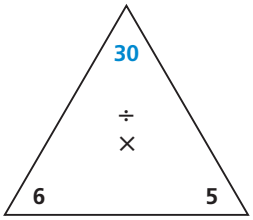
Lesson 2-8

Explaining Multiplication and Division Related Facts

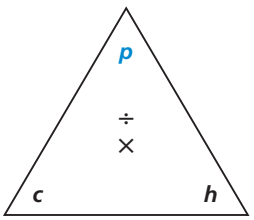
BIG IDEA The Multiplication Property of Equality explains how multiplication and division facts are related and helps to solve equations of the form $ax = b$.

Related Facts for Multiplication

Fact triangles can also be used to represent related facts in multiplication and division. In a multiplication fact triangle, the number at the top is the product of the other two.

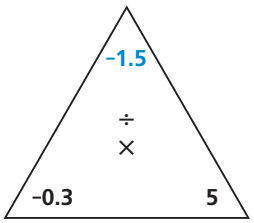


$6 \cdot 5 = 30$
$5 \cdot 6 = 30$
$30 \div 6 = 5$
$30 \div 5 = 6$

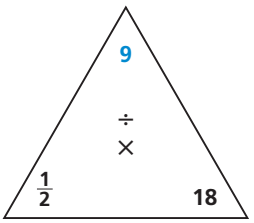


$c \cdot h = p$
$h \cdot c = p$
$p \div c = h$
$p \div h = c$

The related facts that can be obtained from these triangles are shown above. As with addition, the first two facts in each list are equivalent by the Commutative Property of Multiplication. Again, as with addition, the fact triangles work for any nonzero real numbers. Here are some other fact triangles and related facts with positive and negative numbers and with fractions.



$5 \cdot -0.3 = -1.5$
$-0.3 \cdot 5 = -1.5$
$-1.5 \div -0.3 = 5$
$-1.5 \div 5 = -0.3$



$18 \cdot \frac{1}{2} = 19$
$\frac{1}{2} \cdot 18 = 9$
$9 \div \frac{1}{2} = 18$
$9 \div 18 = \frac{1}{2}$

Mental Math

Calculate.

a. $2 \cdot -5 - 10 + 60$

b. $2 \cdot -5 - (10 + 60)$

c. $2 \cdot (-5 - 10 + 60)$

Just as you can add the same number to equal quantities and get equal quantities as a result, you can multiply (or divide) equal quantities by the same number and get equal quantities. For example, we know that $-0.3 \cdot 5 = -1.5$.

$$-0.3 \cdot 5 = -1.5$$
$$10 \cdot -0.3 \cdot 5 = 10 \cdot -1.5 \quad \text{Multiply each side by 10.}$$
$$-15 = -15 \quad \text{Simplify.}$$

This property is called the *Multiplication Property of Equality*.

Multiplication Property of Equality

For all real numbers a, b, and c, if $a = b$, then $ca = cb$.

Because $x \div c = x \cdot \frac{1}{c}$ for all values of x and c (provided c is not zero), every division can be converted to multiplication. So to divide both sides of an equation by c, you can multiply both sides by $\frac{1}{c}$. For this reason, the Multiplication Property of Equality means that there is also a *Division Property of Equality*.

Division Property of Equality

For all real numbers a, b, and all real nonzero numbers c, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

The Related Facts Property of Multiplication and Division

The Multiplication Property of Equality also explains why related facts work. Write down a multiplication fact, as shown below.

$$8 \cdot 45 = 360$$

Now divide each side by 8. Do the computation on the left side only.

$$\frac{8 \cdot 45}{8} =$$
$$45 = \frac{360}{8}$$

The result is a related division fact. You can divide each side of the original fact by 45 to find the other related division fact.

$$8 = \frac{360}{45}$$

Related multiplication and division facts cannot be found if one of the numbers being multiplied is 0 because division of 0 is undefined. However, related facts can be found with nonzero numbers. We call this the *Related Facts Property of Multiplication and Division*.

Related Facts Property of Multiplication and Division

For all nonzero real numbers a, b, and c, if $ab = c$, then $ba = c$, $\frac{c}{b} = a$, and $\frac{c}{a} = b$.

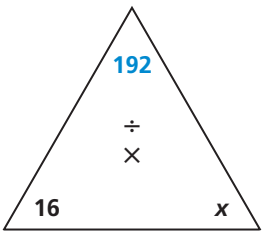
Multiplication equations can be solved by using related facts, or by performing the same operation on each side of the equation.

Example 1

Solve $16x = 192$.

Solution 1 Use a fact triangle.

The fact triangle shows $16x = 192$.



$$16x = 192$$
$$192 \div 16 = x$$
$$12 = x$$

Solution 2 Divide each side by 16.

$$16x = 192$$
$$\frac{16x}{16} = \frac{192}{16}$$
$$x = 12$$

Solution 3 Multiply each side by $\frac{1}{16}$.

$$16x = 192$$
$$\frac{1}{16} \cdot 16x = \frac{1}{16} \cdot 192$$
$$x = 12$$

STOP **QY1**

▶ **QY1**

Solve $\frac{3}{5}x = 60$ by using

a. a fact triangle.

b. the Multiplication Property of Equality.

The Role of Zero in Multiplication and Division

In many ways, the operations of addition and multiplication behave in similar ways. Both have commutative and associative properties. In both, fact triangles illustrate four related number facts. But there is one special case that arises for multiplication which has no parallel in addition. Zero is special in multiplication. You know that whenever 0 is multiplied by a number, the result is zero.

▶ **READING MATH**

In some countries, zero is called the annihilator.

Multiplication Property of Zero

For any real number a, $a \cdot 0 = 0 \cdot a = 0$.

You have also learned that you cannot divide by 0. Another way of putting it is that a fraction cannot have zero in its denominator. This can be explained by using related facts.

Suppose you tried to divide 0 by 0. You write $\frac{0}{0} = b$. What is the value of b? Using related facts, you get $0 \cdot b = 0$. But any value of b would work to make the equation true. Since there is no unique value for $\frac{0}{0}$, we say it is undefined.

Now suppose you tried to divide some nonzero number a by 0. You write $\frac{a}{0} = b$. Then by related facts, you get $0 \cdot b = a$. The Multiplication Property of Zero says that a would have to be 0. But a was specifically indicated as being a nonzero number. So there is no value of b that makes the equation true.

Therefore, an attempt to divide a number by 0 can never give exactly one answer. And since operations must give a single answer and the same answer each time they are performed, division by zero is not allowed.

When zero is involved in multiplication or division, we do not draw a fact triangle. But zero can still be involved in sample multiplication equations. Three types of equations are possible.

Example 2

Solve each equation.

a. $0x = 0$
b. $0x = 4$
c. $13x = 0$

Solutions Each equation uses zero in a slightly different way.

a. Zero times x is zero. All real numbers are solutions.
b. Zero times x is a nonzero number. Since $0 \cdot x$ is always 0, it cannot be 4. There is no solution.
c. A nonzero number times x is zero. There is exactly one solution, 0.

Part c of Example 2 illustrates a simple fact.

Zero Product Property

If the product of two real numbers a and b is 0, then $a = 0$, $b = 0$, or both a and b equal 0.

STOP QY2

Some Properties of the Number One

Another number that has special properties involving multiplication is 1.

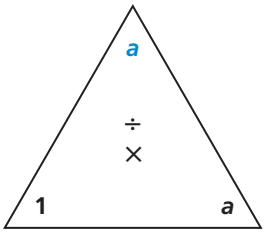
There are two cases to investigate.

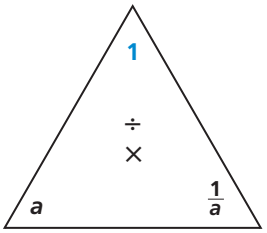
Case 1: One is a factor.

Whatever number you choose for the second factor, the result is that same number.

Case 2: One is the product.

What pairs of numbers multiply to give 1 as a result? They must be reciprocals, like 3 and $\frac{1}{3}$, or $\frac{2}{11}$ and $\frac{22}{4}$, or -8 and -0.125.





The two fact triangles above illustrate two properties of multiplication.

Multiplicative Identity Property

For any real number a, $a \cdot 1 = 1 \cdot a = a$.

The Multiplicative Identity Property is also true for zero, since $0 \cdot 1 = 1 \cdot 0 = 0$.

The second property involves the reciprocals or *multiplicative inverses* of a and $\frac{1}{a}$, whose product is 1.

Multiplicative Inverse Property

For any real number a, where $a \neq 0$, $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$.

GUIDED

Example 3

Find the reciprocal of each number.

a. 1.5625 b. $\frac{4}{15}$ c. -34

▶ QY2

a. Find an equation (not in this lesson) that has no solution.

b. Find an equation (not in this lesson) that is true for every real number.

Solutions

a. The reciprocal of 1.5625 is $\frac{1}{1.5625}$. To find the decimal for $\frac{1}{1.5625}$, you can divide 1 by 1.5625 or find 1.5625^{-1} on your graphing calculator. $\frac{1}{1.5625} =$?

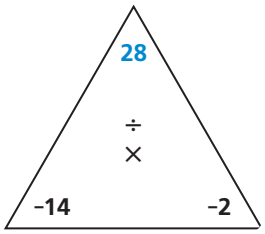
b. The reciprocal of $\frac{4}{15}$ is ? $= 1 \cdot$? $= \frac{15}{4}$. So taking the reciprocal inverts a fraction.

c. The reciprocal of -34 is ?.

Questions

COVERING THE IDEAS

1. Write the related facts for the fact triangle shown below.



2. a. Make a fact triangle with the numbers 275, 25, and $\frac{1}{11}$.
   b. Write the four related facts.
3. If a, b, and c are not equal to zero and $a = bc$, find the other three related facts.
4. Use the equation $\frac{2}{3}p = 96$.
   a. Make a fact triangle for this equation.
   b. Use the fact triangle to solve the equation for p.
   c. Solve the equation using the Multiplication Property of Equality.
   d. Solve the equation using the Division Property of Equality.
5. **Fill in the Blank** If $a = b$, then $6a =$?.

In 6–8, find the reciprocal.

6. 0.8
7. -6
8. $\frac{4}{7}$
9. What number is the multiplicative identity?
10. a. If $ab = 1$, then what is true about a and b?
    b. If $ab = 0$, then what is true about a and b?

11. Explain in your own words why 0 does not have a reciprocal.
12. What number(s) satisfy the following sentences?
    a. Zero times a number is eight.
    b. Zero times a number is zero.
    c. Seven times a number is zero.

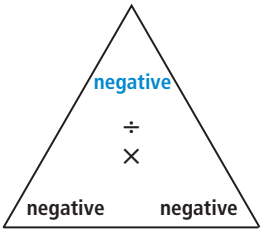
In 13–16, solve the equation.

13. $4x = 0$
14. $0y = \frac{1}{2}$
15. $0 \cdot a = 0$
16. $3b - 3b = 0$

APPLYING THE MATHEMATICS

In 17 and 18, what property is shown by the statement?

17. All the books on the table are free, so a book is free.
18. Since two packages of batteries cost \$7.98, one package costs \$3.99.
19. Illustrate the rules for multiplying and dividing positive and negative numbers by drawing fact triangles. Label corners "pos" for positive number and "neg" for negative numbers. Draw all the possible triangles and give the rules that each triangle illustrates.
20. Explain why the fact triangle below is not possible.



In 21 and 22, find the quotient by multiplying by the reciprocal.

21. $\dfrac{\frac{6}{7}}{\frac{3}{21}}$
22. $\dfrac{9}{\frac{3}{8}}$
23. A rectangle has a length of 50 centimeters and an area of 1 square centimeter.
    a. Is this rectangle possible?
    b. If so, what is the width of the rectangle? If not, why is it not possible?

24. Refer to the pattern in the table.

Quotient	$\frac{p}{10}$	$\frac{p}{1}$	$\frac{p}{0.1}$	$\frac{p}{0.01}$	$\frac{p}{0.001}$	$\frac{p}{0.0001}$
Equal Expression	$0.1p$	$1p$	$10p$	?	?	?

a. Copy and complete the pattern in the table.

b. Use the pattern to rewrite $\frac{p}{0.000000001}$ as a multiple of p.

25. Consider the formula $d = rt$, where d is distance, r is rate, and t is time.

a. Write the related facts for the formula.

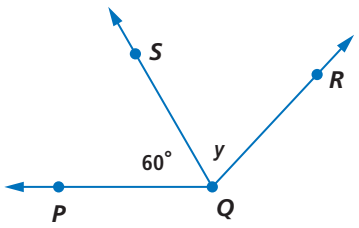
b. Suppose Sam D. Yago is traveling from his home to Santa Clara, California, a distance of 160 miles. To the nearest mile, how long will the trip take him if he can average 42 miles per hour?

26. Meli went grocery shopping. Her least expensive purchase was a drink. She bought bread which cost twice as much as the drink, salad that was four times as much as the drink, and laundry detergent that was five times as much as the drink. Her bill came to \$18. How much did each item cost Meli?

REVIEW

27. The formula $P = 2a + 2b$ gives the perimeter P of a rectangle with sides a and b. Create a fact triangle using this formula, and list the other three related facts. **(Lesson 2-7)**

28. The measure of $\angle PQR$ equals 133°. Find y, the measure of $\angle SQR$. **(Lesson 2-7)**



29. Evaluate the expressions $18p^2 + p$ and $9p^2 + p^3 + 19p$ when $p = 6$. Are the expressions equivalent? Why or why not? **(Lessons 2-6, 1-3)**

30. **a.** Evaluate $-(-194) + -(-(-194))$.

b. Explain how you found your answer. **(Lesson 2-4)**

In 31–34, suppose that x is positive and y is negative. Tell whether the value of the expression is *always positive*, *sometimes positive*, or *never positive*. (Lessons 2-4, 1-1)

31. $\frac{x}{y}$ **32.** $x \cdot y$ **33.** $x - y$ **34.** $x + y$

35. To estimate the number of bricks N needed in a wall, some bricklayers use the formula $N = 7LH$, where L and H are the length and height of the wall in feet. If a wall is to be 8.25 feet high and 27.5 feet long, about how many bricks would a bricklayer need? **(Lesson 1-1)**

36. The value of Birchmere stock went down 1.64 on March 30. On March 31 and April 1 it went up 0.88 of a point each day. Find the net change in Birchmere stock over this 3-day period. **(Previous Course)**

EXPLORATION

37. Find single values for a, x, w, and z that make all five of these equations true. (*Hint*: a, x, w, and z are all different numbers.)

$a^2 = a$

$xw = x$

$z \cdot z = z + z$

$x + w = w$

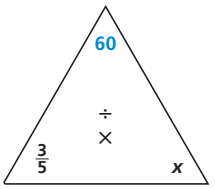
$\frac{z}{a} = z$

Nearly 1 in 3 bricklayers are self-employed.

Source: Bureau of Labor Statistics

QY ANSWERS

1. a.



$x = \frac{60}{\frac{3}{5}} = 60 \cdot \frac{5}{3} = \frac{300}{3} = 100$

1. b. $\frac{3}{5}x = 60$ Multiply both sides by $\frac{1}{\frac{3}{5}}$, or $\frac{5}{3}$; the Multiplication Property of Equality tells us $\frac{5}{3} \cdot \frac{3}{5}x = \frac{5}{3} \cdot 60$, so $x = \frac{300}{3} = 100$.

2. Answers vary. Sample answers:

a. $0 \cdot x = 17$

b. $0 \cdot 3 \cdot x = 2 \cdot 0$