Lesson

2-2 Inverse Variation

Vocabulary

inverse-variation function

varies inversely as

inversely proportional to

BIG IDEA When two variables x and y satisfy the equation $y = \frac{k}{x^n}$ for some constant value of k, we say that y varies inversely as x^n.

The Condo Care Company has been hired to paint the hallways in a condominium community. A few years ago, it took 8 workers 6 hours (that is, 48 worker-hours) to do this job. If w equals the number of workers and t equals the time (in hours) that each worker paints, then the product wt is the total number of hours worked. Since it takes 48 worker-hours to finish the job,

$$wt = 48, \text{ or } t = \frac{48}{w}.$$

Certain combinations of w and t that could finish the job are given below.

Number of Workers w	1	3	5	6	8	12	15
Time t (hr)	48	16	9.6	8	6	4	3.2

STOP QY1

Mental Math

Let $g(x) = 2x^2$. Find:

a. $g(2)$

b. $g(0.4)$

c. $g(3n)$

d. $g(3n) - g(2) + g(1)$

QY1

If 20 workers were to divide the painting job equally, how many hours would each one have to paint?

Inverse-Variation Functions

The formula $t = \frac{48}{w}$, which determines the values in the table above, has the form $y = \frac{k}{x^n}$ where $k = 48$ and $n = 1$. This is an example of an *inverse-variation function*.

Definition of Inverse-Variation Function

An **inverse-variation function** is a function that can be described by a formula of the form $y = \frac{k}{x^n}$, with $k \neq 0$ and $n > 0$.

For the inverse-variation function with equation $y = \frac{k}{x^n}$, we say y **varies inversely as** x^n, or y is **inversely proportional to** x^n. In an inverse variation, as either quantity increases, the other decreases. In the painting example, as the number of workers increases, the number of hours each must work decreases.

As with direct variation, inverse variation occurs in many kinds of situations.

Example 1

The speed S of a bike varies inversely with the number B of back-gear teeth on the rear wheel. Write an equation that expresses this relationship.

Solution Use the definition of an inverse-variation function. In this case, $n = 1$. So,

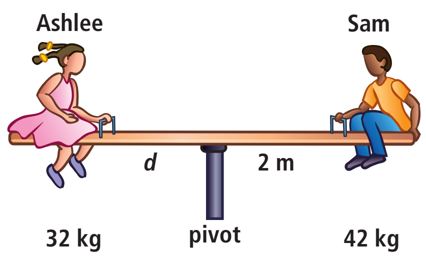
$$S = \frac{k}{B}.$$

Solving Inverse-Variation Problems

Many scientific principles involve inverse-variation functions. For example, imagine that a person is sitting on one end of a seesaw. According to the *Law of the Lever*, in order to balance the seesaw another person must sit a certain distance d from the pivot (or fulcrum) of the seesaw, and that distance is inversely proportional to his or her weight w. Algebraically, $d = \frac{k}{w}$. Since d is inversely proportional to w, as d increases, w will decrease. This means a lighter person can balance the seesaw by sitting farther from the pivot, or a heavy person can balance the seesaw by sitting closer to the pivot.

Example 2

Ashlee and Sam are trying to balance on a seesaw. Suppose Sam, who weighs 42 kilograms, is sitting 2 meters from the pivot. Ashlee weighs 32 kilograms. How far away from the pivot must she sit to balance Sam?



Solution Let d = a person's distance (in meters) from the pivot. Let w = that person's weight (in kilograms).

First write a variation equation relating d and w. From the *Law of the Lever*,

$$d = \frac{k}{w}.$$

To find k, substitute Sam's weight and distance from the pivot into $d = \frac{k}{w}$ and solve for k.

$$2\text{m} = \frac{k}{42\text{ kg}}$$
$$k = 2\text{ m} \cdot 42\text{ kg}$$
$$k = 84\text{ meter-kilograms}$$

Substitute the value found for k into the formula.

$$d = \frac{84}{w}$$

Substitute Ashlee's weight into the formula above to find the distance she must sit from the pivot.

$$d = \frac{84}{32} = 2.625\text{ m}$$

Ashlee must sit about 2.6 meters away from the pivot to balance Sam.

Check Since $d = \frac{k}{w}$, $k = dw$. So the product of Ashlee's distance from the pivot and her weight should equal the constant of variation. Does 2.625 meters • 32 kilograms = 84 meter-kilograms? Yes, the numbers and the units agree.

STOP **QY2**

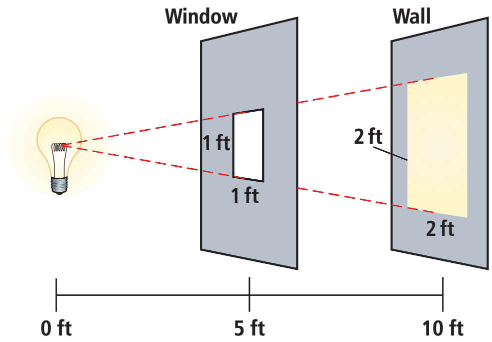
An Inverse-Square Situation

Just as one variable can vary directly as the square of another, one variable can also vary inversely as the square of another. For example, in the figure on the next page, a spotlight shines onto a wall through a square window that measures 1 foot on each side. Suppose the window is 5 feet from the light and the wall is 10 feet from the light. The light that comes through the window will illuminate a square on the wall that is 2 feet on a side. The same light that comes through the 1-square foot window now covers 4 square feet.

QY2

If Saul takes Sam's place on the seesaw and Saul weighs 55 kg, what is the new constant k of variation?

Since the same amount of light illuminates four times the area, the intensity of light on the wall is $\frac{1}{4}$ of its intensity at the window. As distance from the light source increases, the area the light illuminates increases, and the intensity of the light decreases. This is an example of inverse variation: the intensity I of light is inversely proportional to the square of the distance d from the light source.



$$I = \frac{k}{d^2}$$

GUIDED

Example 3

Suppose the intensity of the light 4 meters from a light source is 40 lumens. (A lumen is the amount of light that falls on a 1-square foot area that is 1 foot from a candle.) Find the constant of variation and determine the intensity of the same light 6 meters from its source.

Solution Write an equation relating d and I, where d = the distance from the light source in meters and I = the light's intensity in lumens.

$$I = \frac{k}{?}$$

To find k, substitute d = __?__ and I = __?__ into your equation and solve for k.

$$\underline{\ ?\ } = \frac{k}{?}$$

$$\underline{\ ?\ } \cdot \underline{\ ?\ } = k$$

$$\underline{\ ?\ } = k$$

Substitute k back into the equation to find the inverse-variation formula for this situation.

$$I = \frac{?}{d^2}$$

Evaluate this formula when d = 6 meters.

$$I = \frac{?}{?}$$

$$I = \underline{\ ?\ } \text{ lumens}$$

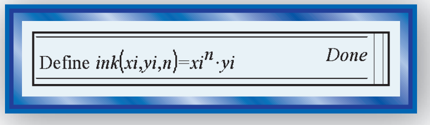
As you did in Lesson 2-1 for direct-variation problems, you can define functions on your CAS to help solve inverse-variation problems.

Activity

MATERIALS CAS

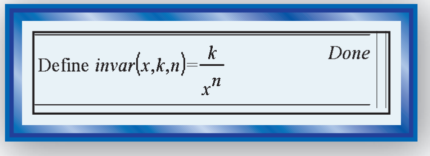
Step 1 Clear all variable values on your CAS.

Define the function $ink(xi, yi, n) = xi^n \cdot yi$.



This function calculates the constant of variation k from three inputs: an *initial* independent variable value xi, an *initial* dependent variable value yi, and the exponent n.

Step 2 Define the function $invar(x, k, n) = \frac{k}{x^n}$.



This function calculates an inverse-variation value from three inputs: *any* independent variable value x, the constant of variation k calculated by *ink*, and the exponent n.

Step 3 Check your solution to Example 3 by using *ink* to find k for $xi = 4$, $yi = 40$ and $n = 2$. Use *invar* with the appropriate inputs to verify the rest of your solution.

Questions

COVERING THE IDEAS

1. **Fill in the Blank** In the Condo Care Company problem at the beginning of this lesson, the time to finish the job varies inversely as the __?__.

2. **Fill in the Blank** The equation $s = \frac{k}{r^2}$ means s varies inversely as __?__.

3. **Multiple Choice** Assume k is a nonzero constant. Which equation does *not* represent an inverse variation?

   A $y = kx$ B $xy = k$ C $y = \frac{k}{x}$ D $y = \frac{k}{x^2}$

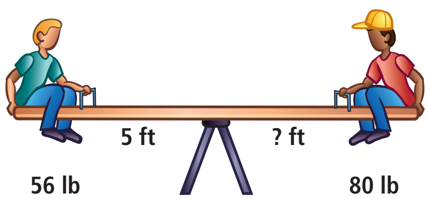
4. Refer to Example 1. Find the constant of variation if you are pedaling 21 mph and have 11 teeth on the back gear.

5. Refer to Example 2. If Sam sits 2.5 meters from the pivot, how far away from the pivot must Ashlee sit to balance him?

6. Suppose the seesaw at the right is balanced.

   a. Find the missing distance.

   b. If the 80 lb person sits farther from the pivot, which side of the seesaw will go up?



7. Refer to Example 3. Find the intensity of the light 9 meters from the source. How does this compare to the intensity of the light 6 meters from the source?

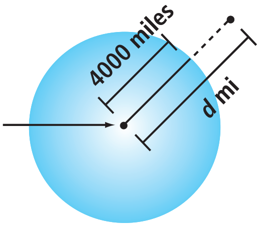
APPLYING THE MATHEMATICS

8. Translate this statement into a variation equation.

   The time t an appliance can run on 1 kilowatt-hour of electricity is inversely proportional to the wattage rating w of the appliance.

9. If y varies inversely as x^3, and $y = 12$ when $x = 5$, find the value of y when $x = 2$.

10. The weight W of a body above the surface of Earth varies inversely as the square of its distance d from the center of Earth. Use 4,000 miles for the radius of Earth.



   a. Write an inverse-variation function to model this situation.

   b. When an astronaut is 300 miles above Earth, what is the value of d?

   c. Suppose an astronaut weighs 170 pounds on the surface of Earth. What will the astronaut weigh in orbit 300 miles above Earth?

   d. What will be the astronaut's weight 2,000 miles above the surface of Earth?

11. Consider again the Condo Care Company situation at the beginning of the lesson.

   a. Complete the table below by filling in the missing values.

w	2	3	4	5	6	7	8	9	10	11	12	15	20
t	?	16	?	9.6	8	?	6	?	4.8	?	?	?	?

   b. **Fill in the Blank** Compare the values of t when $w = 2$ and $w = 4$. Also compare the values of t when $w = 4$ and $w = 8$, and again when $w = 6$ and $w = 12$. Make a conjecture. When the number of people working doubles, the mean time each person needs to work __?__.

   c. **Fill in the Blank** Follow a similar procedure to complete the following conjecture. When the number of people working triples, the mean time __?__.

   d. Prove your conjecture from Part b or Part c.

Fill in the Blank In 12–15, complete the sentence with the word *directly* or *inversely*.

12. The surface area of a sphere varies __?__ as the square of its radius.
13. The number of hours required to drive a certain distance varies __?__ as the speed of the car.
14. My hunger roughly varies __?__ as the time since I last ate.
15. My hunger roughly varies __?__ as the amount of food I have eaten.

REVIEW

16. At Percy's Priceless Pizza, the price of a pepperoni pizza is proportional to the square of its diameter. If Percy charges \$11.95 for a 10-inch diameter pizza, how much does Percy charge for a 14-inch pizza? **(Lesson 2-1)**
17. At 7'7" and 303 pounds, Gheorghe Muresan was one of the tallest people ever to play professional basketball. For people with similar body shapes, weight varies directly with the cube of height. How much would you expect someone with Gheorghe's body shape to weigh if that person were 5'10"? **(Lesson 2-1)**
18. If $f(d) = 3d^3$ for all d, find $f(2x)$. **(Lesson 1-3)**

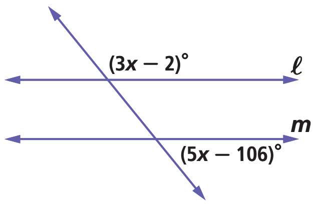
In 19 and 20, simplify and indicate the general property. (Previous Course)

19. $\frac{x^{11}}{x^4}$

20. $(2x)^4$

21. At a certain time of day, a 13' tree casts a shadow 7' long.
    a. Draw a picture of this situation and mark a right triangle in your picture.
    b. A nearby tree is 18' tall. How long would its shadow be at the same time of day? **(Previous Course)**
22. In the figure at the right, line ℓ is parallel to line m. Find x. **(Previous Course)**

Gheorghe Muresan on defense against John Shasky



EXPLORATION

23. The *inverse-square law* in physics governs the way various things happen as distance varies, such as how the light intensity decreases as the distance from the source increases, as discussed in Example 3. Research the inverse-square law, and find three other situations where it applies.

QY ANSWERS

1. 2.4 hours

2. 110 meter-kilograms