how much would be in your account on your 18th birthday?

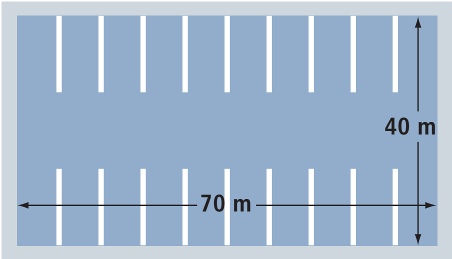
**b.** Evaluate the amount in Part a if $x = 1.05$. (**Lesson 11-1**)

30. Consider the system $\begin{cases} a - 2b = 50 \\ b = -4c \end{cases}$. To solve this system, one student substituted $-4c$ for $b$ in the first equation. The student then wrote $a - 8c = 50$. (**Lessons 10-3, 10-2**)

   a. Is the student's work correct?

   b. If it is correct, finish solving the system. If not, describe the error the student made.

31. A parking lot with length 70 meters and width 40 meters is to have a pedestrian sidewalk surrounding it, increasing its total area to 3,256 square meters. What will be the width of the sidewalk? (**Lesson 9-7**)




**In 32–34, use the Distributive Property to expand the expression.** (**Lesson 2-1**)

32. $4x(x - 9)$

33. $n(n + 52)$

34. $(3m + 19.2)80$

## EXPLORATION

35. Suppose three polynomials of the same degree $n$ are added.

   a. What is the highest possible degree of their sum? Explain your answer.

   b. What is the lowest possible degree of their sum?

**QY ANSWERS**

**1a.** trinomial, degree 7

**b.** binomial, degree 7

**c.** monomial, degree 3

**2.** 69