Lesson 2-6

Equivalent Expressions with Technology

BIG IDEA A computer algebra system (CAS) uses properties of operations to create equivalent expressions.

In this lesson, you will use a computer algebra system (CAS) to test whether expressions are equivalent. Then you will use it to explore a variety of equivalent forms for a single expression. A CAS does algebra just like a calculator does arithmetic. Among the many things that a CAS can do is simplify expressions and solve equations. It has been programmed to use the same algebraic properties that you have been learning. The first computer software with CAS capabilities was created in 1968 at MIT. Until then, mathematicians had to do even the most complicated algebraic calculations by hand.

Mental Math

a. 35 raffle tickets cost \$10. What is the price per ticket?

b. 60 raffle tickets cost \$20. What is the price per ticket?

c. n raffle tickets cost \$15. What is the price per ticket?

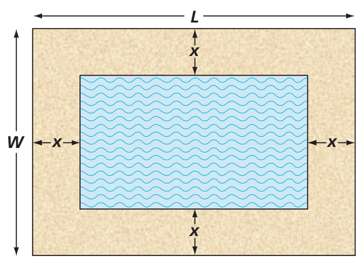
d. y raffle tickets cost x dollars. What is the price per ticket?

Equivalent Formulas in Geometry

In geometry, it is not unusual to see different area formulas for the same type of figure. If the area is found using different formulas, will the results be the same? Where do different formulas come from?

Example 1

The picture below shows the area of a deck surrounding a rectangular swimming pool. The length of the entire region (pool and deck) is *L* and the width of the entire region is *W*. The distance across the deck is *x*.

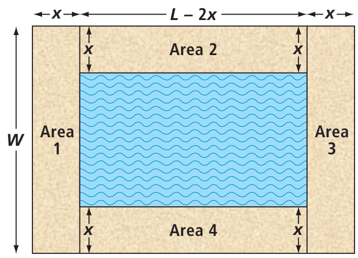


a. Find a formula for the area of the deck by splitting up the deck into 4 rectangles.

b. Split up the deck in another way to find its area.

Solutions Sonia and Roxy both found formulas for the area of the deck, but they are different.

a. Sonia's Method Sonia broke the deck region into four smaller rectangles and then added them. The length of Areas 2 and 4 can be found by taking the total length L and subtracting two lengths of x, or $L - 2x$. She found the areas of Areas 1 through 4 and added them together.



Area 1 $= W \cdot x = Wx$

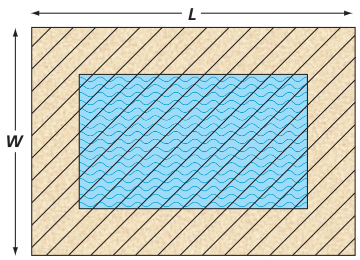
Area 2 $= x \cdot (L - 2x) = x(L - 2x)$

Area 3 $= W \cdot x = Wx$

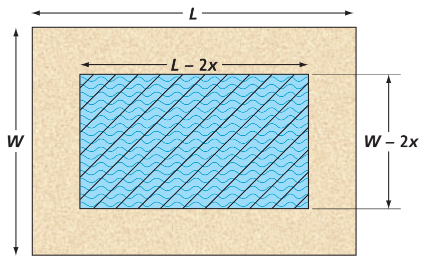
Area 4 $= x \cdot (L - 2x) = x(L - 2x)$

Sonia's Deck Area $= Wx + x(L - 2x) + Wx + x(L - 2x)$

b. Roxy's Method Roxy saw the area of the deck as the area of the pool and deck minus the area of the pool.



Area of Pool and Deck $= W \cdot L = WL$



Area of Pool $= (W - 2x) \cdot (L - 2x)$

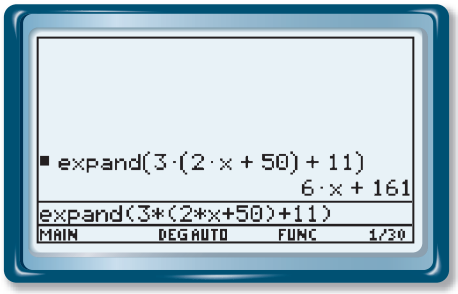
Roxy's Deck Area $= WL - (W - 2x) \cdot (L - 2x)$

Sonia's and Roxy's methods lead to two expressions that appear to be different. But are they? A CAS can answer this question.

Is $Wx + x(L - 2x) + Wx + x(L - 2x)$ equivalent to $WL - (W - 2x) \cdot (L - 2x)$?

The approach that we use is one you saw in the last lesson. We work with the two expressions to see if the result is the same third expression.

Using a CAS to Test for Equivalence

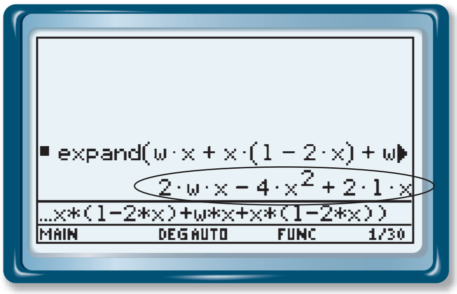


Just as with any new technology, you need to learn the commands that your CAS uses. In this first activity, you will use the Distributive Property to expand multiplication expressions and to combine like terms. Most CAS make both of these changes to an expression by using the `expand` command. The expression that is to be changed appears in parentheses. For example, the command `expand(3(2x+50)+11)` does two operations. It first does the multiplication $3(2x + 50)$, which produces $6x + 150$. Then it adds $6x + 150 + 11$ to get $6x + 161$.

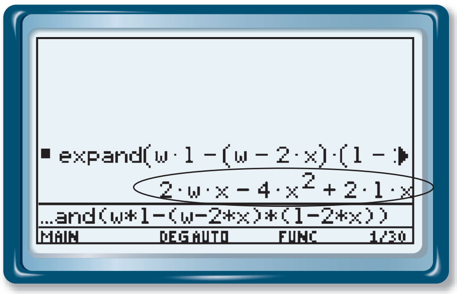
Activity 1

Use a CAS to expand the two expressions from Example 1 to determine whether they are equivalent. Caution: You may need to enter $W \cdot x$ into your CAS so that the CAS will recognize W and x as separate variables.

Step 1 Expand $Wx + x(L - 2x) + Wx + x(L - 2x)$, as shown below.



Step 2 Expand $WL - (W - 2x) \cdot (L - 2x)$, as shown below.



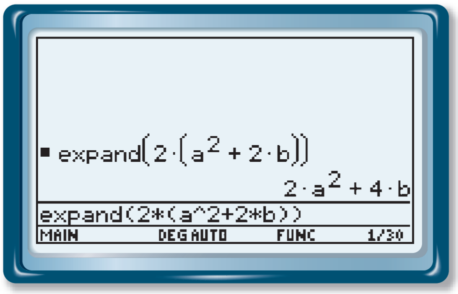
The CAS applied the properties of algebra and found that $Wx + x(L - 2x) + Wx + x(L - 2x)$ and $WL - (W - 2x) \cdot (L - 2x)$ are both equal to $2Wx - 4x^2 + 2Lx$. Therefore, they are equivalent to each other. Both Sonia's and Roxy's approaches will correctly find the area of the deck.

Activity 2

Find three expressions equivalent to $2a^2 + 4b$. In each case, use a CAS to verify the equivalence.

To form equivalent expressions, use the properties you have learned, but in reverse. Instead of trying to make the expression $2a^2 + 4b$ simpler, you need to make it more complicated. There are many approaches to take.

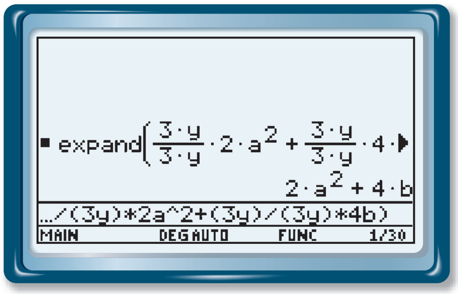
Expression 1 To find a first expression, notice that 2 and 4 are both divisible by 2. Therefore, one possibility is to "undo" the Distributive Property: $2a^2 + 4b = 2(a^2 + 2b)$.
Is $2(a^2 + 2b)$ an equivalent expression? Check using the `expand` command with a CAS. With some problems, you may find using a CAS is slower than doing it yourself. But a good check is one that uses a method different from the one originally used to get the answer.



Using `expand` gives the expression $2a^2 + 4b$.

Thus, $2(a^2 + 2b)$ is equivalent to $2a^2 + 4b$.

Expression 2 To find a second expression, apply the Identity Property of Multiplication. Multiplying a number by 1 does not change its value. The number 1 can take several forms, including $\frac{3y}{3y}$. Therefore, multiplying an expression by $\frac{3y}{3y}$ will result in an equivalent form. Is $\frac{3y}{3y} \cdot 2a^2 + \frac{3y}{3y} \cdot 4b$ an equivalent expression? Using the `expand` command gives $2a^3 + 4b$.



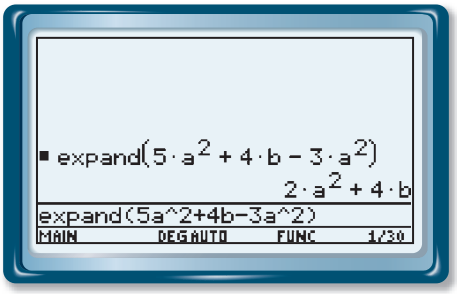
Thus, $\frac{3y}{3y} \cdot 2a^2 + \frac{3y}{3y} \cdot 4b$ is equivalent to $2a^2 + 4b$.

STOP **QY1**

> **QY1**
> Multiply $2a^2 + 4b$ by a different form of 1 to create another equivalent expression.

Expression 3 To find a third expression, apply the Identity Property of Addition and then rearrange the terms. The idea behind this method is to add a "clever" form of zero to the expression. This does not change its value, but produces an expression that looks very different from the original. Here we use $3a^2 + (-3a^2)$.

$2a^2 + 4b + 3a^2 + -3a^2$	Add $3a^2 + -3a^2$, which is 0.
$= (2a^2 + 3a^2) + 4b - 3a^2$	Group two of the like terms.
$= 5a^2 + 4b - 3a^2$	Add the first two like terms.



Is the new expression $5a^2 + 4b - 3a^2$ equivalent to $2a^2 + 4b$? Check using the `expand` command with a CAS.

Using the `expand` command gives the answer $2a^2 + 4b$.
Thus $5a^2 + 4b - 3a^2$ is equivalent to $2a^2 + 4b$.

STOP **QY2**

> **QY2**
> Create a new expression equivalent to $2a^2 + 4b$ by adding another form of zero.

Activity 3

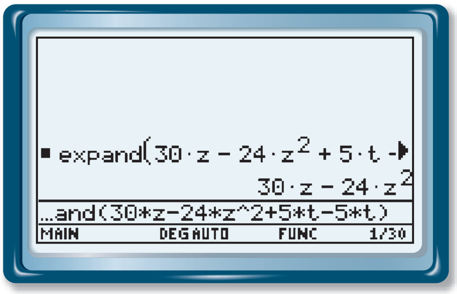
Find at least four equivalent expressions for each of the following expressions. Try to use more than one property on each expression. See how complicated-looking you can make your expression while maintaining equivalence. Use a CAS to verify that your expressions are equivalent.

1. $4(4a^2 - b^2)$
2. $-9m^2 + 12m - 8p$
3. $\frac{1}{2}m \cdot n + \frac{1}{4}$
4. $\frac{6y}{5x}$
5. $100p^2r^2$
6. $2\ell + 2w$

Questions

COVERING THE IDEAS

1. Write an expression equivalent to $5x(7x - 9y)$ by using the Distributive Property.
2. Are $6(x^3 - 5y) + 17x^2 + 12y$ and $6x^3 + 17x^2 - 18y$ equivalent? Why or why not?
3. Explain what the CAS did to simplify the expression on the screen below.



APPLYING THE MATHEMATICS

4. a. Find a third formula for the total area of the deck in Example 1.
   b. Show that it is equivalent to one of the other two formulas in the example.

In 5–8, create three equivalent expressions.

5. $9k^2 - 3k$ 6. $13m^2 + 8m$ 7. $-24y$ 8. $16x^4 + 12xy + 20y$

9. Write a process you could use to convert the expression $5x - 7y$ into the equivalent expression $5(x - 2y) + 3y$.
10. Write an expression equivalent to $34wn + w^2$ using each property.
    a. Commutative Property of Addition
    b. Commutative Property of Multiplication

In 11 and 12, test if the two expressions are equivalent by using a table. Then check your results by simplifying each expression.

11. $3n - 15, 3(n - 4) - 3$

12. $-2(a - 5), -2a - 5$

REVIEW

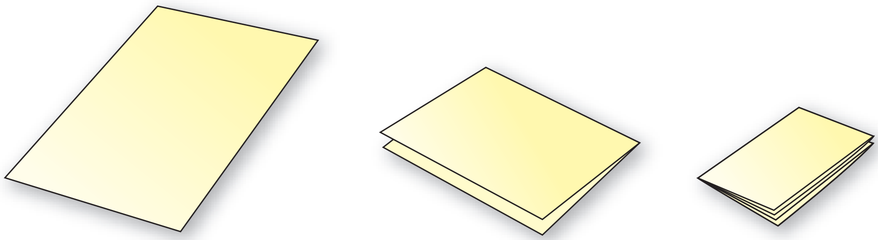
13. **a.** In your own words, state the Multiplication Property of –1.

b. Give an example of this property. (**Lesson 2-4**)

14. **Multiple Choice** Which of the following must be negative? (**Lesson 2-4**)

A $-(-z)$ **B** $-(-6)$ **C** $-x$ **D** $-(-(-\frac{2}{5}))$

15. Take a piece of paper. Fold it in half. The paper now has a thickness twice that of the original piece of paper. Fold the folded paper in half. The paper now has a thickness four times the original piece of paper. (**Lesson 1-2**)



a. Complete the table for the thickness of a piece of paper that is folded 3 and 4 times.

Number of Folds	Thickness of Paper
0	1
1	1 • 2 = 2
2	1 • 2 • 2 = 4
3	?
4	?

b. If a piece of paper is folded six times, what is the thickness of the folded paper?

c. Write an expression to describe the thickness of paper for n folds.

In 16 and 17, let L = the length of a segment. Write an expression for the following. **(Lesson 1-1, Previous Course)**

16. one quarter the length

17. five and one half times the length

18. ***Skill Sequence*** Write each as a decimal. **(Previous Course)**
    a. 1 divided by 5
    b. 1 divided by 0.5
    c. 1 divided by 0.05
    d. 1 divided by 0.00005

19. Convert 0.325823224 to the nearest percent. **(Previous Course)**

EXPLORATION

20. Use a CAS to determine which of these expressions are equivalent.
    A $x^3 + y^3$
    B $(x + y)^3$
    C $(x - y)^3$
    D $x^3 - y^3$
    E $(x + y)(x^2 - xy + y^2)$
    F $(x - y)(x^2 + xy + y^2)$
    G $(x + y)(x + y)(x + y)$
    H $(x - y)(x - y)(x - y)$

QY ANSWERS

1. Answers vary. Sample answer: $\dfrac{2a^3b + 4ab^2}{ab}$
2. $2a^2 + 4b + 5c - 5c$