Chapter 2 Summary and Vocabulary

- The functions studied in this chapter are all based on direct and inverse variation. When $k \neq 0$ and $n > 0$, formulas of the form $y = kx^n$ define **direct-variation functions**, and those of the form $y = \frac{k}{x^n}$ define **inverse-variation functions**. Four special cases of direct and inverse variation commonly occur. The equation, graph, name of the curve, domain D, range R, asymptotes, and symmetries of each special case are summarized on the next page.

- In a formula where y is given in terms of x, it is logical to ask how changing x (the independent variable) affects the value of y (the dependent variable). In direct or inverse variation, when x is multiplied by a constant, the change in y is predicted by the **Fundamental Theorem of Variation**. If y varies directly as x^n, then when x is multiplied by c, y is multiplied by c^n. If y varies inversely as x^n, then when x is multiplied by c, y is divided by c^n. The converse of this theorem is also true.

- The rate of change $\frac{y_2 - y_1}{x_2 - x_1}$ between (x_1, y_1) and (x_2, y_2) is the **slope** of the line connecting the points. For graphs of equations of the form $y = kx$, the rate of change between any two points on the graph is the constant k. For nonlinear curves, the rate of change is not constant, but varies depending on which points are used to calculate it.

- Variation formulas may involve three or more variables, one dependent and the others independent. In a **joint variation**, all the independent variables are multiplied. If the independent variables are not all multiplied, the situation is a **combined variation**. Variation formulas can be derived from real data by examining two variables at a time and comparing their graphs with those given on the following page.

- There are many applications of direct, inverse, joint, and combined variation. They include perimeter, area, and volume formulas, the inverse-square laws of sound and gravity, a variety of relationships among physical quantities such as distance, time, force, and pressure, and costs that are related to these other measures.

Vocabulary

Lesson 2-1
varies directly as
directly proportional to
direct-variation equation
constant of variation
*direct-variation function

Lesson 2-2
*inverse-variation function
varies inversely as
inversely proportional to

Lesson 2-4
*rate of change, slope

Lesson 2-5
parabola
vertex of a parabola
reflection-symmetric
line of symmetry

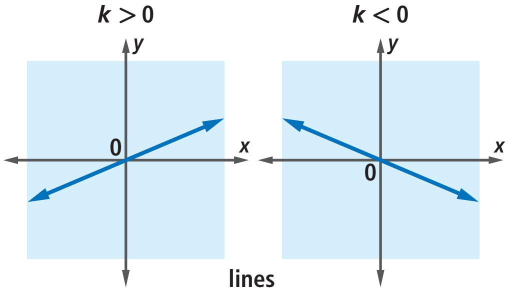
Lesson 2-6
hyperbola
branches of a hyperbola
vertical asymptote
horizontal asymptote
discrete set
inverse-square curve

Lesson 2-9
combined variation
joint variation

- Some **direct-variation functions**

$y = kx$

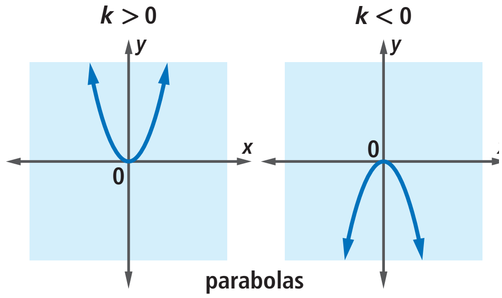
y varies directly as x.



$D = R =$ set of all real numbers

$y = kx^2$

y varies directly as the square of x.

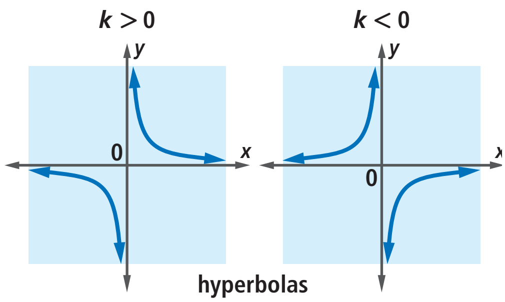


$D =$ set of all reals $D =$ set of all reals
$R = \{y \mid y \geq 0\}$ $R = \{y \mid y \leq 0\}$
reflection-symmetric to the y-axis

- Some **inverse-variation functions**

$y = \frac{k}{x}$

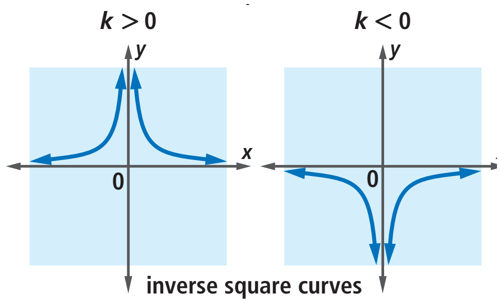
y varies directly as x.



$D = R = \{x \mid x \neq 0\}$
asymptotes: x-axis, y-axis

$y = \frac{k}{x^2}$

y varies inversely as the square of x



$D = \{x \mid x \neq 0\}$ $D = \{x \mid x \neq 0\}$
$R = \{y \mid y > 0\}$ $R = \{y \mid y < 0\}$
asymptotes: x-axis, y-axis
reflection-symmetric to the y-axis

Theorems and Properties

Fundamental Theorem of Variation (p. 88)
Slope of $y = kx$ Throrem (p. 95)
Converse of the Fundamental Theorem of Variation (p. 122)

Chapter 2 Self-Test

Take this test as you would take a test in class. Then use the Selected Answers section in the back of the book to check your work.

In 1 and 2, translate the statement into a variation equation.

1. The number n of items that a store can display on a shelf varies directly as the length ℓ of the shelf.

2. The weight w that a bridge column can support varies directly as the fourth power of its diameter d and inversely as the square of its length L.

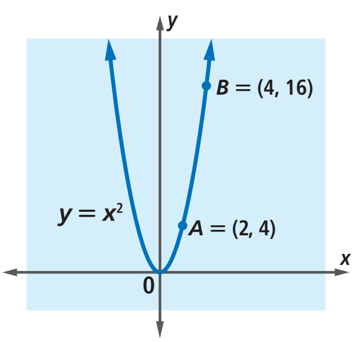
3. Write the variation equation $s = \frac{k}{p^4}$ in words.

4. If T varies directly as the third power of s and the second power of w, and if $T = 10$ when $s = 2$ and $w = 1$, find T when $s = 8$ and $w = \frac{1}{2}$.

5. For the variation equation $y = \frac{5}{x^2}$, how does the y-value change when an x-value is doubled? Give two specific ordered pairs (x, y) that support your conclusion.

6. If y is multiplied by 8 when x is doubled, how is y related to x? Write a general equation to describe the relationship.

7. Find the average rate of change of $y = x^2$ between points A and B shown on the graph.



8. True or False

a. All graphs of variation equations contain the origin.

b. On a direct-variation graph, if you compute the rate of change between $x = 1$ and $x = 2$, and then again between $x = 2$ and $x = 3$, then you will get the same result.

9. a. Fill in the Blanks The graph of $y = kx^2$ is called a __?__ and opens upward if __?__.

b. How does the graph of $y = kx^2$ change as k gets closer to 0?

10. Suppose f is a function with $f(d) = \frac{12}{d^2}$. What is the domain of f?

11. Fill in the Blank Complete each sentence with *inversely, directly,* or *neither inversely nor directly.* Explain your reasoning.

a. The surface area of a sphere varies __?__ as the square of its radius.

b. If you have exactly \$5,000 to invest, the number of shares of a stock you can buy varies __?__ as the cost of each share.

12. a. Make a table of values for $y = -\frac{1}{2}x$. Include at least five pairs.

b. Draw a graph of the equation in Part a.

c. Find the slope of the graph in Part b.

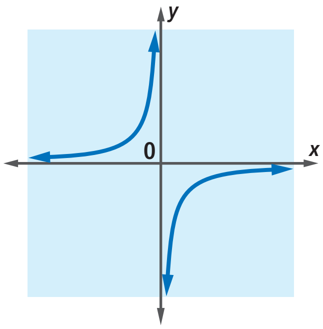
13. **a.** Use a graphing utility to sketch a graph of $y = \frac{4}{x}$.

b. Identify any asymptotes to the graph in Part a.

14. **a. Multiple Choice** Which equation's graph looks the most like the graph below?

A $y = 2x$ B $y = -\frac{2}{x}$

C $y = \frac{2}{x^2}$ D $y = -\frac{x}{2}$



b. State the domain and range of your answer to Part a.

15. Bashir's family is buying a new car. After looking at several models they became interested in the relationship between a car's fuel economy F in miles per gallon (mpg) and its weight w in pounds. Bashir collected data in the table below.

Weight w (lb)	Fuel Economy F (mpg)
2,655	40
2,991	34
3,263	32
3,737	28
3,962	26
5,577	19

a. Graph the data points in the table.

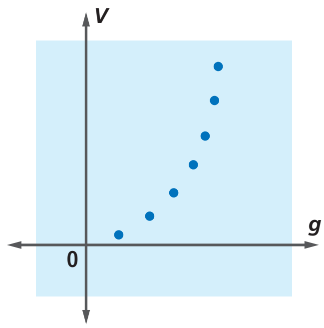
b. Which variation equation is a better model for Bashir's data, $F = \frac{k}{w}$ or $F = \frac{k}{w^2}$? Justify your answer and find the constant of variation.

c. A family is thinking about buying a 5,900-pound sport-utility vehicle (SUV). Based on the model you found in Part b, what would you predict its fuel economy will be?

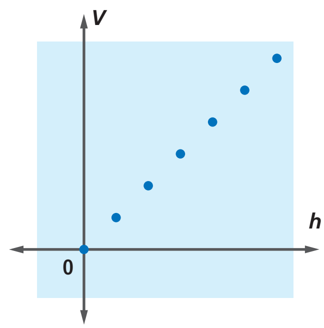
d. In you own words, explain what the model tells you about the relationship between fuel economy and vehicle weight.

16. Suppose that variables V, h, and g are related as illustrated in graphs I and II below. The points on graph I lie on a parabola. The points on graph II lie on a line through the origin. Write a single equation that represents the relationship among V, h, and g.

I



II



17. For a cylinder of fixed volume, the height is inversely proportional to the square of the radius.

a. If a cylinder has height 10 cm and radius 5 cm, what is the height of another cylinder of equal volume and with a 10 cm radius?

b. Generalize Part a. If two cylinders have the same volume, and the radius of one is double the radius of the other, what is the relationship between their heights?

18. Paula thinks the mass of a planet varies directly with the cube of its diameter.

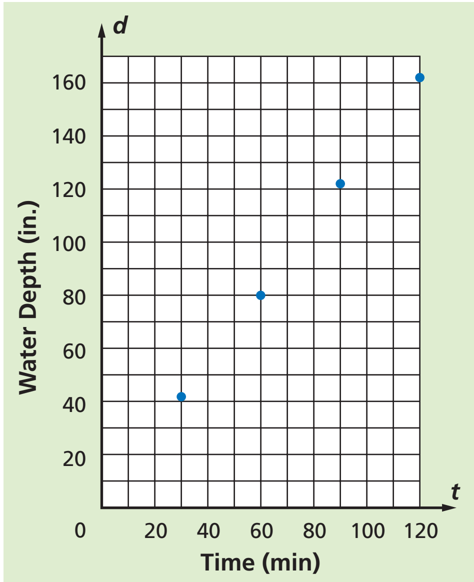
a. Assume Paula is correct. The diameter of Earth is approximately 12,700 km, and its mass is approximately 6.0×10^{24} kg. Compute the constant of variation, and write an equation to model the situation.

b. The diameter of Jupiter is approximately 11 times that of Earth. What does the model from Part a predict Jupiter's mass to be?

c. The mass of Jupiter is actually about 1.9×10^{27} kg. Why do you think this differs from the answer to Part b?

19. A cylindrical tank is being filled with water and the water depth is measured at 30-minute intervals. Below are a table and a graph showing the time passed t and the depth d of the water in the tank.

t (min)	30	60	90	120
d (in.)	41	80	122	162



a. Which variation equation models this data better, $d = kt$ or $d = kt^2$? Justify your answer.

b. Find the constant of variation and write the variation equation.

c. Based on your model, what is the depth of the water after 3 hours?

d. If the tank is 50 feet tall, how long can the water run before the tank is full?

Chapter 2 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Translate variation language into formulas and formulas into variation language. (Lessons 2-1, 2-2, 2-9)

In 1–3, translate the sentence into a variation equation.

1. y varies directly as x.
2. d varies inversely as the square of t.
3. z varies jointly as x and t.
4. **Fill in the Blanks** In the equation $w = kyz$, where k is a constant, __?__ varies __?__ as __?__.
5. **Fill in the Blanks** If $y = \frac{kx}{v^2}$, where k is a constant, then y varies __?__ as __?__, and varies __?__ as __?__.

In 6–8, write each variation equation in words.

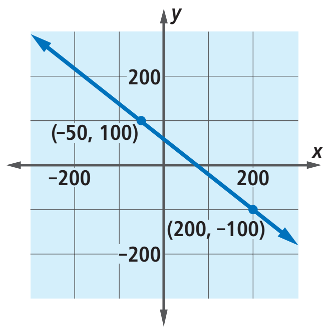
6. $y = kw^3$
7. $y = \frac{k}{w^3}$
8. $y = \frac{kz}{w^3}$

OBJECTIVE B Solve variation problems. (Lessons 2-1, 2-2, 2-9)

9. Suppose y varies directly as x. If $x = 3$, then $y = -21$. Find y when $x = -7$.
10. If y varies directly as the cube of x, and $y = \frac{3}{2}$ when $x = 3$, find y when $x = -6$.
11. Suppose y varies inversely as the square of x. When $x = 4$, $y = \frac{3}{4}$. Find y when $x = -6$.
12. z varies directly as x and inversely as the cube of y. When $x = 12$ and $y = 2$, $z = 39$. Find z when $x = 7$ and $y = 3$.

OBJECTIVE C Find slopes and rates of change. (Lessons 2-4, 2-5)

13. Find the slope of the line passing through (3, -12) and (10, 20).
14. What is the slope of the line graphed below?



In 15 and 16, let $y = 2x^3$.

15. Find the average rate of change from $x = -2$ to $x = 0$.
16. Find the average rate of change from $x = 2$ to $x = 3$.

In 17 and 18, find the average rate of change from $x = 10$ to $x = 20$.

17. $y = \frac{5}{x}$
18. $y = \frac{5}{x^2}$

PROPERTIES Principles behind the mathematics

OBJECTIVE D Use the Fundamental Theorem of Variation. (Lessons 2-3, 2-8)

In 19–21, suppose x is tripled. Tell how the value of y changes under the given condition.

19. y varies directly as x.

20. y varies inversely as x^2.

21. y varies directly as x^n.

22. Suppose y varies inversely as x^2. How does y change if x is divided by 10?

23. **Fill in the Blank** If $y = \frac{k}{x^n}$, and x is multiplied by any nonzero constant c, then y is __?__.

24. **Fill in the Blank** If $y = \frac{k}{x^n}$, and x is divided by any nonzero constant c, then y is __?__.

25. Suppose $y = \frac{kx^n}{z^n}$, x is multiplied by nonzero constant a, and z is multiplied by a nonzero constant b. What is the effect on y?

26. Suppose that when x is divided by 3, y is divided by 27. How is x related to y? Express this relationship in words and in an equation.

OBJECTIVE E Identify the properties of variation functions. (Lessons 2-4, 2-5, 2-6)

27. **Fill in the Blank** The graph of the equation $y = kx^2$ is a __?__.

28. **Fill in the Blanks** Graphs of all direct variation functions have the point (__?__, __?__) in common.

In 29–31, refer to the four equations below.

A $y = kx$ B $y = kx^2$

C $y = \frac{k}{x}$ D $y = \frac{k}{x^2}$

29. Which equations have graphs that are symmetric to the y-axis?

30. The graph of which equation is a hyperbola?

31. **True or False** When $k > 0$, all of the equations have points in Quadrant I.

32. **a.** Identify the domain and range of $f(x) = \frac{k}{x^2}$ when $k < 0$.

b. What are the asymptotes of f if $f(x) = \frac{k}{x}$?

33. For what domain values are the inverse and inverse-square variation functions undefined?

34. Write a short paragraph explaining how the functions $y = -2x^2$ and $y = -\frac{2}{x^2}$ are alike and how they are different.

USES Applications of mathematics in real-world situations

OBJECTIVE F Recognize variation situations. (Lessons 2-1, 2-2)

In 35–37, translate the sentence into a variation equation.

35. The number n of hamsters that can safely live in a square hamster cage is proportional to the square of the length ℓ of a side of the cage.

36. The length of time t required for an automobile to travel a given distance is inversely proportional to the velocity v at which it travels.

37. The electromagnetic force F between two bodies with a given charge is inversely proportional to the square of the distance d between them.

38. Translate Einstein's famous equation for the relationship between energy and mass, $E = mc^2$, into variation language. (E = energy, m = mass, c = the speed of light, which is a constant.)

Fill in the Blank In 39–42, complete each sentence with *directly*, *inversely*, or *neither directly nor inversely*.

39. The number of people that can sit comfortably around a circular table varies ___?___ with the radius of the table.

40. The volume of a circular cylinder of a given height varies ___?___ as the square of the radius of its base.

41. The wall area that can be painted with a gallon of paint varies ___?___ as the thickness of the applied paint.

42. Your height above the ground while on a Ferris wheel varies ___?___ as the number of minutes you have been riding it.

OBJECTIVE G **Solve real-world variation problems. (Lessons 2-1, 2-2, 2-9)**

43. One of Murphy's Laws says that the time t spent debating a budget item is inversely proportional to the number d of dollars involved. According to this law, if a committee spends 45 minutes debating a $5,000 item, how much time will they spend debating a $10,000,000 item?

44. Recall that the weight of an object is inversely proportional to the square of its distance from the center of Earth. If you weigh 115 pounds on the surface of Earth, how much would you weigh in space 50,000 miles from Earth's surface? (The radius of Earth is about 4,000 miles.)

45. Computer programmers are often concerned with the efficiency of the algorithms they create. Suppose you are analyzing large data sets and create an algorithm that requires a number of computations varying directly with n^4, where n is the number of data points to be analyzed. You first apply your algorithm to a test data set with 10,000 data points, and it takes the computer 5 seconds to perform the computations. If you then apply the algorithm to your real data set, which has 1,000,000 data points, how long would you expect the computation to take? (Assume the computer takes the same amount of time to do each computation.)

46. A credit card promotion offers a discount on all purchases made with the card. The discount is directly proportional to the size of the purchase. If you save $1.35 on a $20 purchase, how much will you save on a $173 purchase?

47. The force required to prevent a car from skidding on a flat curve varies directly as the weight of the car and the square of its speed, and inversely as the radius of the curve. It requires 290 lb of force to prevent a 2,200 lb car traveling at 35 mph from skidding on a curve of radius 520 ft. How much force is required to keep a 2,800 lb car traveling at 50 mph from skidding on a curve of radius 415 ft?

48. An object is tied to a string and then twirled in a circular motion. The tension in the string varies directly as the square of the speed of the object and inversely as the length of the string. When the length of the string is 2 ft and the speed is 3 ft/sec, the tension on the string is 130 lb. If the string is shortened to 1.5 ft and the speed is increased to 3.4 ft/sec, find the tension on the string.

OBJECTIVE H Fit an appropriate model to data. (Lessons 2-7, 2-8)

In 49 and 50, a situation and question are given.

a. Graph the given data.

b. Find a general variation equation to represent the situation.

c. Find the value of the constant of variation and use it to rewrite the variation equation.

d. Use your variation equation from Part c to answer the question.

49. Officer Friendly measured the length L of car skid marks when the brakes were applied at different starting speeds S. He obtained the following data.

S (kph)	40	60	80	100	120
L (m)	9	20	36	56	81

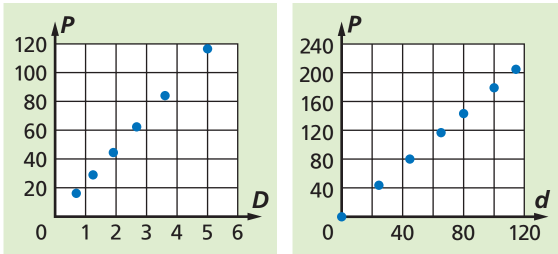
How far would a car skid if the brakes were applied at 150 kph?

50. Two protons will repel each other with a force F that depends on the distance d between them. Some values of F are given in the following table. Note that d is measured in 10^{-10} meters, and F is measured in 10^{-10} newtons.

d	1.5	3	4.5	6	7.5
F	103	26	11	6	4

With what force will two protons repel each other if they are 1×10^{-10}m apart?

51. Jade performed an experiment to determine how the pressure P of a liquid on an object is related to the depth d of the object and the density D of the liquid. She obtained the graph on the left by keeping the depth constant and measuring the pressure on an object in solutions with different densities. She obtained the graph on the right by keeping the density constant and measuring the pressure on an object in solutions at various depths.



Write a general equation relating P, d, and D. Do not find the constant of variation. Explain your reasoning.

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE I Graph variation equations. (Lessons 2-4, 2-5, 2-6)

In 52 and 53, an equation is given.

a. Make a table of values.

b. Graph the equation for values of the independent variable from –3 to 3.

52. $r = -\frac{1}{3}x^2$

53. $p = \frac{2}{q^2}$

In 54 and 55, use graphing technology to graph the equation.

54. $y = \frac{216}{x^3}$

55. $z = -\frac{4.85}{m}$

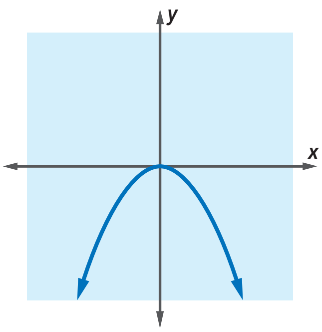
56. Describe how the graph of $y = kx^2$ changes as k decreases from 1 to –1.

57. Describe how the graph of $y = -\frac{k}{x^2}$ changes as k increases from 1 to 100.

OBJECTIVE J Identify variation equations from graphs. (Lessons 2-4, 2-5, 2-6)

Multiple Choice In 58 and 59, select the equation whose graph is most like the one shown. Assume the scales on the axes are the same.

58.



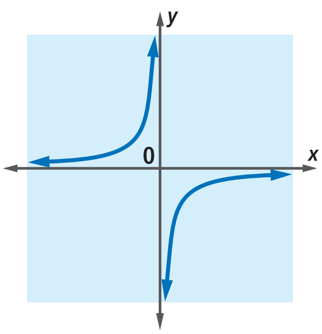
A $y = -x$

B $y = -x^2$

C $y = \frac{1}{x^2}$

D $y = x^2$

59.



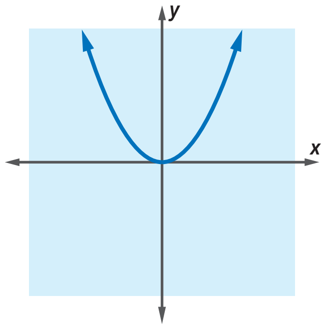
A $y = \frac{x^2}{8}$

B $y = \frac{8}{x}$

C $y = -\frac{8}{x}$

D $y = -\frac{8}{x^2}$

60. In the graph of $y = kx^2$, what type of number must k be?



61. In the graph of $y = \frac{k}{x}$, what type of number must k be?