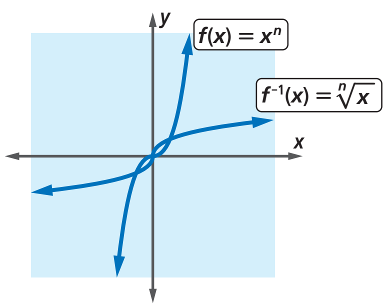
Chapter 8 Summary and Vocabulary

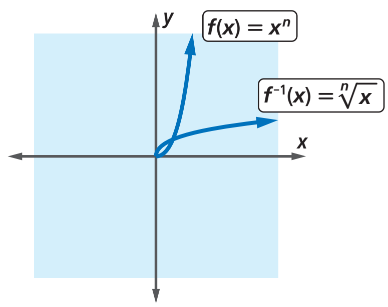
- Every relation has an **inverse** that can be found by switching the coordinates of its ordered pairs. The graphs of any relation and its inverse are reflection images of each other over the line $y = x$.

- Inverses of some functions are themselves functions. A real function graphed on the coordinate plane has an inverse if and only if no horizontal line intersects the function's graph in more than one point. In general, two functions f and g are inverses of each other if and only if $f \circ g$ and $g \circ f$ are defined, $f(g(x)) = x$ for all values of x in the domain of g, and $g(f(x)) = x$ for all values of x in the domain of f.

- Consider the function f with domain the set of all real numbers and equation of the form $y = f(x) = x^n$. If n is an odd integer ≥ 3, its inverse is the nth root function with equation $x = y^n$ or $y = \sqrt[n]{x}$. These two functions are graphed below at the left. When n is an even integer ≥ 2, the inverse of $y = f(x) = x^n$ is not a function. However, if the domain of f is restricted to the set of nonnegative real numbers, the inverse of f is the nth root function with equation $y = \sqrt[n]{x} = x^{\frac{1}{n}}$. The restricted function and its inverse are graphed below at the right.



n is odd



n is even, $x \geq 0$

That is,

1. When $x \geq 0$, $\sqrt[n]{x}$ is defined for any integer $n > 2$. It equals the positive nth root of x.

2. When $x < 0$, $\sqrt[n]{x}$ is defined only for odd integers $n \geq 3$. It equals the real nth root of x, a negative number.

Vocabulary

Lesson 8-1
composite, $g \circ f$
*function composition, composition

Lesson 8-2
inverse of a relation
horizontal-line test

Lesson 8-3
*inverse function, f^{-1}

Lesson 8-4
radical sign, $\sqrt{}$
$\sqrt[n]{x}$ when $x \geq 0$
geometric mean

Lesson 8-6
rationalizing the denominator
conjugate

Lesson 8-7
$\sqrt[n]{x}$ when $x < 0$

Lesson 8-8
extraneous solution

- All properties of powers listed in Chapter 7 apply to radicals when they stand for real numbers. They lead to the following theorems for any real numbers x and y and integers m and n for which the symbols are defined and stand for real numbers.

  Root of a Power Theorem: $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

  Root of a Product Theorem: $\sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y}$

- These properties are helpful in simplifying radical expressions and in solving equations with radicals. To solve an equation involving an nth root, you need to raise each side to the nth power. When you do this, you may gain **extraneous solutions**. Always check every possible answer in the original equation to make sure that extraneous solutions have not been included.

- Radicals appear in many formulas. For example, the nth root of the product of n numbers is the geometric mean of the numbers. When a radical appears in the denominator of a fraction, multiplying both the numerator and denominator by a well chosen number can make the new denominator rational. To **rationalize** a fraction with a denominator of the form $a + \sqrt{b}$, multiply both numerator and denominator by the **conjugate** $a - \sqrt{b}$.

Theorems

Inverse-Relation Theorem (p. 524)
Inverse Functions Theorem (p. 530)
Power Function Inverse Theorem (p. 532)
Root of a Power Theorem (p. 539)
Root of a Product Theorem (p. 546)
*n*th Root of a Product Theorem (p. 559)

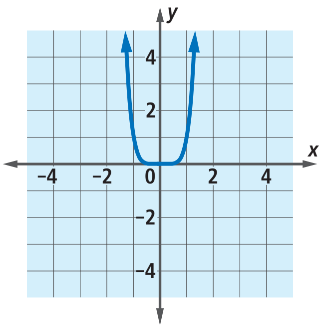
Chapter 8 Self-Test

Take this test as you would take a test in class. You will need a calculator and graph paper. Then use the Selected Answers section in the back of the book to check your work.

In 1–4, let $f(x) = 4 - x^2$ and $g(x) = 3x + 7$.

1. **Fill in the Blank** $f \circ g : -1 \rightarrow$ __?__.
2. Write a formula for $f(g(x))$.
3. Write an equation for $g \circ f$.
4. Are f and g inverses? Justify your answer.
5. a. Find an equation for the inverse of the function r defined by $r(x) = \frac{1}{2}x + 3$.
   b. Is the inverse a function? Justify your answer.

In 6 and 7, refer to the function graphed below.



6. Graph the inverse of the function.
7. How can you restrict the domain of the original function so that the inverse is also a function?
8. **True or False** $\sqrt[4]{16} = -2$. Justify your answer.
9. The formula $r = \sqrt{\frac{S}{4\pi}}$ gives the radius r of a sphere with surface area S. What is the surface area, to the nearest square centimeter, of a spherical balloon with radius 12 cm?

In 10–12, simplify and rationalize all denominators. Assume $x > 0$ and $y > 0$.

10. $\sqrt[3]{-216x^{12}y^3}$
11. $\sqrt[4]{\frac{32x^{12}}{x^4}}$
12. $\frac{3x^2}{\sqrt{81x}}$
13. The top of a 15-foot ladder rests against a window ledge on the side of a building. The height h of the window ledge above the ground is equal to the distance from the bottom of the ladder to the base of the building. On a test, students were asked to find h. Three students' answers are below:

    Deon: $h = \sqrt{\frac{225}{2}}$

    Tia: $h = \frac{15}{2}$

    Carmen: $h = \frac{15\sqrt{2}}{2}$

    Which answer(s) is (are) correct? Justify your answer.
14. Rewrite $\sqrt[5]{\sqrt[3]{13}}$ as a power with a simple fraction exponent in lowest terms.
15. Rationalize the denominator and simplify $\frac{24}{8 - 2\sqrt{3}}$.

16. Use the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$ to approximate the radius r of a sphere with volume $V = 268$ cubic inches to the nearest hundredth.

In 17 and 18, solve for x.

17. $25 = 4\sqrt{7x}$

18. $17 + \sqrt[3]{2x + 7} = 20$

19. For what values of n is the inverse of the function f with equation $y = f(x) = x^n$ also a function?

20. Give the domain and range of the function $y = \sqrt[6]{x}$, if x and y are real numbers.

21. Six samples taken in a water quality survey have the following levels of *E. coli* bacteria.

Sample	1	2	3	4	5	6
Count (per 100 mL)	27	32	144	46	597	1092

Use the geometric mean to compute an average level of bacteria across the samples.

22. Eddie is buying a new pair of eyeglasses. He has a store coupon for $45 off a new pair of glasses. The glasses he chooses have a starting price of P dollars, but they are on sale for 30% off. For what values of P will Eddie get a better deal if he can use the coupon before the discount is taken?

Chapter 8 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Find values and rules for composites of functions. (Lesson 8-1)

1. When applying $f \circ g$, which function is applied last?

In 2–4, let $p(x) = x^2 + x + 1$ and $q(x) = x - 6$.

2. a. Find $p(q(5))$.
   b. Find $p(q(x))$.
3. a. Find $q(p(5))$.
   b. Find $q(p(x))$.
4. The function $p \circ q$ maps -10 onto what number?

In 5 and 6, rules for functions f and g are given. Does $f \circ g = g \circ f$? Justify your response.

5. $f : x \rightarrow -\frac{3}{8}x$; $g : x \rightarrow -\frac{8}{3}x$
6. $f(x) = 3\sqrt{x}$; $g(x) = \frac{9}{x}$, $x > 0$
7. If $h(x) = x^{\frac{2}{3}}$, find an expression for $h(h(x))$.
8. If $r(x) = \frac{2x + 1}{x}$, find an expression for $r(r(x))$.

OBJECTIVE B Find the inverse of a function. (Lessons 8-2, 8-3)

9. A function has equation $y = 6x - 3$. Write an equation for its inverse in slope-intercept form.
10. A function has equation $y = \sqrt{x^2}$. What is an equation for its inverse?
11. **Fill in the Blank** If $f : x \rightarrow 7x + 13$, then $f^{-1} : x \rightarrow$ __?__.
12. Show that $f : x \rightarrow 3x + 2$ and $g : x \rightarrow \frac{1}{3}x - 2$ are not inverse functions.
13. **Fill in the Blank** If $g(t) = -t^2$ for $t \leq 0$, then $g^{-1}(t) =$ __?__.
14. **Multiple Choice** Suppose $f(x) = x^5$. Then $f^{-1}(x) = x^k$, where $k =$
    A -5.
    B $\frac{1}{5}$.
    C $-\frac{1}{5}$.
    D 5.

OBJECTIVE C Evaluate radicals. (Lessons 8-4, 8-7)

In 15–18, write as a whole number or simple fraction.

15. $\sqrt[5]{243}$
16. $\sqrt[3]{-27}$
17. $\sqrt[3]{\left(\frac{64}{343}\right)^2}$
18. $(\sqrt{15} + \sqrt{13})(\sqrt{15} - \sqrt{13})$

In 19–22, approximate to the nearest hundredth.

19. $\sqrt[3]{3}$
20. $\sqrt[4]{81 + 16}$
21. $4\sqrt[3]{-75}$
22. $\sqrt[9]{\sqrt{365}}$

OBJECTIVE D **Rewrite or simplify expressions with radicals.** (Lessons 8-5, 8-6, 8-7)

In 23–30, simplify. Assume that all variables are positive.

23. $\sqrt{b^8}$

24. $\sqrt[3]{57r^3}$

25. $\sqrt[6]{8} \cdot \sqrt[6]{8}$

26. $\sqrt[3]{-60n^{12}}$

27. $\sqrt[7]{-u^{14}v^{28}}$

28. $\sqrt{3x^3} \cdot \sqrt{6x}$

29. $\sqrt{\sqrt{\sqrt{k}}}$

30. $\sqrt{N} \cdot \sqrt[3]{N} \cdot \sqrt[6]{N}$

In 31–34, rationalize the denominator and simplify, if possible.

31. $\frac{13}{\sqrt{13}}$

32. $\frac{8}{\sqrt{2}}$

33. $\frac{7}{\sqrt{3}-1}$

34. $\frac{p}{p+\sqrt{q}}$ $(p > 0, q > 0)$

OBJECTIVE E **Solve equations with radicals.** (Lesson 8-8)

In 35–40, find all real solutions. Round answers to the nearest hundredth where necessary.

35. $\sqrt[3]{y} = 2.5$

36. $13 = 11\sqrt[4]{f}$

37. $12 = \frac{1}{4}\sqrt{16-y}$

38. $\sqrt[3]{x-1} - 9 = 27$

39. $18 + \sqrt[6]{64n} = 12$

40. $\sqrt{6x} + 3\sqrt{6x} = 12$

PROPERTIES Principles behind the mathematics

OBJECTIVE F **Apply properties of the inverse of a function.** (Lessons 8-2, 8-3)

In 41 and 42, state whether the statement is true or false.

41. If functions f and g are inverses of each other, then $f \circ g(x) = g \circ f(x)$ for all x for which these functions are defined.

42. When the domain of f is the set of positive real numbers, then the inverse of $y = x^6$ has equation $y = \sqrt[6]{x}$.

43. Suppose the domain of a linear function L is $\{x \mid x \le 0\}$ and the range is $\{y \mid y \ge -6\}$. What are the domain and range of L^{-1}?

In 44 and 45, suppose f and g are inverses of each other.

44. If (ℓ, m) is a point on the graph of f, what point must be on the graph of g?

45. If the domain of g is the set of all positive integers, what can you conclude about the domain or range of f?

OBJECTIVE G **Apply properties of radicals and *n*th root functions.** (Lessons 8-4, 8-5, 8-7)

46. If x is negative, for what values of n is $\sqrt[n]{x}$ a real number?

47. **Multiple Choice** Which expression is not defined?

A $\sqrt[3]{625}$ B $\sqrt[3]{-625}$

C $\sqrt[4]{625}$ D $\sqrt[4]{-625}$

48. Explain why the statement $\sqrt[7]{a} = a^{\frac{1}{7}}$ is not true for all real numbers a.

49. For what values of x is $\sqrt[5]{x^5} = x$?

50. Give a counterexample to this statement: For all real numbers x, $\sqrt[8]{x^8} = x$.

In 51 and 52, tell whether the statement $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ is true for given conditions. Justify your answer.

51. a and b are negative, $n = 2$

52. a and b are negative, $n = 3$

USES Applications of mathematics in real-world situations

OBJECTIVE H **Solve real-world problems that can be modeled by composite functions.** (Lesson 8-1)

53. An electronics store is having a 25%-off sale on flat-screen televisions. The television Amber wants to buy has a sticker price of $1200, and the sales tax in the state is 9%.

a. How much will Amber pay for the television if the discount is taken before the tax is calculated?

b. How much will she pay if the tax is calculated first?

c. Would Amber get a better deal if the tax was calculated first or if the discount was taken first? Explain your answer.

54. A group goes to a restaurant with a $15-off coupon. The restaurant bill comes to b dollars before the tip and before using the coupon. The group wants to tip the server 20%.

a. Find an expression for $f(b)$, the total cost if the coupon is used before the tip is calculated.

b. Find an expression for $g(b)$, the total cost if the tip is calculated before the coupon is used.

c. Restaurants typically urge patrons to tip on the full bill before any discount is applied. Why do you think restaurants do this?

OBJECTIVE I **Solve real-world problems that can be modeled by equations with radicals.** (Lessons 8-4, 8-8)

55. The maximum distance d you can see from the top of a building with height h is approximated by the formula $d = k\sqrt{h}$. Apartment buildings A and B are 9 and 16 stories high, respectively. If these two apartment buildings have the same height per floor, about how many times farther can you see from the top of apartment B than the top of apartment A?

In 56 and 57, use the following information: To find the speed s (in mph) that a certain car was traveling on a typical dry road, suppose that police use the formula $s = 2\sqrt{5L}$, where L is the length of the skid marks in feet.

56. The car skidded 35 feet before stopping. According to the formula, how fast was the car going?

57. About how far would this car be expected to travel if it skids from 55 mph to a stop?

58. The U.S. Consumer Price Index (CPI) estimates the price of goods and services over time. In the table below, y is the percent change in the CPI from the previous year to the indicated year.

Year	2001	2002	2003	2004	2005	2006	2007
y (percent)	2.8	1.6	2.3	2.7	3.4	3.2	2.8

a. Add a third row to the table in which you convert each percent to a size-change factor. For example, the factor for 2001 is 1.028.

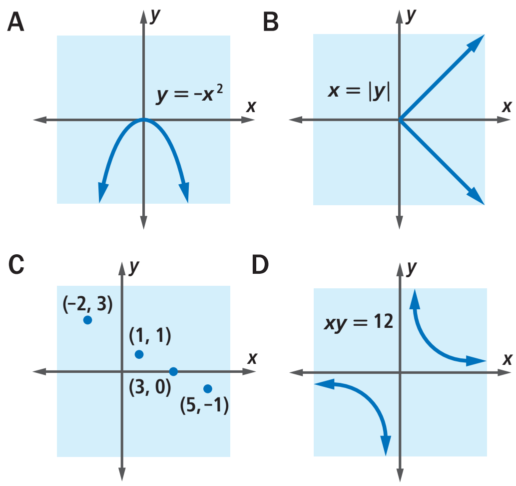
b. Compute the geometric mean of the size-change factors you found in Part a, and determine the average CPI percentage increase over this seven-year time frame.

59. The diameter of a spherical balloon varies directly as the cube root of its volume. If one balloon holds 7 times as much air as a second balloon, how do their diameters compare?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE J **Make and interpret graphs of inverses of relations and functions.** (Lessons 8-2, 8-3)

60. Use the graphs below.



a. **Multiple Choice** Which is the graph of a function whose inverse is not a function?

b. How can you restrict the domain of the function in your answer to Part a so that its inverse is a function?

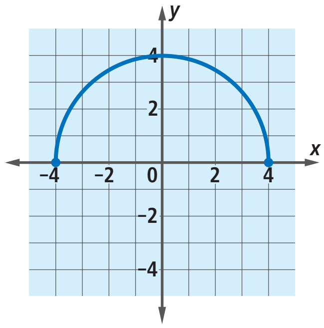
61. Let $f = \{(-3, 6), (-2, 5), (-1, 2), (0, 3)\}$

a. Graph f^{-1}.

b. What transformation maps f onto f^{-1}?

62. Graph the inverse of the function with equation $y = \sqrt{x^2}$.

63. a. Graph the inverse of the function graphed below.



b. Is the inverse a function? Why or why not?

64. Draw a graph of a function with domain $\{x \mid -1 < x < 1\}$ that has an inverse which is not a function.

65. Let $g(x) = x^3$.

a. Graph $y = g(x)$ and $y = g^{-1}(x)$.

b. What is the domain of g^{-1}?

In 66 and 67, an equation for a function is given.

a. Graph the function.

b. State the domain and range of the function.

66. $h(x) = \sqrt[6]{x}$

67. $f(x) = \sqrt[7]{x}$