# Chapter 12 Summary and Vocabulary

- In this chapter, you studied **quadratic relations** in two variables, their graphs, and their geometric properties. Equations for all quadratic relations in two variables can be written in the standard form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where not all of $A$, $B$, and $C$ are zero. The properties of the various quadratic relations are summarized in the table on the next page

- **Conic sections** appear naturally as orbits of planets and comets, in paths of thrown objects, as energy waves radiating from the epicenter of an earthquake, and in many manufactured objects such as tunnels, windows, and satellite receiver dishes.

- Systems of equations with quadratic sentences can be solved in much the same way as linear systems, that is, by graphing, substitution, using linear combinations, or using a CAS. A system of one linear and one quadratic equation may have 0, 1, or 2 solutions; a system of two quadratic equations may have 0, 1, 2, 3, 4, or infinitely many solutions.

## Theorems and Properties

Focus and Directrix of a Parabola Theorem (p. 801)
Circle Equation Theorem (p. 806)
Interior and Exterior of a Circle Theorem (p. 812)
Equation for an Ellipse Theorem (p. 818)
Length of Axes of an Ellipse Theorem (p. 820)
Circle Scale-Change Theorem (p. 826)
Graph Scale-Change Theorem (p. 827)
Area of an Ellipse Theorem (p. 828)
Equation for a Hyperbola Theorem (p. 832)
Asymptotes of a Hyperbola Theorem (p. 834)
Attributes of $y = \frac{k}{x}$ Theorem (p. 841)

## Vocabulary

**Lesson 12-1**
*parabola
focus, directrix
axis of symmetry
vertex of a parabola
paraboloid

**Lesson 12-2**
*circle, radius, center
*concentric circles
semicircle

**Lesson 12-3**
*interior, exterior of a circle

**Lesson 12-4**
*ellipse
foci, focal constant of an ellipse
standard position for an ellipse
standard form of an equation for an ellipse
*major axis, minor axis
center of an ellipse
semimajor axes, semiminor axes

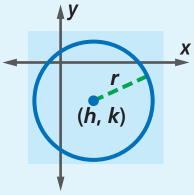
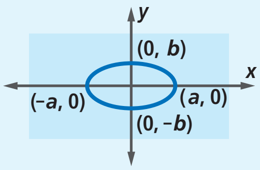
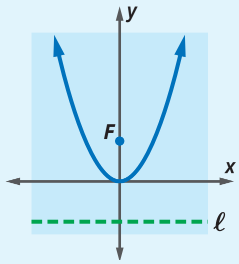
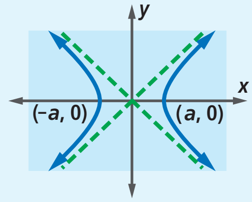
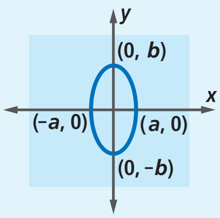
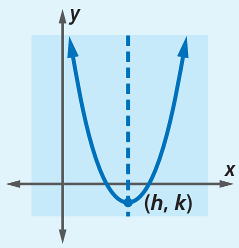
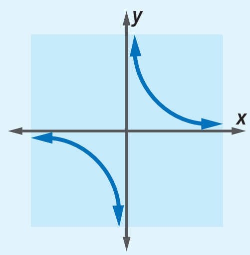
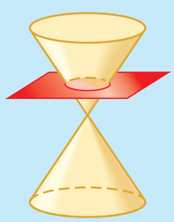
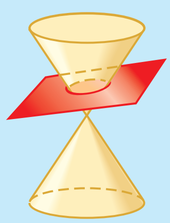
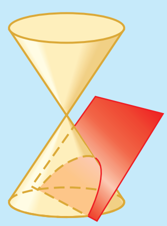
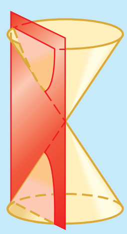
**Lesson 12-5**
eccentricity of an ellipse

**Lesson 12-6**
*hyperbola
foci, focal constant of a hyperbola
vertices of a hyperbola
standard position of a hyperbola
*standard form of an equation for a hyperbola

**Lesson 12-7**
conic section
*standard form of an equation for a quadratic relation

**Lesson 12-8**
quadratic system
quadratic-linear system

**Lesson 12-9**
quadratic-quadratic system

| | Quadratic Relation | | | |
|---|---|---|---|---|
| | **Circle** | **Ellipse** | **Parabola** | **Hyperbola** |
| **Geometric Definition** | given center $C$ and radius $r$, set of points $P$ such that $PC = r$ | given foci $F_1$ and $F_2$ and focal constant $2a$, set of points P such that $PF_1 + PF_2 = 2a$ | given focus $F$ and directrix $\ell$, set of points $P$ equidistant from $F$ and $\ell$ | given foci $F_1$ and $F_2$ and focal constant $2a$, set of points $P$ such that $\lvert PF_1 - PF_2 \rvert = 2a$ |
| **Equation(s) in Standard Form** | $(x-h)^2 +$<br>$(y-k)^2 = r^2$ | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ | $y = ax^2 + bx + c$<br>or<br>$y - k = a(x-h)^2$ | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$<br>or<br>$xy = k$ |
| **Graph** |  <br>center: $(h, k)$<br>radius: $r$ | If $a > b$<br> <br>foci: $(-c, 0)$, $(c, 0)$<br>length of major axis (focal constant): $2a$<br>length of minor axis: $2b$<br>$b^2 = a^2 - c^2$ | $y = ax^2$<br> <br>axis of symmetry: $x = 0$<br>vertex: $(0, 0)$<br>focus: $\left(0, \frac{1}{4a}\right)$<br>directrix: $y = -\frac{1}{4a}$ | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$<br> <br>foci: $(-c, 0)$, $(c, 0)$;<br>$b^2 = c^2 - a^2$<br>asymptotes: $\frac{y}{b} = \pm\frac{x}{a}$ |
| | | If $b > a$<br> <br>foci: $(0, -c)$, $(0, c)$<br>length of major axis (focal constant): $2b$<br>length of minor axis: $2a$<br>$a^2 = b^2 - c^2$ | $y - k = a(x-h)^2$<br> <br>axis of symmetry: $x = h$<br>vertex: $(h, k)$ | $xy = k$<br> <br>foci: $(\sqrt{2k}, \sqrt{2k})$, $(-\sqrt{2k}, -\sqrt{2k})$<br>focal constant: $2\sqrt{2k}$<br>asymptotes: $x = 0, y = 0$ |
| **Conic Section** |  |  |  |  |

# Chapter 12 Self-Test

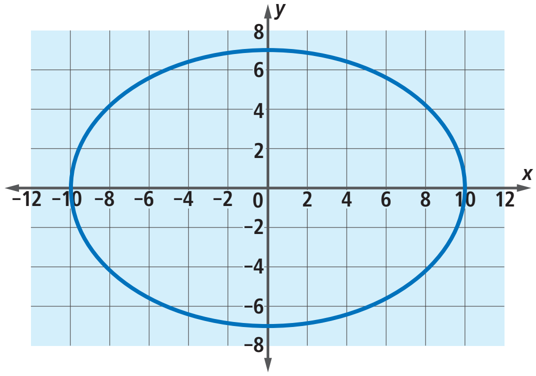
Take this test as you would take a test in class. You will need graph paper and a calculator. Use the Selected Answers section in the back of the book to check your work.

1. a. Rewrite $\frac{x^2}{25} - \frac{y^2}{144} = 1$ in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.
   b. Give the values of $A$, $B$, $C$, $D$, $E$, and $F$.
   c. Identify the conic section represented by the equation in Part a.

In 2 and 3, write an equation or inequality for

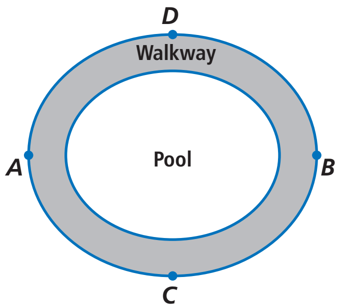
2. the interior of the circle with center (3, -5) and radius 7.
3. the image of the graph of $x^2 + y^2 = 1$ under $T_{-2,5}$.
4. Consider the parabola with equation $y = \frac{1}{11}x^2$.
   a. Give the coordinates of its focus.
   b. Give the coordinates of its vertex.
   c. Write an equation for its directrix.
5. Explain what has to be true about the foci and the major and minor axes of an ellipse for it to be a circle.
6. Ernesto's Eye Extravaganza sold \$5600 worth of sunglasses last year. This year, Ernesto's lowered the prices by two dollars, sold seventy more pairs of sunglasses, and took in \$5880. Assuming sunglasses are all the same price,
   a. how much is he selling his sunglasses for now?
   b. how many pairs did he sell this year?

In 7–9, refer to the ellipse below.



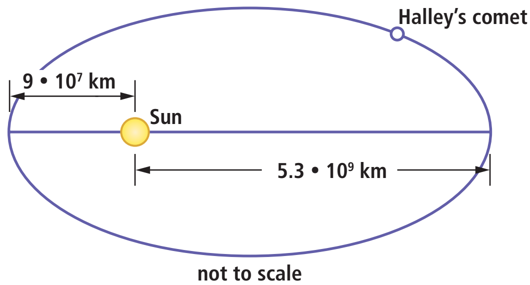

7. Write an equation for the ellipse.
8. Find the area of the ellipse.
9. The ellipse is the image of the circle $x^2 + y^2 = 1$ under what scale change?
10. Graph the conic section with equation $\frac{x^2}{25} - \frac{y^2}{16} = 1$. Identify all its major features.
11. Graph the system $\begin{cases} xy = 2 \\ 2x + 5 = y \end{cases}$ and identify the points of intersection.
12. Solve the system $\begin{cases} y = x^2 - 4x + 3 \\ x^2 + y^2 = 9 \end{cases}$.
13. Consider the equation $xy = -15$.
    a. What type of curve is the graph of this equation?
    b. State the foci and focal constant of the curve.
    c. Does the curve have any lines of symmetry? If so, write an equation for one of them.

14. The elliptically shaped pool shown below is to be surrounded by a tile walkway so that the outer edge of the walkway is also an ellipse. The major axis of the pool has length 15 m, and the minor axis of the pool has length 8 m. The length $AB$ of the major axis of the outer edge of the walkway is 18 m, and the length $CD$ of the minor axis is 11 m. What is the area of the walkway?




15. Colorado has an area of approximately 104,094 square miles and a perimeter of about 1320 miles. Assuming that Colorado is rectangular, find its dimensions.


16. Halley's comet has an elliptical orbit with the Sun at one focus. Its closest distance to the Sun is about $9 \cdot 10^7$ km, while its farthest distance is about $5.3 \cdot 10^9$ km.




Find the length of the major axis of Halley's comet's orbit.

17. Find equations for a quadratic-quadratic system that has exactly three solutions.

18. Graph the set of points that are 8 units from (3, 2.5) and describe the graph with an equation.

19. The entrance to a cave is a semicircular arch that is 14 feet wide at its base. How far from the center of the opening can a 5'8"-tall spelunker stand upright?

20. Graph the set of points that satisfy the inequality $\frac{x^2}{9} + \frac{y^2}{100} \geq 1$.

# Chapter 12 Chapter Review

**S**KILLS
**P**ROPERTIES
**U**SES
**R**EPRESENTATIONS

**SKILLS** Procedures used to get answers

**OBJECTIVE A** Rewrite an equation for a conic section in the standard form of a quadratic equation in two variables. (Lesson 12-7)

In 1–6, rewrite the equation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Give the values of *A*, *B*, *C*, *D*, *E*, and *F*.

1. $(x - 7)^2 + (y + 2)^2 = 81$
2. $y = 6(t - 3)^2 + 4$
3. $\frac{x^2}{36} - \frac{y^2}{16} = 1$
4. $\frac{a^2}{8} + \frac{b^2}{5} = 1$
5. $y = \sqrt{169 - x^2}$
6. $y = \frac{30}{x}$

**OBJECTIVE B** Write equations for quadratic relations and inequalities for their interiors and exteriors. (Lessons 12-1, 12-2, 12-3, 12-4, 12-6, 12-7)

7. Find an equation for the circle with center at the origin and radius 12.
8. Find an equation for the circle with center at (-3, 6) and diameter 7.
9. Give an equation for the upper semicircle of the circle with equation $a^2 + b^2 = 35$.
10. What inequality describes the interior of the circle with equation $x^2 + y^2 = 64$?
11. What sentence describes the exterior of the circle with equation $x^2 + y^2 = 64$?

In 12 and 13, write an equation for the ellipse satisfying the given conditions.

12. foci: (0, 1) and (0, -1); focal constant: 5
13. The endpoints of the major and minor axes are (4, 0), (-4, 0), (0, 7), and (0, -7).
14. Write an equation for the parabola with focus (0, -3) and directrix $y = 3$.

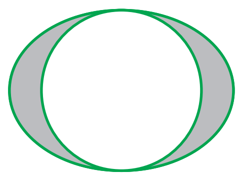
In 15 and 16, find an equation for a hyperbola satisfying the given conditions.

15. vertices: (2, 2) and (-2, -2)
16. foci: (9, 0) and (-9, 0); focal constant: 11

**OBJECTIVE C** Find the area of an ellipse. (Lesson 12-5)

In 17 and 18, find the area of the ellipse satisfying the given conditions.

17. It has equation $\frac{x^2}{100} + \frac{y^2}{49} = 1$.
18. The endpoints of its axes are (3, 0), (-3, 0), (0, 6), and (0, -6).
19. Which has a larger area and by how much: a circle of radius 15 or an ellipse with major and minor axes of lengths 46 and 20?
20. Find the area of the shaded region between an ellipse with major axis of length 7 and minor axis of length 5, and a circle with diameter 5.



**OBJECTIVE D** Solve systems of one linear and one quadratic equation or two quadratic equations with and without technology. (Lessons 12-8, 12-9)

In 21–26, solve.

21. $\begin{cases} y = x^2 + 8 \\ y = -x^2 + 7x + 8 \end{cases}$
22. $\begin{cases} 6x + y = 24 \\ y = x^2 + 4x - 5 \end{cases}$
23. $\begin{cases} t = r^2 + 2r - 8 \\ t = 2r^2 + 2r - 6 \end{cases}$
24. $\begin{cases} (x - 1)^2 + y^2 = 3 \\ x^2 + (y + 1)^2 = 3 \end{cases}$
25. $\begin{cases} ab = 4 \\ b = 3a + 1 \end{cases}$
26. $\begin{cases} p^2 - q^2 = 4 \\ \frac{p^2}{16} + \frac{q^2}{9} = 1 \end{cases}$

27. The product of two numbers is 1073. If one number is increased by 3 and the other is decreased by 7, the new product is 960.

 a. Write a system of equations representing this situation.

 b. Find the numbers.

**PROPERTIES** Principles behind the mathematics

**OBJECTIVE E** **Identify characteristics of parabolas, circles, ellipses, and hyperbolas. (Lessons 12-1, 12-2, 12-4, 12-6, 12-7)**

**In 28 and 29, identify the center and radius of the circle with the given equation.**

28. $(r + 6)^2 + s^2 = 196$ 29. $x^2 + y^2 = 361$

30. Consider the ellipse with equation $\frac{x^2}{169} + \frac{y^2}{64} = 1$.

 a. Name the endpoints of its axes.

 b. State the length of its minor axis.

31. Consider the parabola with equation $y = \frac{1}{7}x^2$.

 a. Give the coordinates of its focus.

 b. Give the coordinates of its vertex.

 c. State the equation of its directrix.

32. Consider the hyperbola with equation $\frac{p^2}{25} - \frac{q^2}{9} = 1$.

 a. Name its vertices.

 b. State equations for its asymptotes.

33. Consider the ellipse with equation $\frac{x^2}{16} + \frac{y^2}{100} = 1$.

 a. Give the coordinates of its foci $F_1$ and $F_2$.

 b. Suppose $P$ is on the ellipse. Find the value of $PF_1 + PF_2$.

34. Identify the asymptotes of the hyperbola with equation $xy = \frac{23}{5}$.

**OBJECTIVE F** **Classify curves as circles, ellipses, parabolas, or hyperbolas using algebraic or geometric properties. (Lessons 12-1, 12-2, 12-4, 12-5, 12-6)**

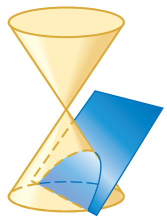
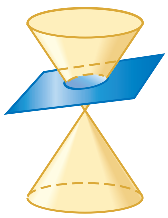
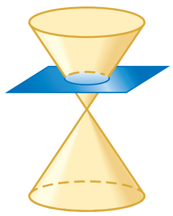
**In 35 and 36, consider two fixed points $F_1$ and $F_2$ and a constant $d$. Name the curve formed by the set of points $P$ satisfying the given conditions.**

35. $F_1P + F_2P = d$, where $d > F_1F_2$

36. $|F_1P - F_2P| = d$, where $d < F_1F_2$

37. Each figure below shows a double cone intersected by a plane. In Figure b, the plane is parallel to the edge of the cone; in Figure d, the plane is perpendicular to the axis of the cone. Identify the curve produced by each intersection.

a.

b.

c.

d.


**In 38–40, answer true or false.**

38. A hyperbola can be considered as the union of two parabolas.

39. All quadratic relations in two variables can be formed by the intersection of a plane and a double cone.

40. The image of an ellipse under a scale change can be a circle.

41. a. What equation describes the image of the circle with equation $x^2 + y^2 = 1$ under the transformation $S : (x, y) \rightarrow (5x, 8y)$?

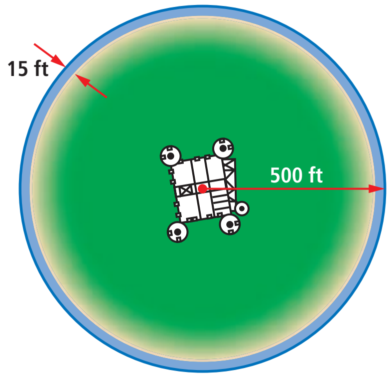
 b. What kind of curve is the image in Part a?

**USES** Applications of mathematics in real-world situations

**OBJECTIVE G** Use circles and ellipses to solve real-world problems. (Lesson 12-2, 12-3, 12-4, 12-5)

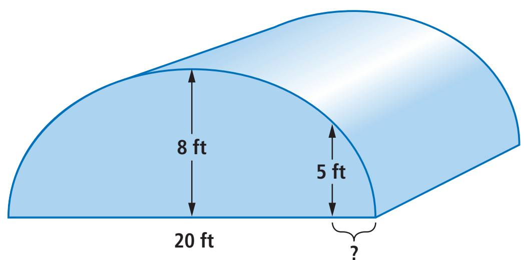
42. An elliptical garden surrounds a circular fountain with diameter 10 feet. The major axis of the garden is 20 feet long, and the minor axis is 10 feet long. What is the area of the garden?

43. A castle is surrounded by a circular moat 15 feet wide. The distance from the center of the castle to the outside of the moat is 500 feet. If the center of the castle is considered the origin, write a system of inequalities to describe the set of points on the surface of the moat.



top view

44. A tent is in the form of half a cylindrical surface. Each cross section is a semiellipse (half an ellipse) with base length 20 feet and height 8 feet. How close to either end can a person 5 feet tall stand straight up?




45. A moving van 6 ft wide and 12 ft tall is approaching a semicircular tunnel with radius 13 ft.
   a. Explain why the truck cannot pass through the tunnel if it stays in its lane.
   b. Can the moving van fit through the tunnel if it is allowed to drive anywhere on the roadway? Justify your answer.

**OBJECTIVE H** Use systems of quadratic equations to solve real-world problems. (Lessons 12-8, 12-9)

46. A piece of paper has an area of 93.5 square inches and a perimeter of 39 inches. Find the dimensions of the piece of paper.

47. Suppose the epicenter of an earthquake is 75 miles away from monitoring stations 1 and 2. Station 2 is 30 miles west and 50 miles north of Station 1. Let Station 1 be at the origin of a coordinate system.
   a. Write a system of equations to describe the location $(x, y)$ of the epicenter.
   b. Find the coordinates of all possible locations of the epicenter relative to Station 1.

48. Perla has 150 meters of fencing material and wants to form a rectangular pen with an area of 1300 square meters.
   a. Let $w$ = the width of the pen and $\ell$ = its length. Write a system of equations to determine $w$ and $\ell$.
   b. Solve the system and interpret your answer in the context of the problem.

49. One month Wanda's Wonderful Wagons took in \$12,000 from sales of wagons. The next month Wanda sold 40 fewer wagons because she had raised the price by \$20. In spite of this, total sales rose to \$12,800. Find the price of wagons in each month.

**REPRESENTATIONS** Pictures, graphs, or objects that illustrate concepts

**OBJECTIVE I** **Graph quadratic relations when given equations for them in standard form, and vice versa. (Lessons 12-2, 12-4, 12-6, 12-7)**

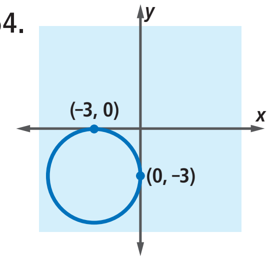
In 50–53, sketch a graph of the equation.

**50.** $\frac{x^2}{25} + \frac{y^2}{64} = 1$  **51.** $\frac{x^2}{25} - \frac{y^2}{64} = 1$

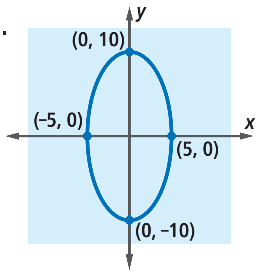
**52.** $xy - 18 = 0$  **53.** $x^2 + y^2 = 16$

In 54 and 55, state an equation for the curve.

**54.**




**55.**

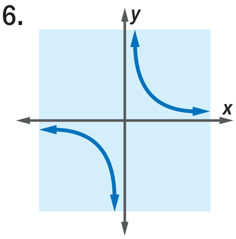



**Multiple Choice** In 56 and 57, select the equation that best describes each graph.

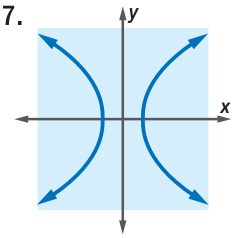
A $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  B $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

C $y = ax^2$  D $xy = a$

**56.**

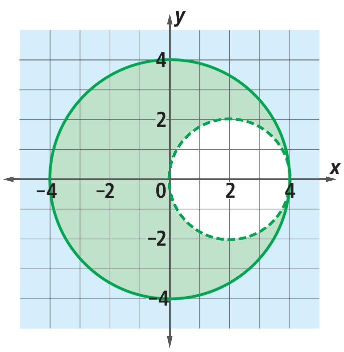



**57.**




**OBJECTIVE J** **Graph interiors and exteriors of ellipses when given inequalities for them, and vice versa. (Lesson 12-3)**

**58.** Write a system of inequalities to represent the points in the shaded region at the right.




In 59 and 60, sketch a graph of the inequality.

**59.** $x^2 + (y - 4)^2 > 25$  **60.** $\frac{x^2}{16} + \frac{y^2}{4} \leq 1$

**OBJECTIVE K** **Interpret representations of quadratic-linear and quadratic-quadratic systems. (Lessons 12-8, 12-9)**

In 61 and 62, give equations for

**61.** a circle and a hyperbola that intersect in exactly three points.

**62.** two hyperbolas that intersect exactly twice.

**63.** Someone claims that the sum of two real numbers is 17 and their product is 73. Use equations and a graph to explain why this is impossible.

**64. a.** Graph $x^2 + y^2 = 5$ and $y = x^2 - 2$ in the same window.

**b.** Find the points of intersection to the nearest tenth.

**OBJECTIVE L** **Draw a graph or interpret drawings or graphs of conic sections based on their definitions. (Lessons 12-1, 12-2, 12-4, 12-6)**

**65.** Graph the set of points equidistant from the point (5, 3) and the line $y = -1$.

**66.** Draw a line $\ell$ and point $F$ not on $\ell$. Sketch 5 points equidistant from $F$ and $\ell$.

**67.** Graph the set of points that are 2.5 units from the origin.

**68.** Draw two points $F_1$ and $F_2$. Sketch 6 more points whose distances from $F_1$ and $F_2$ add up to $2F_1F_2$.

**69.** Graph the set of points whose distances from (0, –3) and (0, 3) add up to 8.

**70.** Draw two points $F_1$ and $F_2$ 4 units apart. Sketch 3 points that are 2 units farther from $F_1$ than from $F_2$ and 3 points that are 2 units farther from $F_2$ than from $F_1$.

**71.** Graph the set of points whose difference of distances from (–6, 0) and (6, 0) is 5.