Lesson

11-1 Investments and Polynomials

Vocabulary

polynomial in x

standard form for a polynomial

BIG IDEA When amounts are invested periodically and earn interest from the time of investment, the total value can be represented by a polynomial.

Among the most important money matters adults commonly deal with are salary or wages, savings, payments on loans for cars or trips or other items, and home mortgages or rent.

Each of these items involves paying or receiving money each month, every few months, or every year. But what is the total amount paid or received? The answer is not easy to calculate because interest starts at different times. Here is an example of this kind of situation.

Mental Math

Find the distance between

a. (x, y) and $(0, 0)$.

b. $(a, 5)$ and $(a, -11)$.

c. $(m, m - n)$ and (m, m).

Example 1

Each birthday from age 12 on, Jessica has received $500 from her grandparents. She saves the money in an account that pays an annual yield of 6%. How much money will she have by the time she is 18?

Solution Write down how much Jessica has on each birthday. On her 12th birthday she has $500. She then receives interest on that $500. She receives an additional $500 on her 13th birthday. So on her 13th birthday she has $500(1.06) + 500 = \$1{,}030.00$.

Each year interest is paid on all the money previously saved and each year another $500 gift is added. The totals for her 12th through 15th birthdays are given below.

Birthday	Expression	Total
12th	500	$= \$500$
13th	$500(1.06) + 500$	$= \$1{,}030.00$
14th	$500(1.06)^2 + 500(1.06) + 500$	$= \$1{,}591.80$
15th	$500(1.06)^3 + 500(1.06)^2 + 500(1.06) + 500$	$= \$2{,}187.31$

from 12th birthday | from 13th birthday | from 14th birthday | from 15th birthday

You can see the pattern. By her 18th birthday, Jessica will have three more gifts of \$500 and earn interest on this money for three more years. The total will be $500(1.06)^6 + 500(1.06)^5 + 500(1.06)^4 + 500(1.06)^3 + 500(1.06)^2 + 500(1.06) + 500 = \$4{,}196.91$.

This total of \$4,196.91 that she has by her 18th birthday is \$696.91 more than the total \$3,500 she received as gifts because of the interest earned.

Letting $x = 1.06$, the amount of money Jessica has (in dollars) after her 18th birthday is given by the polynomial $500x^6 + 500x^5 + 500x^4 + 500x^3 + 500x^2 + 500x + 500$.

This expression is called a *polynomial in x*. A **polynomial in *x*** is a sum of multiples of powers of *x*. In this situation the polynomial is useful because if the interest rate is different, you only have to substitute a different value for *x*. We call *x* in this situation a *scale factor*. For example, had Jessica invested her money at an annual yield of 4%, the scale factor would be 104% = 1.04. At the end of 6 years, Jessica's investment (in dollars) would be $500(1.04)^6 + 500(1.04)^5 + 500(1.04)^4 + 500(1.04)^3 + 500(1.04)^2 + 500(1.04) + 500$.

You should verify with a calculator that this sum equals \$3,949.14.

GUIDED

Example 2

Suppose Rajib's parents gave him \$100 on his 12th birthday, \$120 on his 13th, \$140 on his 14th, and \$160 on his 15th. If he invests all the money in an account with a yearly scale factor *x*, how much money will he have on his 15th birthday?

Solution By his 15th birthday, the ___?___ from Rajib's 12th birthday will earn 3 years' worth of interest. It will have grown to ___?___ • $x^{?}$.

The ___?___ from his 13th birthday will have grown to ___?___.

The ___?___ from his 14th birthday will have grown to ___?___.

On his 15th birthday he receives ___?___.

Through his 15th birthday, the total dollar amount Rajib will have from his birthday gifts is $100x^3 + 120x^2 + 140x + 160$.

Rajib's aunt gave him \$50 on each of these 4 birthdays. If he puts this money into the same account, the amount available from the aunt's gifts would be $50x^3 + 50x^2 + 50x + 50$.

The total amount he would have from all these gifts is found by adding these two polynomials.

$$(100x^3 + 120x^2 + 140x + 160) + (50x^3 + 50x^2 + 50x + 50)$$

Recall that this sum can be simplified. First, use the Associative and Commutative Properties of Addition to rearrange the polynomials so that like terms are together.

$$= (100x^3 + 50x^3) + (120x^2 + 50x^2) + (140x + 50x) + (160 + 50)$$

Then use the Distributive Property to add like terms.

$$= (100 + 50)x^3 + (120 + 50)x^2 + (140 + 50)x + (160 + 50)$$
$$= 150x^3 + 170x^2 + 190x + 210$$

Notice what the answer means in relation to Rajib's birthday presents. The first year he got \$150 (\$100 from his parents, \$50 from his aunt). The \$150 has 3 years to earn interest. The \$170 from his next birthday earns interest for 2 years. And so on. Also notice that in these examples we have written the polynomials in the form of decreasing powers of x. This is called **standard form for a polynomial.** Polynomials are often written in standard form.

When comparing investments, it is often useful to make a table or construct a spreadsheet.

Example 3

Kelsey and Chip plan to save money for a round-the-world trip when they retire 10 years from now. Kelsey plans to save \$2,000 per year for the first 5 years, and then will stop making deposits. Chip plans to wait 5 years to begin saving, but then hopes to save \$2,500 per year for 5 years. They will each deposit their savings at the beginning of the year into a special account earning 6% interest compounded annually. How much will each have after 10 years?

Solution Make a spreadsheet showing the amount of money each person will have at the end of each year. At the end of the first year Kelsey will have 1.06(2,000) = 2,120. At the end of the second year, she will have 106% of the sum of the previous balance and the new deposit of \$2,000. In all, she will have 1.06(2,120 + 2,000) = \$4,367.20. This pattern continues. But after 5 years, she deposits no more money. So her money only accumulates interest. Kelsey's end-of-year balance in the spreadsheet on the next page was computed by entering the formula =1.06*B2 into cell C2 and the formula =1.06*(C2+B3) into cell C3. The formula in cell C3 was then replicated down column C to C11. A similar set of formulas generated Chip's end-of-year balance.

Miami was the top cruise ship departure port of the United States in 2004 with 641 departures.

Source: Bureau of Transportation Statistics

◇	A	B	C	D	E
1	Year	Kelsey's Deposits ($)	Kelsey's End-of-Year Balance ($)	Chip's Deposits ($)	Chip's End-of-Year Balance ($)
2	1	2,000	2,120.00	0	0
3	2	2,000	4,367.20	0	0
4	3	2,000	6,749.23	0	0
5	4	2,000	9,274.19	0	0
6	5	2,000	11,950.64	0	0
7	6	0	12,667.68	2,500	2,650.00
8	7	0	13,427.74	2,500	5,459.00
9	8	0	14,233.40	2,500	8,436.54
10	9	0	15,087.40	2,500	11,592.73
11	10	0	15,992.65	2,500	14,938.30

Ten years from now Kelsey will have about $16,000 and Chip will have about $15,000.

In Example 3, notice that even though Kelsey deposits $10,000 and Chip deposits $12,500 at the same rate of interest, compounding interest over a longer period of time gives Kelsey about $1,055 more than Chip. Here is what has happened. After 10 years:

Kelsey has $2{,}000x^{10} + 2{,}000x^9 + 2{,}000x^8 + 2{,}000x^7 + 2{,}000x^6$.

Chip has $2{,}500x^5 + 2{,}500x^4 + 2{,}500x^3 + 2{,}500x^2 + 2{,}500x$.

When $x = 1.06$, Kelsey has more than Chip.

Questions

COVERING THE IDEAS

1. Refer to Example 1. Suppose Jessica is able to get an annual yield of 5% on her investment.
   a. How much money will she have in her account by her 18th birthday?
   b. How much less is this than what she would have earned with a 6% annual yield?
2. Mary's grandfather will receive a $3,000 bonus from his employer on each of his birthdays from age 61 to age 65 if he indicates he will retire at 65.
   a. If he saves the money in an account paying an annual yield of 6.5%, how much will he have by the time he retires at age 65?
   b. How much will he have accumulated by the time he retires at age 65 if his investment grows by a scale factor of x each year?

3. Refer to Example 2. Suppose Rajib also gets \$75, \$85, \$95, and \$105 from cousin Lilly on his four birthdays. He puts this money into his account also.
   a. By his 15th birthday, how much money will Rajib have from just his cousin?
   b. What is the total Rajib will have saved by his 15th birthday from all of his birthday presents?

In 4–6, refer to Example 3.

4. Explain why Chip had less money saved than Kelsey at the end of the 10-year period even though he put more money into his account than Kelsey.
5. Chip said to Kelsey, "I might have less money now, but I am catching up to you, and even if we put no more money into our accounts, the amount in my account will be greater after a few more years." Kelsey said, "You have less now and you will always have less." Who is right?
6. Suppose Kelsey and Chip were able to earn 3% on their investments. Recalculate the balances in the spreadsheet and describe the end result.
7. Refer to page 654. Write the number 84,267 as a polynomial in base 10.

APPLYING THE MATHEMATICS

In 8–11, Clara, Mona, and Odella are friends who have the same birthday. They received the following cash presents on their birthdays. Each put all her money into a bank account that paid a 6% annual yield.

	Clara	Mona	Odella
In 2003	\$200	\$250	\$100
In 2004	\$300	\$250	\$500
In 2005	\$250	\$250	nothing

8. How much money did Clara have on her birthday in 2003?
9. How much did Mona have on her birthday in 2004?
10. How much did Odella have on her birthday in 2005?
11. In 2006, Clara received \$300 on her birthday. If all the money from 2003 to 2006 had been and remains in an account with scale factor x, how much would she have had by her birthday in each of the following years?
    a. 2005
    b. 2006
    c. 2007
    d. 2008

Out of a class of 24 students, the probability of any 3 that share a birthday is about 16.6%.

12. Suppose in 1999 Tanya received \$100 on her birthday. From 2000 to 2003 she received \$150 on her birthday. She put the money in a shoe box. The money is still there.
    a. How much money did Tanya have after her 2003 birthday?
    b. How much more would she have had if she had invested her money at an annual yield of 4% each year?

13. **Multiple Choice** Which is the sum of $x^4 + x^3 + x^2$?
    A x^9 B $3x^9$
    C x^{24} D None of these

In 14–17, simplify the expression.

14. $(2y^2 + 13y - 14) + (4y^2 - 3y - 24)$
15. $6(11n + 8n^2 - 2) + (6n^2 - n - 9)$
16. $(7w^2 - 2w + 16) - 4(7w^2 + 15)$
17. $(x^3 + 2x^2 + 8) - (2x - 5x^3 + 6)$
18. Solve the equation $(3x^2 + 2x + 4) + (3x^2 + 11x + 2) = 0$.
19. Solve the equation $(3x^2 + 2x + 4) - (3x^2 + 11x + 2) = 0$.

In 20 and 21, find the missing polynomial.

20. $(91x^2 + 4x - 15) + (\_?\_) = 110x^2 + 62$
21. $(3y^2 - 2y - 1) - (\_?\_) = -4y^2 - 6y + 21$
22. A *cord* of wood is an amount of wood equal to about 128 cubic feet. A wood harvester has planted trees in a forest each spring for four years, as shown in the table at the right.

Year	Number of Trees Planted
1	10,000
2	15,000
3	20,000
4	18,000

Suppose each tree contains 0.01 cord of wood when planted, and the cordage grows with a scale factor x each year. How many cords of wood are in the forest after planting the fourth spring?

Forests cover 747 million acres in the United States.

Source: U.S. Department of Agriculture

REVIEW

23. If 8 pencils and 5 erasers cost \$4.69 and 3 pencils and 4 erasers cost \$2.80, find the cost of 2 pencils. **(Lesson 10-2)**
24. Write an equation for the line which passes through the points (4, –8) and (–10, 6). **(Lesson 6-6)**

25. Consider the equation $\frac{30(4h-2)}{6(2h-1)} = 10$. **(Lesson 5-5)**

    a. For what value of h is $\frac{30(4h-2)}{6(2h-1)}$ undefined?

    b. Solve for h.

26. In one of his studies, discussed in the book *The Effects of Cross- and Self-Fertilization in the Vegetable Kingdom,* Charles Darwin compared the heights of two groups of plants. One group was cross-pollinated, meaning they were fertilized by pollen from other plants. The other group was self-fertilized. For the cross-fertilized plants, the mean height was 20.2, and for the self-fertilized plants the mean height was 17.6. Darwin's data are shown below. (*Note:* The values were rounded before plotting in Question 27.) **(Lessons 1-7, 1-6)**

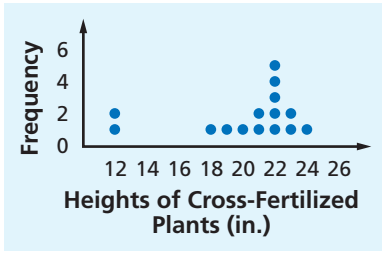
Cross-Fertilized (in.)	23.5	12.0	21.0	22.0	19.1	21.5	22.1	20.4	18.3	21.6	23.3	21.0	22.1	23.0	12.0
Self-Fertilized (in.)	17.4	20.4	20.0	20.0	18.4	18.6	18.6	15.3	16.5	18.0	16.3	18.0	12.8	15.5	18.0

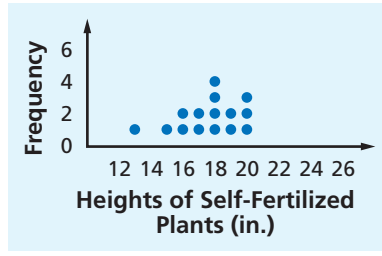
    a. Find the range of the data for each type of plant.

    b. Find the mean for each type of plant.

    c. Find the mean absolute deviation for each type of plant.

27. Use the two dot plots below. **(Lesson 1-7)**





    a. Classify each dot plot as skewed right, skewed left, symmetric, or uniform.

    b. Based on Darwin's data, what advice would you give to a farmer who wants to grow tall plants?

EXPLORATION

28. Refer to Kelsey and Chip in Example 3 and Questions 4–6.

    a. For what interest rates does Kelsey end up with more money after 10 years than Chip?

    b. For what interest rates does Chip end up with more money than Kelsey?