Chapter 13 Summary and Vocabulary

- Some sums and products are denoted by special symbols.

  For instance, the sum $x_1 + x_2 + ... + x_n$ is represented by $\sum_{i=1}^{n} x_i$.

  The product $n(n-1)(n-2) \cdot ... \cdot 2 \cdot 1$ is represented by **$n!$** (**n factorial**).

- A **series** is an indicated sum of terms of a sequence. Values of finite arithmetic or geometric series may be calculated from the following formulas:

  For an **arithmetic sequence** $a_1, a_2, ..., a_n$ with common difference d:

  $$\sum_{i=1}^{n} a_i = \frac{1}{2}n(a_1 + a_n) = \frac{n}{2}(2a_1 + (n-1)d).$$

  For a finite **geometric sequence** $g_1, g_2, ..., g_n$ with common ratio r:

  $$\sum_{i=1}^{n} g_i = g_1\frac{(1-r^n)}{1-r} = g_1\frac{(r^n-1)}{r-1}.$$

- The **mean absolute deviation** and the **standard deviation** of a data set are measures of the spread, or dispersion, of the data in the set. For the data set $\{x_1, ..., x_n\}$, the mean μ is $\frac{1}{n}\sum_{i=1}^{n} x_i$, the mean absolute deviation is $\frac{1}{n}\sum_{i=1}^{n} |x_i - \mu|$, and the standard deviation is $\sqrt{\frac{1}{n}\sum_{i=1}^{n} (x_i - \mu)^2}$.

- **Pascal's Triangle** is a 2-dimensional sequence. The $(r+1)$st element in row n of Pascal's Triangle is denoted by $\binom{n}{r} = \frac{n!}{r!(n-r)!}$. The expression $\binom{n}{r}$, also denoted ${}_nC_r$, appears in several other important applications. It is the coefficient of $a^{n-r}b^r$ in the binomial expansion of $(a+b)^n$. It is the number of subsets, or **combinations**, with r elements taken from a set with n elements. If a situation consists of n trials with two outcomes, and the probability of one of these outcomes is p, then the probability of that outcome occurring exactly r times is $\binom{n}{r}p^r(1-p)^{n-r}$. This is a **binomial probability.**

Vocabulary

Lesson 13-1
*series
*arithmetic series
∑, sigma
∑-notation, sigma notation, summation notation
index variable, index

Lesson 13-2
*geometric series

Lesson 13-3
*mean
measure of center, measure of central tendency
absolute deviation
*mean absolute deviation, *m.a.d.*
*standard deviation, *s.d.*

Lesson 13-4
permutation
!, factorial symbol
n!
*combination

Lesson 13-5
*Pascal's Triangle

Lesson 13-6
*binomial coefficients

Lesson 13-7
trial
binomial experiment

Lesson 13-8
lottery
unit fraction

Lesson 13-9
probability function, probability distribution
binomial probability distribution, binomial distribution
normal distribution
normal curve
standard normal curve
standardized scores

- The number of **permutations** of n objects is $n!$. By using permutations and combinations, the probabilities of winning many games of pure chance, such as **lotteries**, can be calculated.

- Distributions of binomial probabilities are related to Pascal's Triangle. As the number of the row of Pascal's Triangle increases, the graph of the distribution takes on a shape more and more like a **normal curve**. Some tests are standardized so that graphs of their scores fit that shape. In a **normal distribution**, about 68% of the data are within 1 standard deviation of the mean, and about 95% are within 2 standard deviations. The equation $y = \frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ for a normal distribution with mean μ and standard deviation σ combines many of the ideas of this book in one place.

Theorems

Arithmetic Series Formula (p. 871)
Finite Geometric Series Formula (p. 878)
Number of Permutations Theorem (p. 890)
Factorial Product Theorem (p. 890)
Combination Counting Formula (p. 892)
Pascal's Triangle Explicit Formula (p. 899)
Binomial Theorem (p. 905)
Binomial Probability Theorem (p. 913)

Chapter 13 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

1. Write using summation notation: $1^4 + 2^4 + 3^4 + \ldots + 17^4$.

2. Evaluate $\sum_{j=0}^{100} (5j + 1)$.

In 3 and 4, seven members of the Stern Rowing Team weigh in before a race. In kilograms, their weights are 68, 69, 70, 71, 71, 74, 74.

3. Find the mean absolute deviation and the standard deviation of the Stern Team's weights.

4. Suppose the Jolly Rowing Team has seven members with a mean team weight identical to the Stern Team's. However, Jolly's standard deviation is 0. What are the seven weights of Jolly's team members?

5. Expand $(x^2 + 2)^5$ using the Binomial Theorem.

6. Find the coefficient of x^3 in the expansion of $(x + 3)^4$.

7. To celebrate the end of the school year, you buy a bowl with three scoops of ice cream from a shop that sells 17 different flavors. How many combinations of three different flavors are possible?

8. Tyler buys 9 different textbooks his freshman year of college. In how many ways can they be arranged on his bookshelf?

9. Evaluate.

a. $\binom{7}{4}$ **b.** ${}_{164}C_4$

10. Calculate ${}_{37}C_{36}$ and explain why your answer makes sense.

11. Rewrite $117 \cdot 116 \cdot 115 \cdot 114 \cdot 113$ as a quotient of factorials.

12. a. Calculate $\binom{6}{3}$ and describe its position in Pascal's Triangle.

b. Calculate the coefficient of p^5q^2 in the expansion of $(p + q)^7$ and describe its position in Pascal's Triangle.

13. Francisco gets a summer job on a trial basis. The first day he is paid \$30, the second day he is paid \$32, and he continues to get a \$2 raise each day. How much will Francisco be paid for 30 days of work?

14. a. Write the arithmetic series $3 + 7 + 11 + \ldots + 87$ using Σ-notation.

b. Calculate the sum in Part a.

15. Find the sum of the integer powers of 5 from 5^0 to 5^{19}.

16. Find the sum of the first 20 terms of the sequence $\begin{cases} a_1 = 50 \\ a_n = \frac{4}{5}a_{n-1}, \text{ for integers } n \geq 2 \end{cases}$ to the nearest hundredth.

17. A fair coin is flipped five times. Find the probability of each outcome: 0 heads, 1 head, 2 heads, and so on to 5 heads.

18. In Lottery A, you need to match six numbers picked at random from the integers 1 to 50. In Lottery B, you need to match five numbers picked at random from the integers 1 to 70. Order does not matter in either lottery. In which lottery do you have a higher probability of winning? Explain your reasoning.

19. On a recent administration of the SAT test, the mean mathematics score was 515, and the standard deviation was 114.

    a. If test scores are normally distributed, about what percent of scores are within 1 standard deviation of the mean?

    b. To what range of scores on this test does your answer in Part a correspond?

    c. About what percent of scores are at or above 743?

20. Suppose a coin is biased so that there is a 65% chance that the coin will show tails when tossed. Find the probability that there will be 2 heads in 3 tosses.

21. Let $P(n) = \frac{1}{2^6}\binom{6}{n}$.

    a. Make a table of values for this function.

    b. Graph this function.

    c. Describe in words what $P(n)$ represents in the context of a coin toss.

22. Find a value of x such that $\binom{n}{2} = \binom{n}{n-x}$ for all integers $n \geq 2$.

Chapter 13 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Calculate values of arithmetic series. (Lesson 13-1)

In 1–4, evaluate the arithmetic series.

1. $1 + 2 + 3 + \ldots + 123$
2. $2 + 8 + 14 + \ldots + 92$
3. the sum of the smallest 49 positive integers that are divisible by 7
4. the sum of the first 9 terms of the sequence $\begin{cases} a_1 = 120 \\ a_n = a_{n-1} - 6 \text{ for integers } n \geq 2 \end{cases}$
5. If $1 + 2 + 3 + \ldots + k = 1653$, what is the value of k?

OBJECTIVE B Calculate values of finite geometric series. (Lesson 13-2)

In 6–9, evaluate the geometric series.

6. $7 + 2.1 + 0.63 + \ldots + 7(0.3)^7$
7. $3 - 12 + 48 - 192 + \ldots + 196{,}608$
8. the sum of integer powers of 4 from 4^0 to 4^{17}
9. the sum of the first 11 terms of the sequence $\begin{cases} g_1 = 21 \\ g_n = \frac{3}{7}g_{n-1}, \text{ for integers } n \geq 2 \end{cases}$
10. A geometric series has 18 terms. The constant ratio is 1.037, and the first term is 1313. Estimate the value of the series to the nearest integer.

OBJECTIVE C Use summation ($\sum$) and factorial (!) notation. (Lesson 13-1, 13-2, 13-3, 13-4)

In 11 and 12, write the terms of the series, and then evaluate the series.

11. $\sum_{n=1}^{5} (3n - 6)$
12. $\sum_{i=-3}^{2} 5 \cdot 7^i$
13. **Multiple Choice** Which equals the sum $1 + 8 + 27 + \ldots + 1{,}000{,}000$?

    A $\sum_{n=1}^{10} n^6$
    B $\sum_{n=1}^{100} 3^n$
    C $\sum_{n=1}^{100} n^3$
    D $\sum_{n=1}^{1000} n^2$

14. Suppose $a_1 = 23$, $a_2 = 24$, $a_3 = 25$, $a_4 = 26$, $a_5 = 27$. Evaluate $\frac{1}{5}\sum_{i=1}^{5} a_i$.

In 15 and 16, rewrite using $\sum$-notation.

15. $3 + 6 + 9 + \ldots + 123$
16. $\mu = \frac{y_1 + y_2 + y_3 + \ldots + y_n}{n}$
17. If $g(n) = n! - n$, calculate $g(3) - g(8)$.
18. Rewrite $41 \cdot 42 \cdot 43 \cdot 44$ as a quotient of factorials.
19. **Multiple Choice** $\frac{(n-1)!}{n!} =$

    A -1
    B n
    C $n - 1$
    D $\frac{1}{n}$

OBJECTIVE D **Calculate permutations and combinations.** (Lessons 13-4, 13-5)

20. Interpret the symbol $\binom{n}{r}$ in terms of Pascal's Triangle.

21. **Multiple Choice** Which of the following is equal to $\frac{13!}{10! \cdot 3!}$?

A $_{10}C_3$ B $\binom{13}{3}$

C $\binom{10}{3}$ D $13 \cdot 12 \cdot 11$

In 22 and 23, consider the set {V, E, R, T, I, C, A, L}.

22. How many permutations of the letters in VERTICAL are possible?

23. a. How many subsets have 3 elements?

b. What is the total number of subsets that can be formed?

In 24–27, evaluate.

24. $\binom{13}{6}$

25. $\binom{432}{432}$

26. $_7C_5$

27. $_{532}C_{531}$

OBJECTIVE E **Use the Binomial Theorem to expand binomials.** (Lesson 13-6)

In 28–31, expand using the Binomial Theorem.

28. $(x + y)^4$

29. $(t - 3)^5$

30. $(2a^2 - 3)^3$

31. $\left(\frac{p}{2} + 2q\right)^4$

True or False In 32 and 33, if the statement is false, change the statement to make it true.

32. One term of the binomial expansion of $(17x + z)^8$ is $17x^8$.

33. One term of the binomial expansion of $(43a - b)^{15}$ is $\binom{15}{2}(43a)^{13}(-b)^2$.

34. **Multiple Choice** Which equals $\sum_{r=0}^{n} \binom{n}{r} x^{n-r} 7^r$?

A $(x + n)^7$ B $(x + r)^n$

C $(x + 7)^r$ D $(x + 7)^n$

PROPERTIES Principles behind the mathematics

OBJECTIVE F **Recognize properties of Pascal's Triangle.** (Lesson 13-5, 13-6)

In 35 and 36, consider the top row in Pascal's Triangle to be row 0.

	row
1	0
1 1	1
1 2 1	2
	.
	.
	.

35. Write row 7 of Pascal's Triangle.

36. What is the sum of the numbers in row n?

37. Find a solution to $\binom{10}{6} + \binom{10}{7} = \binom{x}{y}$

38. **True or False** For all positive integers n, $\binom{n}{1} = \binom{n}{n-1}$. Justify your answer.

39. Describe the coefficient of $r^2 s^{13}$ in the expansion of $(r + s)^{15}$ in terms of Pascal's Triangle.

USES Applications of mathematics in real-world situations

OBJECTIVE G **Solve real-world problems using arithmetic or geometric series.** (Lessons 13-1, 13-2)

40. A bank stacks rolls of quarters in the following fashion: one roll on top, two rolls in the next layer, three rolls in the next layer, and so on.

a. If there are 10 layers of quarters, how many rolls are in the stack?

b. If you want to stack 120 rolls as described above, how many rolls will you need to put on the bottom layer?

41. In a non-leap year, Kevin saved \$1 on January 1, \$2 on January 2, and \$3 on January 3. Each day Kevin saved one dollar more than the previous day.
 a. How much did Kevin save on February 15?
 b. How much did he save in total by February 15?
 c. How many days will it take Kevin to save a total of \$10,000?

42. A ball is dropped from a height of 2 meters and bounces to 90% of its previous height on each bounce. When it hits the ground the eighth time, how far has it traveled?

43. A concert hall has 30 rows. The first row has 12 seats. Each row thereafter has 2 more seats than the preceding row. How many seats are in the concert hall?

OBJECTIVE H **Solve real-world counting problems involving permutations or combinations.** (Lesson 13-4)

44. The visible spectrum is associated with the acronym ROY G BIV (red, orange, yellow, green, blue, indigo, violet). How many ways can these letters be rearranged (ignoring spaces)?

45. A used car dealer has 10 cars that he can line up next to the street. How many different ways can he arrange his cars?

46. There are 25 students in a class. How many handshakes will take place if every student shakes hands with everyone else exactly once?

In 47 and 48, use the fact that the Senate of the 110th Congress had 49 Republicans, 49 Democrats, and 2 Independents.

47. How many choices were there for forming a 5-member committee of Senators?

48. How many 7-member committees could be formed with Independents and Democrats?

OBJECTIVE I **Use measures of central tendency or dispersion to describe data or distributions.** (Lessons 13-3, 13-9)

In 49 and 50, consider these 2007 profits of the ten largest companies in the United States.

Company	Profit (millions of \$)
Wal-Mart Stores	12,731
Exxon Mobil	40,610
Chevron	18,688
General Motors	–38,732
ConocoPhillips	11,891
General Electric	22,208
Ford Motor	–2,723
Citigroup	3,617
Bank of America Corp.	14,982
AT&T	11,951

49. Find the mean absolute deviation for this data set.

50. Find the standard deviation.

In 51–53, use this information: Johns Hopkins University compared the SAT math scores for the incoming 1989 freshman class to the scores for the incoming 2006 freshman class. Some of the data are presented below.

Year	Number of Students in Class	Mean	Standard Deviation
1989	831	662.6	68.2
2006	1211	664.9	62.5

51. Which class shows a greater spread of scores?

52. Assume that the scores for the 2006 class are normally distributed. Within what interval would you expect the middle 68% of the class scores to occur?

53. When the scores for both classes are pooled into one data set, the mean SAT math score is 664.0, which is not the average of the means of the two classes when considered separately. Explain why.

54. Consider the test scores {93, 71, 78, 83, 93, 72, 99, 85}. Give the range of possible values for the mean if a ninth score ranging from 50 to 100 is added to the data set.

OBJECTIVE J **Solve problems using combinations and probability.** (Lessons 13-7, 13-8)

In 55 and 56, suppose that a fair coin is tossed 6 times. Calculate the probability of each event.

55. getting exactly 1 head

56. getting exactly 3 heads

In 57–59, suppose a coin is biased so there is a 70% chance that the coin shows tails when tossed. Find the probability of each event to the nearest thousandth.

57. There are 3 heads in 3 tosses.

58. When tossed twice, the coin shows heads the first time and tails the second time.

59. When tossed twice, the coin shows heads once and tails once.

60. If 7 out of 10 is a passing score on a true-or-false quiz with 10 questions and a student guessed at every answer, what is the probability that he passed the test?

61. If 70 out of 100 is a passing score on a true-or-false test and a student guesses at every answer, will he have the same probability of passing the test as the quiz in Question 60? Explain your reasoning.

62. In Illinois' Lotto game, a participant chooses six numbers from 1 to 52. To win the jackpot, the participant must match all six winning numbers. (Order does not matter.) What is the probability of this occurring?

63. The Virginia "Pick 3" lottery requires that a participant choose three numbers, each a digit from 0 to 9. To win the grand prize, the participant must match all three numbers in the order drawn. What is the probability of winning this lottery?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE K **Graph and analyze binomial and normal distributions.** (Lesson 13-9)

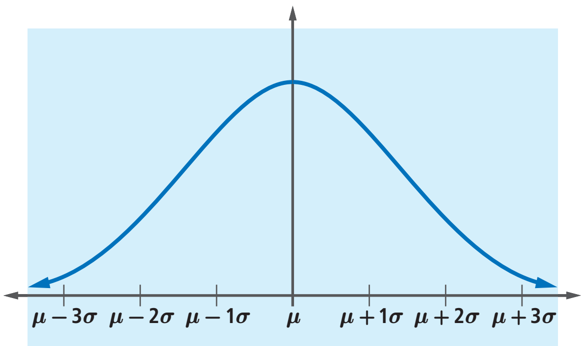
64. Consider the function $P(n) = \frac{1}{2^8}\binom{8}{n}$.

a. Evaluate $P(n)$ for integers 0, 1, …, 8.

b. Graph this function.

c. What name is given to this function?

65. Below is pictured a normal distribution with mean μ and standard deviation σ.



a. What percent of the data are greater than or equal to μ?

b. About what percent of the data are between $\mu - 1\sigma$ and $\mu + 1\sigma$?

c. About what percent of the data are more than 2 standard deviations away from μ?