Lesson

13-8 Lotteries

Vocabulary

lottery

unit fraction

BIG IDEA The probability of winning a lottery prize can be calculated using permutations and combinations.

A **lottery** is a game or procedure in which prizes are distributed by pure chance. The simplest lotteries are raffles in which you buy tickets that are put in a bin, and winning tickets are picked from that bin. In recent years, however, more complicated lotteries have been designed. These lotteries pay out large amounts of money to a few individuals in order to attract participants. Today, several countries run lotteries themselves or allow private lotteries. In the United States, about 94% of people live in states that run lotteries, and participants must be 18 years of age or older.

Lotteries are designed to make money, so they always take in more than they pay out. This means that many more people will lose money in a lottery than will win. Still, the possibility, however remote, of winning a huge amount of money is very attractive.

Consider a typical lottery. To participate in the Mega Millions game, which is played across several states, a participant pays \$1 and picks five numbers out of the set of consecutive integers {1, 2, 3, …, 56} and one number (called the "Mega Ball") from the set of consecutive integers {1, 2, 3, …, 46}. Twice a week, five balls are picked at random from balls numbered 1 through 56, and one Mega Ball is randomly picked from a different set of 46 balls. These balls show the winning numbers. For instance, the winning numbers in one Mega Millions game were 1, 5, 13, 18, 33, and 30. The people who pick all six winning numbers split the grand prize, which is always at least \$12 million.

Mental Math

Given that sin 52° ≈ 0.788, find

a. sin(–52°)

b. sin 232°

c. sin 128°

d. cos 38°

GUIDED

Example 1

What is the probability of picking the six winning numbers in the Mega Millions lottery?

Solution The number of possible combinations for the first five numbers that could be chosen is $\binom{56}{?} = \frac{56!}{? \cdot ?} = \underline{\quad ? \quad}$.

(continued on next page)

There are 46 choices for the Mega Ball number. So there are a total of $\_?\_ \cdot 46 = \_?\_$ possible combinations of numbers to choose. Because each combination has the same chance of being drawn, the probability that a particular combination will appear is $\frac{1}{?}$. So, the probability of picking the six winning numbers is $\frac{1}{?}$ or about $\_?\_$.

The answer to Example 1 explains why there is often no winner in a lottery. The chance of choosing all the winning numbers is quite small.

Because the probability of winning the grand prize is so low, most lotteries give smaller prizes to participants who pick almost all of the numbers. For instance, in Mega Millions, there is a *much* smaller prize for picking the five regular winning numbers but not the Mega Ball number. There is an even smaller prize given for picking four of the five regular winning numbers.

Example 2

What is the probability of picking exactly four of the five regular winning numbers in Mega Millions?

Solution Of the five regular numbers, four must be picked from the five winning regular numbers, one must be picked from the 51 incorrect regular numbers that remain, and the Mega Ball number must be chosen incorrectly.

There are $\binom{5}{4}$ different sets of 4 numbers from the 5 winning regular numbers.

There are 51 ways to choose one incorrect number from the remaining regular numbers.

There are 45 non-winning Mega Ball numbers.

Winners are chosen from the 175,711,536 possible combinations found in Example 1. So, the probability of picking four of the five regular winning numbers is

$$\frac{\binom{5}{4} \cdot 51 \cdot 45}{175{,}711{,}536} = \frac{11{,}475}{175{,}711{,}536} \approx 0.000065 \approx \frac{1}{15{,}385}.$$

A **unit fraction** is a simple fraction with 1 in its numerator. In Example 2, the probability 0.000065 is approximated by the unit fraction $\frac{1}{15{,}385}$ to make it easier to interpret the answer as "about 1 chance in 15,385" of picking four winning balls. This fraction was found by calculating the reciprocal of 0.000065. That reciprocal is about 15,385. The reciprocal of the reciprocal equals the original number.

STOP QY

Because probabilities of winning are so low in many lotteries, people can get discouraged from entering repeatedly. Consequently, states also have lotteries with fewer numbers to match in which more people win, but the payouts are much lower. Some of these lotteries require that three or four digits (from 0 to 9) be matched exactly, in order. Because the order matters, the probability of winning cannot be calculated using combinations.

▶ QY

A probability of 0.0028 means "about 1 in ___?___."

Example 3

To play Florida's basic "Play 4" lottery, a participant picks four digits, each from 0 to 9, and must match a 4-digit number. What is the probability of matching the four winning digits?

Solution 1 There is a probability of $\frac{1}{10}$ that each digit will be matched. Because these events are independent, the probability of matching all four numbers is $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$, or $\frac{1}{10,000}$.

Solution 2 Think of the four digits as forming one number. There are ten thousand numbers from 0000 to 9999. The probability of matching one of these is $\frac{1}{10,000}$.

Some people have computers pick lottery numbers for them. Others study past winning numbers to look for patterns. Still others use their "lucky" numbers or their birthdays. None of these strategies changes the probability of winning. The probability is always tiny, and there is no systematic way to win lotteries like these.

Questions

COVERING THE IDEAS

1. What is a lottery?

In 2 and 3, consider the Mega Millions game.

2. What must a participant do in order to win the grand prize in this lottery?
3. What is the probability of picking only the five regular winning numbers in this lottery?
4. New York's "Take 5" lottery gives participants a chance to win a jackpot by matching five numbers (in any order) from a set of 39. What is the probability of winning this lottery?

5. Refer to Example 3. If Florida's "Play 4" lottery had an option where a participant must match the first three of four numbers (each of which can be any digit from 0 through 9) in exact order, what would be the probability of winning?

APPLYING THE MATHEMATICS

6. Another multi-state lottery, Powerball, is played by picking five numbers from balls numbered 1 to 55 and one additional Powerball number from balls numbered 1 to 42. A smaller prize is awarded to participants who match four of the regular winning numbers and the Powerball number.
   a. In how many ways can a participant win the smaller prize?
   b. What is the probability of winning the smaller prize?
7. The European lottery "Euro Millions" is played by choosing five numbers from a set of 50 and two "star" numbers from a set of nine (the digits 1 to 9). What is the probability of picking all seven winning numbers?
8. Consider New York's "Take 5" lottery in Question 4.
   a. How many different tickets are possible for this lottery? Remember that order does not matter, so all tickets listing the numbers 12, 32, 11, 19, and 6, for instance, are considered the same.
   b. How many tickets from your answer to Part a are losers?
   c. A special promotion advertises that a participant with a losing ticket can enter for a second chance prize. What is the probability of being eligible for this drawing?
9. To play the "Cash 5" lottery in Connecticut, a participant pays \$1 and picks five numbers from a set of 35. For 50¢ more, the participant can add a "kicker" number picked from the remaining 30 numbers not already picked. One way to win a kicker prize is to match the kicker number and four out of five of the regular numbers. What is the probability of winning a kicker prize in this way?
10. In the Quinto lottery formerly played in Washington state, a participant picked five cards from a standard 52-card deck. The lottery paid \$1000 for a ticket matching four of the five cards, and \$20 for a ticket matching three of the five. Does this mean that participants were $\frac{1000}{20} = 50$ times as likely to match three of five winning cards as to match four of five winning cards? Explain your answer.

REVIEW

11. A golden retriever has a litter of 8 puppies. If males and females are equally likely to be in the litter, what is the probability that 4 of the puppies are male and 4 are female? **(Lesson 13-7)**

12. Sam Dunk makes 75% of the free throws he attempts. What is the probability that he will make at least 7 out of 10 free throws? **(Lesson 13-7)**

13. You pick four numbers out of a set of 30. If four numbers are selected at random from this set, what is the probability that the four numbers you picked are selected? **(Lesson 13-7)**

14. Solve ${}_nC_3 = 84$. **(Lessons 13-4, 11-6)**

In 15 and 16, do not solve the equation.

a. State the number of roots each equation has.

b. State the number of positive roots the equation has. **(Lesson 11-6)**

15. $x^5 + 8x^3 + x = 0$

16. $13t^2 + 6t^7 + it^3 = 14$

17. **Multiple Choice** Choose the equation for an exponential decay function and explain why it is that kind of function. **(Lesson 9-2)**

A $f(x) = \frac{1^x}{5}$ B $f(x) = 5^{-x}$ C $f(x) = 5^x$ D $f(x) = \left(\frac{1}{5}\right)^{-x}$

18. Write the reciprocal of $2 + i$ in $a + bi$ form. **(Lesson 6-9)**

19. Assume that the cost of a spherical ball bearing varies directly as the cube of its diameter. What is the ratio of the cost of a ball bearing 6 mm in diameter to the cost of a ball bearing 3 mm in diameter? **(Lesson 2-3)**

EXPLORATION

20. The term "odds" is often used interchangeably (and often incorrectly) with the term "probability" when discussing probability. Look up information about odds and find out how odds are related to, but are not the same thing as, probability.

QY ANSWER

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