Chapter 2

Summary and Vocabulary

- With algebra, a wide variety of relationships among numbers can be explained. An important tool in these explanations is the **Distributive Property.** This property says that for all real numbers a, b, and c, $c(a + b) = ca + cb$. Moving from $c(a + b)$ to $ca + cb$ is called expanding the expression. Moving from $ca + cb$ to $c(a + b)$ is called factoring the expression $ca + cb$.
- In an equation, the expression on the left side is equivalent to the expression on the right. Algebraic properties provide a way of determining whether expressions are equivalent and, at the same time, explaining why they are equivalent. This makes the use of algebraic properties a more powerful method than using tables or graphs, which can suggest that expressions are equivalent but cannot prove them equivalent. Yet tables and graphs can be useful when algebraic methods are not available. Also, by using algebraic properties, you can create your own equivalent forms of algebraic expressions.
- Among the earliest properties you ever learned were the **related facts of addition and subtraction, and of multiplication and division.** The related facts properties and the other algebraic properties can also help explain why various number puzzles work. Algebra is a language that helps you better understand the world.

Vocabulary

2-2
like terms
coefficient
factoring

2-4
additive inverse
opposite

Theorems and Properties

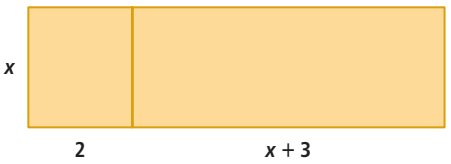
Distributive Property of Multiplication over Addition (p. 66)
Distributive Property of Multiplication over Subtraction (p. 67)
Opposite of Opposites Property (p. 85)
Multiplication Property of –1 (p. 86)
Opposite of a Sum Property (p. 86)
Opposite of a Difference Property (p. 87)
Addition Property of Equality (p. 106)
Subtraction Property of Equality (p. 106)
Related Facts Property of Addition and Subtraction (p. 107)
Additive Identity Property (p. 108)
Additive Inverse Property (p. 109)
Multiplication Property of Equality (p. 113)
Division Property of Equality (p. 113)
Related Facts Property of Multiplication and Division (p. 114)
Multiplication Property of Zero (p. 115)
Zero Product Property (p. 115)
Multiplicative Identity Property (p. 116)
Multiplicative Inverse Property (p. 116)

Chapter 2 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

In 1–7, simplify the expression.

1. $5w - 2w$
2. $\frac{6}{7}(3v + 78 + v)$
3. $-7h + 2(h - 4)$
4. $3(k + 10) - (11 - 4k)$
5. $-(-(-(r)))$
6. $\frac{5x + 7}{6} + \frac{2x}{6}$
7. $\frac{2}{3x} - \frac{1}{x}$
8. Carlos bought seven shirts at $19.98 each. Show how Carlos could use the Distributive Property to calculate the total cost of the shirts in his head.
9. Write two expressions to describe the total area of the figure below.



10. **Fill in the Blank** If $2 \cdot n = 4$, then $2 \cdot n \cdot 6 =$ ___?___.
11. Write the related facts for $C = \pi \cdot d$, where C is the circumference of a circle and d is its diameter.
12. If you know the circumference of a circle and want to find the diameter, which of the related facts from Question 11 should you use?
13. Make a fact triangle for $-12 = -4 + -8$, and write the related facts.

In 14 and 15, find the opposite of the expression.

14. $-3p$
15. $b + 2$

In 16 and 17, find the reciprocal of the expression.

16. -2.5
17. $\frac{4}{11d}$
18. **True or False** Is $(-2)^5 = 2^5$? Justify your response in one or two sentences.
19. Let n be any real number. Use algebra to show how the number puzzle below works.
    Step 1 Pick a number.
    Step 2 Multiply by 10.
    Step 3 Add 30.
    Step 4 Divide by 5.
    Step 5 Subtract your original number.
    Step 6 Add 1.
    Step 7 Subtract 7.
    You have your original number.
20. Use a graphing calculator to determine whether $w + (2w - 1) + 3(w + 6)$ and $6w - 5$ seem to be equivalent.
21. Use properties of operations to determine whether $w + (2w - 1) + 3(w + 6)$ and $6w - 5$ are equivalent.
22. Four siblings worked on a project. Darryl worked the shortest amount of time. Carol worked twice as long as Darryl. Beryl worked three times as long as Darryl, and Errol worked four times as long as Darryl. They decided to split up the $1,500 they earned from the project based on the amount of time each of them worked. How much did each sibling get?
23. Make a table to show that $2x + 1$ and $|-2x - 1|$ are not equivalent expressions.

Chapter 2 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Use the Distributive Property to expand and combine like terms. (Lessons 2-1, 2-2)

In 1–12, simplify the expression by distributing and/or combining like terms.

1. $3(x + 4)$
2. $(2a - 1.3)10$
3. $5(3x + 7) - 11(x + 6)$
4. $\frac{2}{5}(10 + -15w + 4w)$
5. $4(y - 13) + 6(3y - 3y)$
6. $1.5x + -4x + 17x$
7. $7x + -2x + 13x$
8. $-6m + 5m + -m$
9. $\frac{3x}{4} - \frac{3}{8} + 2x$
10. $\frac{2x}{5} + \frac{4z}{5} - \frac{2x}{5}$
11. $\frac{n+1}{3} + \frac{5}{3}$
12. $\frac{3}{2x} + \frac{1}{x}$

OBJECTIVE B Use the Opposite of Opposites Property, the Opposite of a Sum, and the Opposite of a Difference Property to simplify expressions. (Lesson 2-4)

In 13–20, remove the parentheses and then combine like terms, if possible.

13. $-(7a + 4)$
14. $-(3h - 7g + 8)$
15. $1 - (1 - z)$
16. $7x - (4x - 8)$
17. $1\frac{1}{2} - (\frac{3}{4} - y)$
18. $3(b - 2) - 5(3 + 2e)$
19. $(-2)^3$
20. $(-x)^4$
21. Evaluate each expression.
    a. $(-3)^4$
    b. -3^4
    c. $(-3)^5$
    d. -3^5

In 22–24, determine whether the expression is positive or negative. How do you know?

22. $-3(7.4)(-237)(-2)$
23. $(-1{,}135)4$
24. $x \cdot -x$

OBJECTIVE C Use related facts to solve sentences. (Lessons 2-7, 2-8)

In 25–27, identify all the real numbers that complete the sentence.

25. $0n = 3$
26. $-2 + k = 0$
27. $0g = 0$

In 28–31, find the related facts for the sentence.

28. $d = cg$
29. $\frac{1}{2} = \frac{1}{6} + \frac{1}{3}$
30. $a + b = 5$
31. $317.23 = 1 \cdot 317.23$

In 32–34, use related facts to find the value of the variable.

32. $2.6 = 13 + a$
33. $56 = x \cdot 0.8$
34. $0 = 1853.42b$

PROPERTIES The principles behind the mathematics

OBJECTIVE D Apply and recognize the following multiplication properties: Multiplicative Identity Property, Multiplicative Inverse Property, Multiplication Property of Zero, Multiplication Property of Equality, and the Zero Product Property. (Lessons 2-4, 2-8)

In 35–37, write the reciprocal of the number.

35. -5
36. 0.513
37. $\frac{1}{8x}$
38. Write the following statement in symbols: *The product of a number and its reciprocal is the multiplicative identity.*
39. Of what property is this an example? If $w = x$, then $w - 1.7 = x - 1.7$.

40. **Fill in the Blank** Multiplication by –1 changes a number to its ___?___.

In 41–43, evaluate the expression $(x + 5)(x + 4)(x + 3)$ for the given value of x.

41. $x = -3$ 42. $x = 3$ 43. $x = -2$

OBJECTIVE E Apply and recognize the following properties: Additive Identity Property, Additive Inverse Property, and Addition Property of Equality. (Lesson 2-7)

44. If $a + b = 0$, how are a and b related?

45. If $a + b = a$, what is the value of b?

46. **Fill in the Blank** Let $r = t$. Then $r + 2.576 = t +$ ___?___.

In 47–49, write the additive inverse of the number.

47. –7.536 48. 0 49. $-(-x)$

50. Write the following statement in symbols: *If two numbers are additive inverses, their sum is the additive identity.*

OBJECTIVE F Use and apply the Distributive Property to perform calculations in your head. (Lesson 2-1)

51. In the sentence $2(r + w) = 2r + 2w$, what property has been applied?

In 52–55, explain how the Distributive Property can be used to do the calculations in your head.

52. $7 \cdot \$5.95$

53. $103 \cdot 36$

54. $4 \cdot 59$

55. the cost of 11 shirts if each one costs \$3.50

USES Applications of mathematics in real-world situations

OBJECTIVE G Use algebra to explain how number puzzles work. (Lesson 2-3)

In 56–58, let n be the number used to solve the number puzzle. Use algebra to explain the result.

56. **Step 1** Pick a number.
Step 2 Subtract 5.
Step 3 Multiply by 4.
Step 4 Add 4.
Step 5 Divide by 2.
Step 6 Add 10.
Step 7 Subtract twice your original number.
You will always end up with 2.

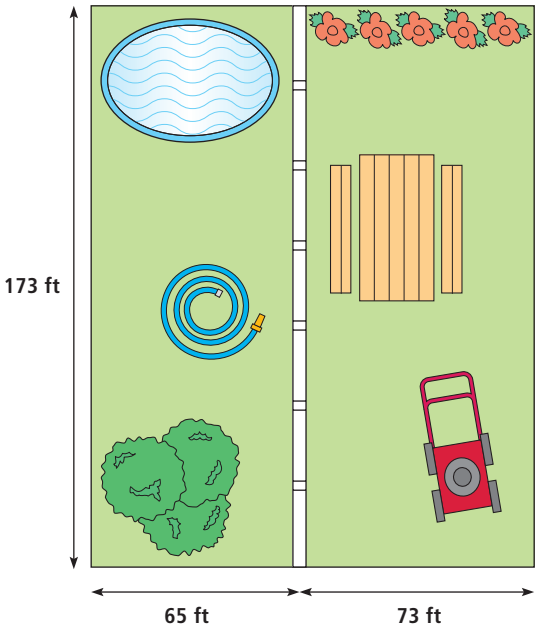
57. **Step 1** Pick a number.
Step 2 Multiply by 4.
Step 3 Add 10.
Step 4 Add 2 more than your original number.
Step 5 Divide by 4.
Step 6 Subtract 3.
You will always end up with your original number.

58. **Step 1** Pick a number.
Step 2 Add 11.
Step 3 Multiply by 6.
Step 4 Subtract 12.
Step 5 Divide by 2.
Step 6 Subtract 30.
Step 7 Add 3.
You will always end up with 3 times your original number.

OBJECTIVE H Apply the Distributive Property in real-world situations. (Lessons 2-1, 2-2)

59. A $150,000 estate is to be split among 4 children, 2 grandchildren, and a charity. Each child gets the same amount, while the grandchildren get half as much. If the charity receives $5,000, how much will each child receive?

60. Two next-door neighbors' yards are pictured below. Find the total area of both yards.



61. Suppose a taxicab driver is allowed to keep $\frac{2}{5}$ of all fares collected. The remaining fares go to the company. If a driver makes F dollars in fares, what is the driver's share?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE I Use a spreadsheet or table to test the equivalence of expressions. (Lesson 2-5)

62. Make a table of values to show that $3(2x + 4)$ and $6x + x + 12 - x$ are equivalent expressions.

63. Make a table of values to show that x^2 and $2x$ are not equivalent expressions. Circle a counterexample in the table.

64. A table of values generated by two expressions is shown below. Do the expressions seem to be equivalent? Why or why not?

Expression 1		Expression 2	
x	y	x	y
–5	–4	–5	4
–4	–2	–4	2
–3	0	–3	0
–2	2	–2	2
–1	4	–1	4

OBJECTIVE J Use technology to test for equivalence of expressions. (Lessons 2-5, 2-6)

In 65–67, use a CAS or a graphing calculator to determine whether the expressions are equivalent.

65. $n - (1 - (2 - (3 - n)))$ and $2n + 2$

66. $x^2 - 4x + 4$ and $(x - 2)(x - 2)$

67. $(x + 1)(x)(x - 1)$ and x^3

68. Suppose two different students use the formulas $A = \pi r^2$ and $A = \frac{\pi d^2}{4}$ to find the area of a circle. Remember, the diameter d of a circle is twice as long as the radius r. Use a graphing calculator or CAS to determine if the two formulas are equivalent.