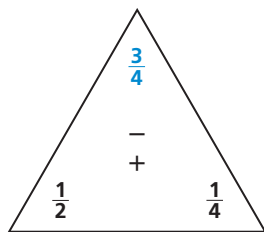# 2-7

Explaining Addition and Subtraction Related Facts

▶ **BIG IDEA** The Addition Property of Equality explains how addition and subtraction facts are related and helps solve equations of the form $a + x = b$.
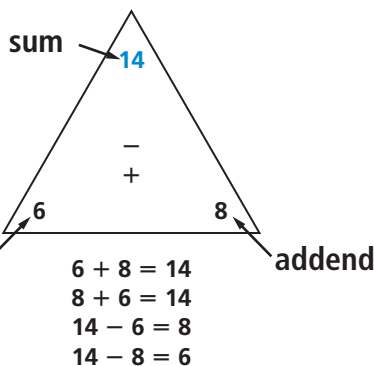
A diagram called a *fact triangle* is shown at the right. Any pair of numbers in the triangle can be combined with addition or subtraction to produce the third number. The numbers 6, 8, and 14 produce the four related number facts listed below the triangle. You know that the first two facts are equivalent because $6 + 8 = 8 + 6$ by the Commutative Property of Addition.

sum — 14

−
+

6   8

addend   addend

$6 + 8 = 14$
$8 + 6 = 14$
$14 − 6 = 8$
$14 − 8 = 6$

Mental Math

a. 1 km $\approx \frac{3}{5}$ mile, so
1 mile $\approx$ __?__ km.

b. 1 in. $\approx$ 2.5 cm, so
1 cm $\approx$ __?__ in.

Fact triangles can be used to show addition/subtraction related facts with *any* kinds of numbers, including fractions and negative numbers. Here are two examples below. The fact triangle on the left uses fractions and the fact triangle on the right uses negative numbers. Each fact triangle has two addition facts and two subtraction facts.

$\frac{3}{4}$

−
+

$\frac{1}{2}$ $\frac{1}{4}$

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$
$\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$
$\frac{3}{4} − \frac{1}{2} = \frac{1}{4}$
$\frac{3}{4} − \frac{1}{4} = \frac{1}{2}$

−5

−
+

−12   7

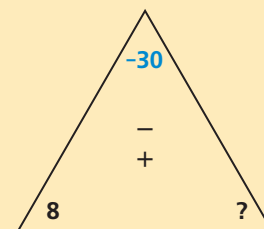$−12 + 7 = −5$
$7 + −12 = −5$
$−5 − −12 = 7$
$−5 − 7 = −12$

STOP QY1

▶ **QY1**

a. What number goes in the empty corner of the fact triangle below?

b. Write the four related facts.

−30

−
+

8   ?

The Addition Property of Equality

You know from arithmetic that if you start with equal quantities and add the same amount to each, the resulting quantities are still equal. For example, we know that $4 + 3 = 7$.

Adding 5 to each side gives $4 + 3 + 5 = 7 + 5$.

$$12 = 12$$

Note that the value of each side changes from 7 in the first equation to 12 in the second equation. But in each case, the two sides of the equation are equal. The idea that adding a number to both sides produces another true equation is called the *Addition Property of Equality*. It is a basic property of addition.

Addition Property of Equality

For all real numbers a, b, and c, if $a = b$, then $a + c = b + c$.

By the Definition of Subtraction, $a - c = a + -c$ for all real numbers. Every subtraction can be converted to an addition. So if you wanted to subtract a number, say 40, from both sides of an equation, you could add -40 instead. For this reason, the Addition Property of Equality means also that there is a *Subtraction Property of Equality*.

Subtraction Property of Equality

For all real numbers a, b, and c, if $a = b$, then $a - c = b - c$.

The Related Facts Property of Addition and Subtraction

These properties also explain why related facts work. Write down a general addition fact. The sum of two numbers is a third number. The result is a related subtraction fact.

$$h + m = S$$
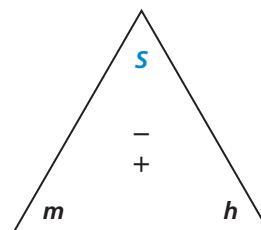$$h + m - m = S - m \quad \text{Subtract } m \text{ from both sides.}$$
$$h = S - m$$

If instead you subtract h from both sides of the original addition fact, you get the other related subtraction fact.
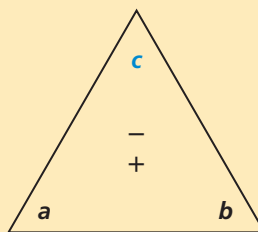
$$h + m - h = S - h$$
$$m = S - h$$

In this way, algebra explains why fact triangles work. Either addend is equal to the sum minus the other addend. We call this the *Related Facts Property of Addition and Subtraction*.

Related Facts Property of Addition and Subtraction

For all real numbers a, b, and c,
if $a + b = c$, then $b + a = c$,
$c - b = a$, and $c - a = b$.

STOP QY2

> ▶ **QY2**
>
> Write the two related subtraction facts for $5a + x = k$.

Example 1

Solve the equation $x + -8 = -30$
a. using a fact triangle (the Related Facts Property).
b. using the Subtraction Property of Equality.

Solutions

a. Draw a fact triangle. Place x and -8 at the lower corners. The sum at the top is -30, as shown at the right.

The triangle shows that $x = -30 - (-8)$ is a related fact.
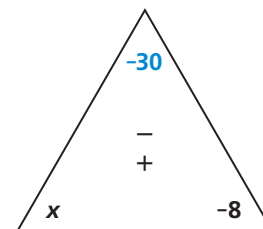
$x = -30 - -8 = -30 + 8 = -22$

b. Subtract -8 from both sides.

$$x + -8 = -30$$
$$x + -8 - -8 = -30 - -8$$
$$x + -8 + 8 = -30 + 8$$
$$x = -22$$

Example 2

Solve the equation $-3.4 - y = 6.1$
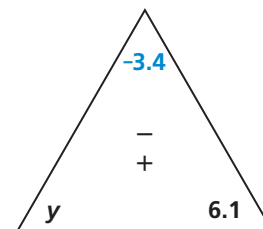a. using a fact triangle.   b. using properties.

Solutions

a. Because y is subtracted from -3.4, it must be that -3.4 is the sum. So put -3.4 in the upper corner and y in one of the lower corners. Put 6.1 in the third corner.

$$y = -3.4 - 6.1$$
$$y = -3.4 + -6.1$$
$$y = -9.5$$

(continued on next page)

b. $-3.4 - y = 6.1$ Write the equation.

$-3.4 + \text{-}y = 6.1$ Definition of Subtraction

$-3.4 + \text{-}y + y = 6.1 + y$ Add y to each side since we are looking for y and not the opposite of y.

$-3.4 = 6.1 + y$ Simplify.

$-3.4 - 6.1 = 6.1 - 6.1 + y$ Subtract 6.1 from each side.

$-9.5 = y$ Simplify.

Check Substitute –9.5 for y in the original equation.
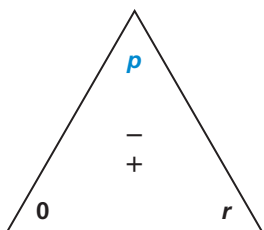
Does $-3.4 - -9.5 = 6.1$?

$-3.4 + 9.5 = 6.1$
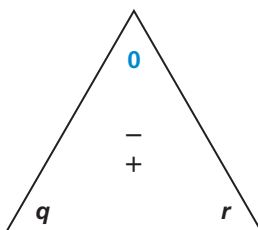
$6.1 = 6.1$ Yes.

Special Numbers for Addition

Important properties can be seen from a fact triangle in which one of the numbers is zero. Two cases are possible.

Case 1: Zero is one of the addends.   **Case 2:** Zero is the sum.

$0 + r = p$ $q + r = 0$
$r + 0 = p$ $r + q = 0$
$p - 0 = r$ $0 - q = r$
$p - r = 0$ $0 - r = q$

When 0 is an addend (Case 1), one of the related facts is $p - 0 = r$. This means that $p = r$. So the first related fact $0 + r = p$ can be rewritten as $0 + p = p$ or as $0 + r = r$. Adding 0 to a number keeps the *identity* of that number. So 0 is called the *additive identity*.

> **Additive Identity Property**
>
> For any real number a, $a + 0 = 0 + a = a$.

When 0 is the sum (Case 2), a related subtraction fact is $0 - q = r$.

$0 + (\text{-}q) = r$ Definition of Subtraction

$\text{-}q = r$ Additive Identity Property

Since $-q = r$, $q + r = 0$ becomes $q + (-q) = 0$. As you know from Lesson 2-4, q and $-q$ are *additive inverses,* or *opposites.* When you add two inverses, the sum is 0, the additive identity. Every number, including 0, has exactly one additive inverse.

> **Additive Inverse Property**
>
> For any real number a, $a + -a = -a + a = 0$.

Questions

COVERING THE IDEAS

1. What are the related facts for the fact triangle shown at the right?

In 2–4, make a fact triangle to fit the equation. Then write the other three related facts.

2. $82 + -5 = 77$ 3. $-\frac{1}{8} - 3\frac{7}{8} = -4$ 4. $x + 5x = 6x$

(Fact triangle: 0.67 at top; − + in middle; 0.23 at bottom left; 0.44 at bottom right)

5. Solve the equation $y + (-6) = 14$
   a. using a fact triangle (the Related Facts Property).
   b. using the Addition Property of Equality.

6. a. Write the other three related facts of $7 - b = -8$.
   b. What is the value of b?

7. **Fill in the Blank** $0 + -10 = \underline{\ ?\ }$

8. Why is zero called the additive identity?

9. Give an example of two numbers that are additive inverses.

10. What is another name for an additive inverse?

11. What is the additive inverse of $-x$?
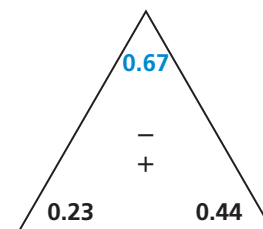
APPLYING THE MATHEMATICS

12. Use $-7 + 7 = 0$ to describe a real situation.

13. Make a fact triangle where the sum is
    a. $\frac{5}{9}$.   b. $2x + 9$.

14. Write a note to a friend explaining how to make a fact triangle for $x + 10 = -19$.

15. Show all possible fact triangles that can be made where two of the numbers are
    a. 3 and –3.   b. n and $-n$.

**16.** Consider this expression:
$$-0.3 + 1.7 - 14.2 - 1.7 + 0.3 - 2.8 + 14.2$$
Use the Associative and Commutative Properties of Addition to first group the additive inverses together. Then evaluate the expression.

17. At 11:00 A.M., the temperature was 15°F. By the time the football game started at 7:00 P.M., the temperature had dropped to –4°F. By how much did the temperature drop?

REVIEW

18. The figure below shows a rectangle with width $40 + a$ and height b. Find two different expressions that describe the area of the figure. Verify that the expressions are equivalent using properties. (**Lessons 2-5, 2-2, 2-1**)
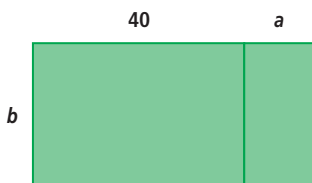
In the United States, there were a total of 1,071,775 high school football players during the 2005–2006 season.

Source: National Federation of State High School Associations

19. A farmer owns a piece of land that is 350 yards wide by 200 yards long, as shown below. She uses one part to harvest wheat and another to harvest corn. She does not use the brown part. Let W = the area of the wheat field and C = the area of the cornfield. The farmer decides to sell the wheat and corn fields. Write an expression for the area of the land she will own after the sale. (**Lesson 2-4**)
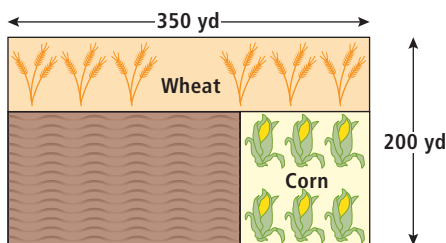
20. *Skill Sequence* Simplify without using a calculator. (**Lesson 2-4, Previous Course**)

   a. $23.7 + -23.7$ **b.** $-(-23.7 + 23.7)$ **c.** $-23.7(23.7 + -23.7)$

In 21 and 22, use the Distributive Property to write an equivalent expression. (**Lessons 2-2, 2-1**)
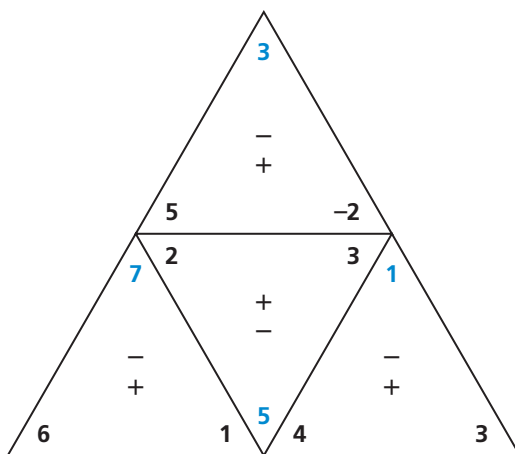
21. $-3p + 4p - 8$ **22.** $-2(3 - 4d)$

23. Two times a number is increased by 5. The resulting quantity is tripled. If m is the original number, write the final result without parentheses. (**Lesson 2-1**)

24. **Multiple Choice** Which formula was used to create the table at the right? (**Lessons 1-3, 1-1**)

    A $y = 4x$

    C $y = 2x + 2$

    B $y = x + 2$

    D $y = 2x + 1$

x	y
0	2
1	4
2	6
3	8
4	10

25. Evaluate the following expressions. (**Lesson 1-1**)

    a. $n^3 - \frac{(-1)^n}{2+n}$ for $n = -5$

    b. $a\left(a + \frac{1}{a}\right)$ for $a = 2$

EXPLORATION

26. A tetrahedron is a 3-dimensional solid whose four faces are equilateral triangles. A net to construct a tetrahedron is given below. A set of related facts has been formed within each triangle and at each vertex of the tetrahedron. Create a tetrahedron of your own with these properties. Describe the method you used to create it.