

Lesson

9-4

Fitting Exponential Models to Data

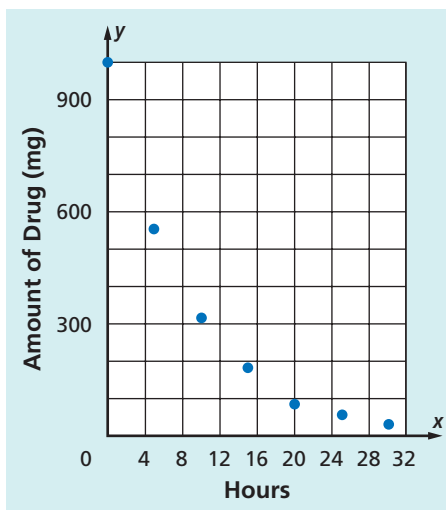
► **BIG IDEA** When a set of data points in a situation seems to be showing exponential growth or decay, exponential regression can fit an exponential function to the data points.

After a person takes medicine, the amount of drug left in the person's body decreases over time. When testing a new drug, a pharmaceutical company develops a mathematical model to quantify this relationship. To find such a model, suppose a dose of 1000 mg of a certain drug is absorbed by a person's bloodstream. Blood samples are taken every five hours, and the amount of drug remaining in the body is calculated.

Possible data from an experiment are shown in the table and scatterplot below. The scatterplot suggests that an exponential model might be appropriate. Exponential models can be fit to data using methods similar to those that you used to find linear and quadratic models in earlier chapters.

Drug Absorption Data

Hours Since Drug was Administered	Amount of Drug in Body (mg)
0	1000
5	550
10	316
15	180
20	85
25	56
30	31



Finding Exponential Models for Data Believed to Be Exponential

As you know, exponential functions have the form $y = ab^x$, where a is the value of y when $x = 0$ and b is the growth factor during each unit period of time.

Mental Math

Alberto is putting his books onto a new bookshelf. Each hardcover book is about 2 inches wide and each paperback book is about 1.5 inches wide. Is one 24-inch shelf enough space for his

- 8 paperback and 4 hardcover mystery novels?
- 3 paperback and 10 hardcover biographies?
- 12 paperback and 2 hardcover science fiction books?



Example 1

Find an exponential model to fit the drug absorption data using the initial condition and one other point in the table.

Solution You need to find a and b in the equation $y = ab^x$. The initial condition occurs when $x = 0$, so $1000 = ab^0$.
 Since $b^0 = 1$, $a = 1000$. So the equation is of the form $y = 1000b^x$.
 Choose another point from the table and substitute. We chose $(20, 85)$.

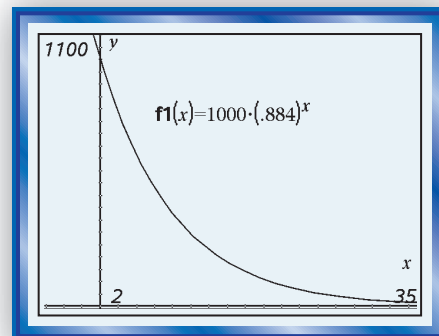
$$85 = 1000b^{20} \quad \text{Substitute.}$$

$$0.085 = b^{20} \quad \text{Divide each side by 1000.}$$

$$0.884 \approx b \quad \text{Take the 20th root of each side (or raise each side to the } \frac{1}{20} \text{ power).}$$

So, one model for the data is $y = 1000 \cdot (0.884)^x$.

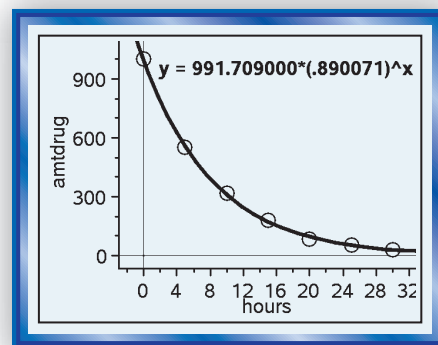
Check Graph this model on a graphing utility. It looks like an exponential decay function with y -intercept 1000, as you would expect from the data.



You can also use your graphing utility to find an exponential regression model.

Activity

- Step 1** Enter the drug absorption data into your calculator. Apply the exponential regression option to find a and b and then write an equation to model these data.
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- Step 2** Your calculator probably gives a and b to many digits. The accuracy of the experimental data suggests that a more sensible model rounds a and b to three digits each. Rewrite your equation after rounding.
-
- Step 3** Use your regression model from Step 2 and the two-point model $y = 1000 \cdot (0.884)^x$ from Example 1 to estimate the amount of drug left in a body after 5 hours. Which model better fits the actual 550 mg value from the experiment?
-
- Step 4** Compare the accuracy of the two models by making a table of values for each equation and comparing these values to the actual data. Which model appears to be more accurate overall?



Recall that, by design, regression is intended to fit a model as close as possible to *all* the data in a set. The Activity shows that a regression model is indeed a better fit to the drug absorption data than a model based on only two points.

Deciding Whether an Exponential Model Is Appropriate

For some data you may not be sure that an exponential model is appropriate. In that case, consider two things. First, look at a scatterplot of your data to see if it has the general shape of an exponential function. This is a quick way to check if the growth factor between various data points is relatively constant. Second, find an exponential model, then look at a table of values or the graph of your model to see how well it fits the data.

Example 2

Below are the U.S. box office gross for the first eleven weekends of the release of the movie *Mission Impossible III* in 2006.

Weekend	1	2	3	4	5	6	7	8	9	10	11
U.S. Box Office Gross (millions of dollars)	47.70	25.00	11.35	7.00	4.68	3.02	1.34	0.72	0.49	0.31	0.20

- Fit an exponential model for box office gross based on the number of weekends the movie has been out and determine if the exponential model is appropriate.
- Use the model to estimate the movie's gross in its 12th weekend.

Solution

- The scatterplot appears to have an exponential shape so an exponential model seems appropriate. Perform exponential regression to obtain

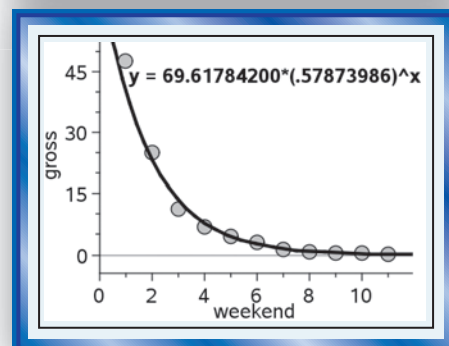
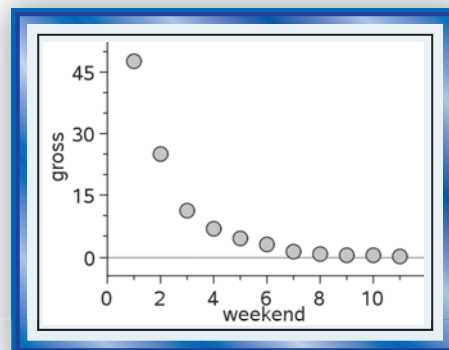
$$y = 69.617842 \cdot (0.57873986)^x.$$

Round a and b based on the actual data's accuracy.

$$y = 69.6 \cdot 0.579^x$$

When it is added to the scatterplot, the graph of the model closely follows the pattern of the data points. It appears to be a good model.

Exponential decay models the data closely, but could a parabola fit the data better?



Graph the data together with the graph of a quadratic regression equation.

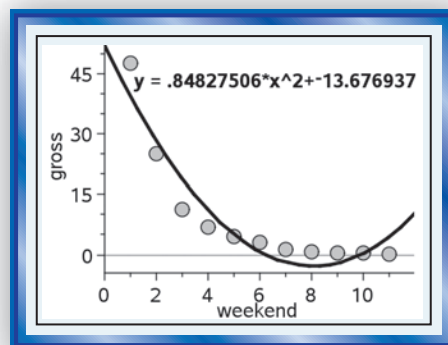
Exponential decay is a better fit. This makes sense because you know that movie gross earnings typically continues to decrease each successive week after release. Gross earnings do not reach a minimum and then continue to climb indefinitely, so a quadratic model is illogical.

- b. Substitute $x = 12$ in for x in the model.

$$y = 69.6 \cdot 0.579^{12}$$

$$y \approx 0.099$$

According to this model, the movie grossed about \$99,000 in its 12th week.



The weekend box office gross earnings of many movies decline according to an exponential pattern. This helps theater managers estimate how long they should continue to show a movie.

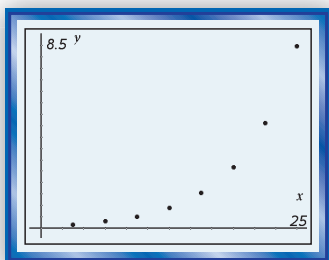
Questions

COVERING THE IDEAS

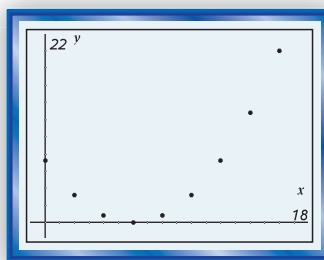
In 1 and 2, refer to the drug absorption data given at the beginning of the lesson.

- Use the point (10, 316) and the method of Example 1 to find an equation to model the data.
 - Use your equation from Part a to predict when there will be less than 10 mg of the drug left in the patient's body.
- Use the model developed in the Activity to predict when the amount of the drug in the patient's body will fall below 10 mg.
- Can you use the method of Example 1 to find an equation to model the data in Example 2? Why or why not?
- Multiple Choice** Which scatterplot(s) below show(s) data that could be reasonably represented by an exponential model?

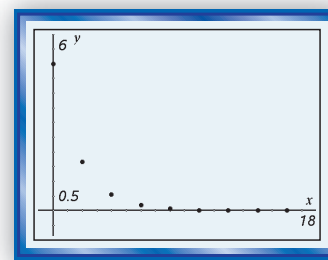
A



B



C



5. Recall the chart of cell phone subscriptions from the opening of this chapter. The table at the right presents the data for seven years.
- Find an exponential model for the growth of cell phone subscriptions.
 - Compare cell phone growth and population growth. The population of the United States was 301.1 million in 2007 and growing exponentially at 0.89% per year. In which year does your model first predict that there will be at least one cell phone subscription per person?

Year	Subscribers (millions)
2001	128.4
2002	140.8
2003	158.7
2004	182.1
2005	207.9
2006	233.0
2007	255.4

APPLYING THE MATHEMATICS

6. The gross earnings for the first three weekends of a popular movie are given in the table at the right.
- Find an exponential model for the decline in weekend gross.
 - Studio executives want to pull the movie from theaters before its weekend gross drops below \$1 million. How many weekends should the studio expect to keep the movie in theaters?
7. Deven dropped 3728 pennies on the floor. He picked up all the coins showing heads and set them aside. Then he counted the tails, put them in a container, mixed them up, and dropped them. He repeated this several times and made a table of the results as shown at the right. Unfortunately, he forgot to record a couple of entries.
- Create an exponential model of the data using the 5 completed row entries.
 - Use the model to estimate in the missing data.
 - Deven's brother Tyler said the model looked like a half-life model for radioactive elements. Explain whether Tyler is correct.
8. The table at the right contains the cumulative monthly English-language article total for the website *Wikipedia*. The timetable ranges from March, 2005 to February, 2006.
- Develop an *exponential* model for the number y of English-language articles in *Wikipedia* in month x . Does this model seem reasonable based on the graph?
 - Develop a *quadratic* model for the number y of English-language articles in *Wikipedia* in month x . Does this model seem reasonable based on the graph?
 - Develop a *linear* model for the number y of English-language articles in *Wikipedia* in month x . Does this model seem reasonable based on the graph?
 - Use each model to predict how many articles will be on *Wikipedia* in the current year. Check your results on the Internet to determine the most accurate model.

Weekend	Gross earnings (millions of dollars)
1	135
2	62
3	35

Drop Number	Number of Tails
1	1891
2	?
3	470
4	229
5	118
6	?
7	27

Month	Articles (thousands)
1	507
2	539
3	573
4	609
5	655
6	703
7	744
8	791
9	836
10	886
11	942
12	992

