Chapter 9 Summary and Vocabulary

- A function with an equation of the form $y = ab^x$, where $b > 0$ and $b \neq 1$, is an **exponential function**. All geometric sequences are also exponential functions. In the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, when P, r, and n are given, A is an exponential function of t.

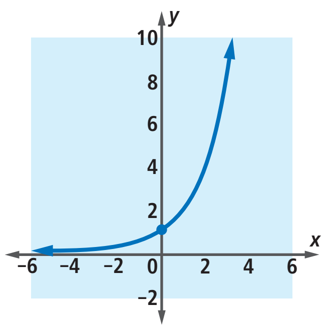
- The exponential function with base b has an equation of the form $f(x) = ab^x$. Some exponential functions represent **exponential growth** or **decay** situations. In an exponential growth situation, the growth factor b is greater than one. In an exponential decay situation, b is between 0 and 1. Over short periods of time, many populations grow exponentially. The value of many items depreciates exponentially. Quantities that grow or decay exponentially have a constant doubling time or **half-life**, respectively. Real data from these and other contexts can be modeled using exponential functions.

- When an initial amount of \$1.00 is continuously compounded at 100% interest, the value of the investment after one year is $e \approx 2.71828$. Like π, the number e is an irrational number. In general, the formula $A = Pe^{rt}$ can be used to calculate the value A of an investment of P dollars at r% interest **compounded continuously** for t years.

- The inverse of the exponential function $f: x \to b^x$ is $f^{-1}: b^x \to x$, the **logarithm function with base *b***. Thus, $b^x = a$ if and only if $x = \log_b a$. Because exponential and logarithm functions are inverses, their graphs are reflection images of each other over the line $y = x$. Properties of logarithm functions can be derived from the corresponding properties of exponential functions.

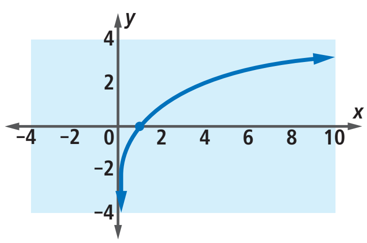
Exponential Growth Function

$y = b^x, b > 1$



Logarithmic Function

$y = \log_b x, b > 1$



Vocabulary

Lesson 9-1
*exponential function
exponential curve
growth factor

Lesson 9-2
exponential decay
depreciation
half-life

Lesson 9-3
e
compounded continuously

Lesson 9-5
logarithm of *x* to the base 10, log of *x* to the base 10, log base 10 of *x*
common logarithm, common log
logarithmic function to the base 10, common logarithm function

Lesson 9-6
logarithmic scale
decibel, dB

Lesson 9-7
*logarithm of *a* to the base *b*
logarithm function with base *b*

Lesson 9-8
*natural logarithm of *m*, ln *m*

Exponential Growth Function		Logarithmic Function
all real numbers	**Domain**	all positive reals
all positive reals	**Range**	all real numbers
y-intercept is 1, no x-intercept	**Intercepts**	x-intercept is 1, no y-intercept
the x-axis ($y = 0$)	**Asymptotes**	the y-axis ($x = 0$)

- Logarithmic functions are used to scale data having a wide range (for example, **decibel** levels or earthquake magnitudes) and to solve equations of the form $b^x = a$, where b and a are positive and $b \neq 1$. One way to solve equations of this type is to take the logarithm of both sides; the solution is $x = \frac{\log a}{\log b}$.
- The base of a logarithmic function can be any positive real number not equal to 1, but the most commonly used bases are 10 and e. When the base is 10, the values of the log function are called **common logarithms**. When the base is e, the values of the log function are called **natural logarithms**.
- The basic properties of logarithms correspond to properties of powers. Let $x = b^m$ and $y = b^n$, and take the logarithms of both sides of each power property. The result is a logarithm property.

Power Property	**Logarithm Property**
$b^0 = 1$	$\log_b 1 = 0$
$b^m \cdot b^n = b^{m+n}$	$\log_b(xy) = \log_b x + \log_b y$
$\frac{b^m}{b^n} = b^{m-n}$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
$(b^m)^a = b^{am}$	$\log_b(x^a) = a \log_b x$
$b^x = a$	$\log_b a = \frac{\log_t a}{\log_t b}$

Theorems

Continuously Compounded Interest Formula (p. 597)
Log$_b$ of b^n Theorem (p. 636)
Logarithm of 1 Theorem (p. 636)
Logarithm of a Product Theorem (p. 637)
Logarithm of a Quotient Theorem (p. 638)
Logarithm of a Power Theorem (p. 639)
Change of Base Theorem (p. 646)

Chapter 9 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

1. What are the domain and range of the function f defined by $f(x) = \log_{13} x$?

2. The half-life of uranium-237 is about 7 days. What percent of the original amount of uranium will remain in an artifact after three weeks?

3. Let $f(x) = 8^x$.

   a. Graph f on the interval $-2 \le x \le 2$.

   b. Approximate $f(\pi)$ to the nearest tenth.

   c. Does the graph have any asymptotes? If it does, state the equations for all asymptotes. If it does not, explain why not.

4. a. Give a value of b for which the equation $y = ab^x$ models exponential decay.

   b. What are the domain and range of the function in Part a for your value of b?

In 5–7, explain how you would evaluate each expression exactly without using a calculator.

5. $\log 100{,}000{,}000$

6. $\log_2\left(\frac{1}{16}\right)$

7. $\ln e^{-4}$

8. Rewrite $\log a + 2 \log t - \log s$ as a single logarithm.

9. Without natural predators, the number of a certain species of bird will grow each year by 12%. A colony of 50 birds is started in a predator-free area. What is the expected number of birds of this species after 3 years?

In 10–12, solve. If necessary, round solutions to the nearest hundredth.

10. $\log_x 27 = \frac{3}{4}$

11. $5^y = 40$

12. $\ln(7z) = \ln 3 + \ln 21$

13. **True or False** $\log_5 a + \log_3 b = \log_{15}(ab)$. Justify your answer.

14. Write an equation for the inverse of the function with equation $y = \log_4 x$.

15. The Henderson-Hasselbalch formula $pH = 6.1 + \log\left(\frac{B}{C}\right)$ can be used to find the pH of a patient's blood as a function of the bicarbonate concentration B and the carbonic-acid concentration C. A patient's blood has a bicarbonate concentration of 23 and a pH reading of 7.3. Find the concentration of carbonic acid.

16. To the nearest thousandth, what is $\log_{14} 24.72$?

17. State the general property used in simplifying the expression $\log_{19} 19^{23}$.

18. Consider the function defined by $y = \log_5 x$.

    a. State the coordinates of three points on the graph.

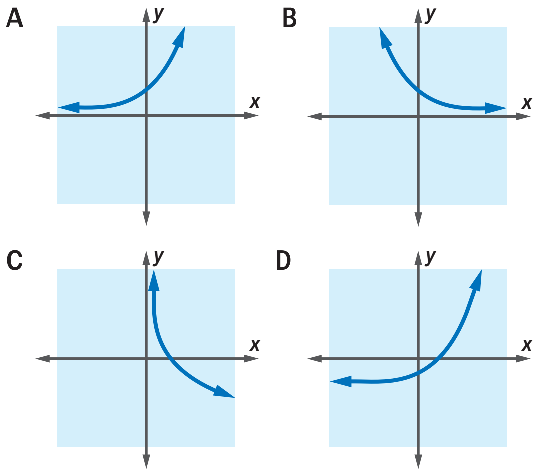
    b. State the domain and range of the function.

    c. Graph the function.

    d. State an equation for its inverse.

    e. Graph the inverse on the same axes you used in Part c.

19. Multiple Choice Assume that the value of an investment grows according to the model $y = I(1.075)^x$, where I is the original investment and y is the amount present after x years. Which graph below could represent this situation?



20. The Population Reference Bureau reports the following data about the population P of the United States.

Population mid-2007	302,200,000
Births per 1000 population	14
Deaths per 1000 population	8
Rate of natural increase/year	$0.6\% = 0.006$
Projected population, mid-2025	349,400,000

Assume the continuous growth model $P = 302{,}200{,}000e^{rt}$, where r is the annual rate of increase and t is the number of years after 2007.

a. Use the model to show that the reported rate of natural increase of 0.6% is *not* the rate leading to the projected population in 2025.

b. Calculate the growth rate to the nearest 0.1% that will give the reported projected population for 2025 of 349,400,000. Show how you arrived at your answer.

21. For the equation $27^x = 14$,

a. find the exact solution.

b. find a decimal solution rounded to the nearest thousandth.

22. The table below shows the total number of German-language articles in *Wikipedia* each month from March 2007 to February 2008.

Month	Articles (thousands)
March 2007	570
April 2007	584
May 2007	599
June 2007	612
July 2007	626
August 2007	640
September 2007	655
October 2007	668
November 2007	681
December 2007	695
January 2008	711
February 2008	726

a. Fit an exponential model to these data.

b. The number of English articles in *Wikipedia* surpassed 1 million in March, 2006. Use your model from Part a to predict when the number of German articles will reach this benchmark.

Chapter 9 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Determine values of logarithms. (Lessons 9-5, 9-7, 9-8, 9-10)

In 1–8, find the exact value of each logarithm without using a calculator.

1. $\log 10{,}000$
2. $\log 0.00001$
3. $\ln e^8$
4. $\log_4 1024$
5. $\log_{13}(13^{15})$
6. $\ln 1$
7. $\log_{\frac{1}{3}} 27$
8. $\log_9 \sqrt[3]{9}$

In 9–14, approximate each logarithm to the nearest hundredth.

9. $\log 98{,}765$
10. $\ln 10.95$
11. $\ln(-3.7)$
12. $\log 0.003$
13. $\log_7 25$
14. $\log_3 12.3$

OBJECTIVE B Use logarithms to solve exponential equations. (Lesson 9-10)

In 15–22, solve. If necessary, round to the nearest hundredth.

15. $\log_6 5 = t$
16. $\log_{12} 9900 = s$
17. $2000(1.06)^n = 6000$
18. $13 \cdot 2^x = 1$
19. $e^z = 44$
20. $(0.8)^w = e$
21. $11^{a+1} = 1011$
22. $3^{-2a} = 53$

OBJECTIVE C Solve logarithmic equations. (Lessons 9-5, 9-6, 9-7, 9-9)

In 23–30, solve. If necessary, round to the nearest hundredth.

23. $\log_x 33 = \log_{11} 33$
24. $\ln(4y) = \ln 9 + \ln 12$
25. $\log z = 18$
26. $\log x = 3.71$
27. $3 \ln 5 = \ln x$
28. $\log_8 x = \frac{3}{7}$
29. $\log_x 347 = 3$
30. $\log_x 5 = 10$

PROPERTIES Principles behind the mathematics

OBJECTIVE D Recognize properties of exponential functions. (Lessons 9-1, 9-2, 9-3)

31. What are the domain and range of the function f defined by $f(x) = e^x$?

32. What are the domain and range of the function g defined by $g(x) = 2^x$?

33. When does the function $f: x \rightarrow a^x$ describe exponential growth?

34. What must be true about the value of b in the equation $y = ab^x$, if the equation models exponential decay?

35. Write the equation(s) of the asymptote(s) to the graph of $y = 27(1.017)^x$.

36. **Multiple Choice** Which situation does the equation $y = e^{-x}$ describe?

   A constant increase
   B constant decrease
   C exponential growth
   D exponential decay

OBJECTIVE E Recognize properties of logarithmic functions. (Lessons 9-5, 9-7, 9-8)

37. What is the inverse of f, when $f(x) = e^{-x}$?

38. Give an equation of the form $y =$ __?__ for the inverse of the function with equation $y = \log_3 x$.

39. **True or False** The domain of the log function with base 12 is the range of the exponential function with base 12.

40. **True or False** Negative numbers are not included in the domain of f when $f(x) = \log_b x$.

OBJECTIVE F **Apply properties of logarithms. (Lessons 9-9, 9-10)**

In 41–44, write in exponential form.

41. $\log_3\left(\frac{1}{243}\right) = -5$ **42.** $\ln 23.14 \approx \pi$

43. $\log m = n$ **44.** $\log_b p = q$

In 45–48, write in logarithmic form.

45. $10^{-1.8} \approx 0.01585$ **46.** $e^5 \approx 148.413$

47. $x^y = z, x > 0, x \neq 1$ **48.** $4^a = 18$

In 49–56, rewrite the expression as a whole number or a single logarithm and state the theorem or theorems you used.

49. $\ln 17 + \ln 12$ **50.** $\log 50 - \log 5$

51. $-2 \log_{12} 11$ **52.** $\ln e$

53. $\log_{107} 107^{79}$ **54.** $\log_{6.3} 1$

55. $\log a - 3 \log b$

56. $\log u + \log v + 0.7 \log w$

USES Applications of mathematics in real-world situations

OBJECTIVE G **Create and apply exponential growth and decay models. (Lessons 9-1, 9-2, 9-3, 9-10)**

57. In 2005 the population of the Tokyo-Yokohama region in Japan was about 35.327 million, the largest metropolitan area in the world. The average annual growth rate was 0.43%. Assuming this growth rate continues, find the population of the Tokyo-Yokohama area in 2020.

58. In 2005 the sixth largest metropolitan area in the world was that of Mumbai, India, with 18.202 million people. Mumbai was growing at an average rate of 1.96% annually. Suppose this rate continues indefinitely.

a. Find the population of this area in 2020.

b. In what year will the population of Mumbai reach 30 million?

59. Refer to Questions 57 and 58. Estimate the year in which Mumbai's population will first exceed Tokyo-Yokohama's population.

60. The population of a certain strain of bacteria grows according to $N = C \cdot 3^{0.593t}$, where t is the time in hours. How long will it take for 30 bacteria to increase to 500 bacteria?

61. The amount A of radioactivity from a nuclear explosion is given by $A = Ce^{-0.2t}$, where t is measured in days after the explosion. What percent of the original radioactivity is present 9 days after the explosion?

62. Strontium-90 (^{90}Sr) has a half-life of 29 years. If there was originally 25 grams of ^{90}Sr,

a. how much strontium will be left after 87 years?

b. how much strontium will be left after t years?

63. A new car costing $28,000 is predicted to depreciate at a rate of 14% per year. About how much will the car be worth in six years?

OBJECTIVE H **Fit an exponential model to data. (Lesson 9-4)**

64. Find an equation for the exponential function $f: x \rightarrow ab^x$ passing through (0, 1.2) and (3, 25).

65. A bacteria population was counted every hour for a day with the following results.

Hour h	1	2	3	4	5	6	7
Population p (hundreds)	5	13	25	49	103	211	423

a. Construct a scatterplot of these data.

b. Fit an exponential model to these data.

c. Use your model to estimate the population at the 11th hour.

66. A hypothetical new substance, mathium, was manufactured and experiments showed that it decayed at the following rate.

Days	Amount Present (g)
1	1156
2	907
3	715
4	660
5	432
6	340
7	273
8	210
9	168
10	129

a. Construct a scatterplot of these data.

b. From the data in the scatterplot, what is the approximate half-life of this new substance? Explain your answer.

c. Fit an exponential model to these data.

d. On the 20th day, how much of the substance will be present?

OBJECTIVE I Apply logarithmic scales, models, and formulas. (Lessons 9-6, 9-8)

In 67–69, use the formula $D = 10 \log\left(\frac{I}{10^{-12}}\right)$ to convert sound intensity I in $\frac{\text{W}}{\text{m}^2}$ into relative intensity D in decibels.

67. Find D when $I = 3.88 \cdot 10^9$.

68. What sound intensity corresponds to a relative intensity of 80 decibels?

69. How many times as intense is a 60 dB sound as a 20 dB sound?

70. Baking soda has a pH value of 8, while pure water has a pH value of 7. How many times as acidic is water than baking soda?

71. The boiling point T of water in degrees Fahrenheit at barometric pressure P in inches Hg (inches of mercury) is given by the model

$$T = 49.161 \cdot \ln P + 44.932.$$

At what temperature does water boil in Colorado if the average barometric pressure is 27 inches Hg?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE J Graph exponential functions. (Lessons 9-1, 9-2)

72. Graph $y = 3^x$ using at least five points.

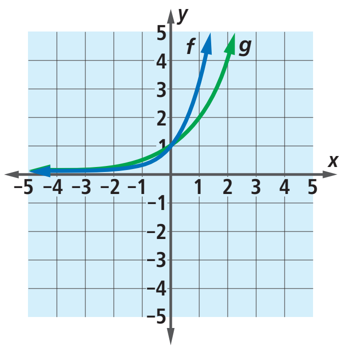
73. Graph $y = \left(\frac{1}{3}\right)^x$ using at least five points.

74. Graph $g(x) = \left(\frac{1}{5}\right)^x$ and $h(x) = \left(\frac{1}{5}\right)^{2x}$ on the same set of axes.

a. Which function has greater values when $x > 0$?

b. Which function has greater values when $x < 0$?

75. Below are the graphs of the equations $y = 2^x$ and $y = 3^x$.



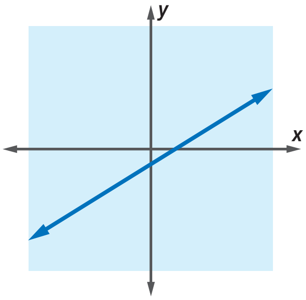
a. Which equation corresponds to the graph of f?

b. Which equation corresponds to the graph of g?

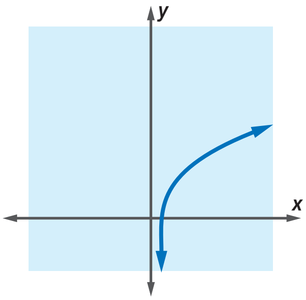
c. Describe how the graph of $y = e^x$ is related to the graphs of f and g.

76. **Multiple Choice** Which graph below represents exponential decay?

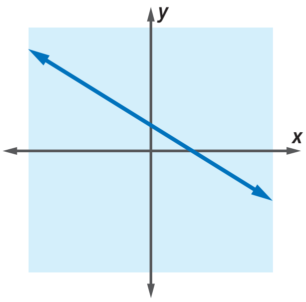
A



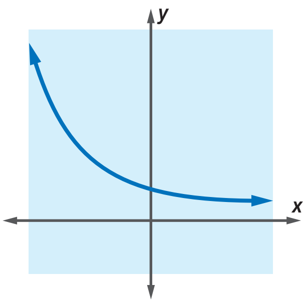
B



C



D



OBJECTIVE K **Graph logarithmic curves.** **(Lessons 9-5, 9-7)**

77. **a.** Graph $y = 4^x$ using at least five points.

b. Use the results of Part a to plot at least five points on the graph of $y = \log_4 x$.

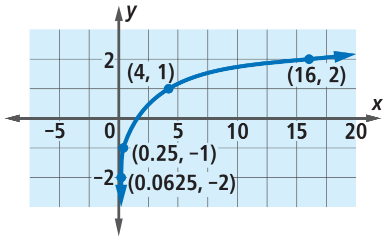
78. **a.** Plot $y = 10^x$ and $y = \log_{10} x$ on the same set of axes.

b. Identify all intercepts of these curves.

79. **a.** Graph $y = \ln x$ using at least five points.

b. Give an equation for the inverse of this function.

80. The graph below has the equation $y = \log_Q x$. Find Q.



81. What is the x-intercept of the graph of $y = \log_b x$, where $b > 1$?