Lesson 2-4 Opposites

Vocabulary

additive inverse

opposite

BIG IDEA The opposite $-k$ of a number k has properties relating it to addition, multiplication, and subtraction: $-k + k = 0$; $-k = -1 \cdot k$; and $a - k = a + -k$.

The real numbers k and $-k$ are opposites. You have dealt with opposites in previous lessons. In this lesson, we look at some of the basic properties of opposites.

Mental Math

a. a. How many games must a team win to win a best-of-5 playoff series?

b. How many games must a team win to win a best-of-7 playoff series?

c. How many games must a team win to win a best-of-n playoff series?

Opposites as Additive Inverses

Suppose you walk forward 10 steps and then walk backward 10 steps. We signal these opposite directions by calling the walk forward 10 and the walk backward -10. The result of doing these actions one after the other is to end in the same place from where you started. So $10 + -10 = 0$. When two numbers add to zero, they are called **additive inverses,** or **opposites.** Because they signal opposite actions, another name for additive inverse is opposite. So, if two real numbers x and y are opposites, then their sum is zero ($x + y = 0$). Reversing this, if you know that $x + y = 0$, then x and y must be opposites ($x = -y$ or $y = -x$).

STOP **QY1**

QY1

What is the opposite of each number?

a. 3.5 **b.** $\frac{-17}{8}$

The numbers k and $-k$ are opposites regardless of the value of k. If $k = 5$, then $-k = -5$. If $k = -10$, then $-k = -(-10) = 10$. *When k is negative, −k is positive.*

What is the opposite of $-k$? One way to denote the opposite of $-k$ is as $-(-k)$. Yet we know that the opposite of $-k$ is k. This tells us that these two expressions are equivalent. We call this the *Opposite of Opposites Property.*

Opposite of Opposites Property

For any real number a, $-(-a) = a$.

Taking the Opposite by Multiplying by –1

One way of changing a number to its opposite is to multiply it by –1. The Distributive Property explains why this works. If the sum of two numbers is 0, then they must be opposites. Add k to $-1 \cdot k$ and see if the result is 0.

$$\begin{aligned} k + (-1) \cdot k &= 1 \cdot k + (-1) \cdot k \\ &= (1 + (-1))k \\ &= 0k \\ &= 0 \end{aligned}$$

So k and $-1 \cdot k$ are opposites. In symbols, this can be written $-1 \cdot k = -k$. We call this the *Multiplication Property of –1.*

Multiplication Property of –1

For any real number a, $a \cdot -1 = -1 \cdot a = -a$.

The Opposite of Opposites Property and the Multiplication Property of –1 can be used to rewrite and simplify algebraic expressions.

An Example of the Opposite of a Sum

Suppose Marisol has $800 in her savings account. She withdraws a dollars from her account. Deciding that this is not enough, she makes another withdrawal of b dollars. The amount of money left in her savings account can be expressed in several different ways. One way is to think that Marisol withdrew a total of $(a + b)$ dollars. So she has $800 - (a + b)$, or $800 + -(a + b)$ dollars left.

Another way is to think that Marisol withdrew a dollars, then withdrew b dollars. So she has $800 - a - b$, or $800 + -a + -b$ dollars left.

The fact that $-(a + b)$ is the same as $-a - b$ is due in part to the Distributive Property.

$-(a + b) = -1(a + b)$	Multiplication Property of –1
$= -1a + -1b$	Distributive Property
$= -a + -b$	Multiplication Property of –1
$= -a - b$	Definition of Subtraction

The opposite of a sum is the sum of the opposites of its terms.

Opposite of a Sum Property

For all real numbers a and b, $-(a + b) = -a + (-b) = -a - b$.

For example, $-(15y + 3) = -15y - 3$.

The first cash dispenser in use was at Chemical Bank, Long Island, New York in 1969.

Source: www.atmwarehouse.com

STOP QY2

▶ QY2

Simplify $-(a^2 + 2b)$.

Opposite of a Difference

Suppose the expression in parentheses involves subtraction rather than addition. How can you rewrite its opposite? Again, the Distributive Property can be used.

$-(a - b) = a + -b$	Definition of Subtraction
$= -a + -(-b)$	Opposite of a Sum Property
$= -a + b$	Opposite of Opposites Property

Opposite of a Difference Property

For all real numbers a and b, $-(a - b) = -a + b$.

Some problems involve subtracting an expression with two or more terms. Begin by rewriting the subtraction as adding the opposite.

Example 1

Simplify $4x - (3x + 7)$.

Solution Rewrite the subtraction as adding the opposite.

$4x - (3x + 7) = 4x + -(3x + 7)$	Definition of Subtraction
$= 4x + -3x + (-7)$	Opposite of a Sum Property
$= x + -7$	Add like terms.
$= x - 7$	Definition of Subtraction

STOP QY3

▶ QY3

Graph $y = 4x - (3x + 7)$ and $y = x - 7$ in the same window. Do both equations have the same graph?

GUIDED

Example 2

Simplify $(x + 6) - 7(2x - 3)$.

Solution

$(x + 6) +$ _?_ $(2x - 3)$	Definition of Subtraction
$= x + 6 +$ _?_ $+$ _?_	Distributive Property
$=$ _?_ $+$ _?_	Add like terms.

Check When you are asked to simplify expressions, you can make a quick check to see if your answer is equivalent to the given expression by substituting a value for the variable. If the given expressions have one variable each, you can graph the expressions or generate a table of values to check that they are equivalent.

Questions

COVERING THE IDEAS

Research suggests that nutrients in apples and apple juice improve memory and learning.

Source: www.applejuice.org

1. A bottle of apple juice contains 48 fluid ounces. You pour f ounces into a glass and drink it. Then you pour n ounces more into the glass.
   a. Express the amount of juice left in the bottle in two different ways.
   b. Check that the two expressions in Part a are equal by letting $f = 12$ ounces and $n = 5$ ounces.
2. Simplify $-(-(-w))$.

In 3–5, find the opposite of the expression.

3. $-2n$
4. $6p - 8$
5. $-2a^2 + 28a - 15$
6. **Multiple Choice** The opposite of $-x$ is *not*
   A $-x$. B x. C $-(-x)$. D $-(-(-(x)))$
7. **Multiple Choice** Which of the following is equal to $-(P + 7)$?
   A $-P + 7$ B $P + -7$ C $-P + -7$ D $P - 7$
8. **Multiple Choice** Which expression does *not* equal $-(x - y)$?
   A $-x + y$ B $-1x + -1y$ C. $-x - (-y)$ D $y - x$

In 9–16, write an equivalent expression without parentheses.

9. $-(x + 15)$
10. $-(4n - 3m)$
11. $x - (x + 2)$
12. $3y - 5(y + 1)$
13. $(3k^4 + 4) - (7k^4 - 9)$
14. $-(5 + k^3) + (k^3 - 18)$
15. $a^2 + b - c - (a^2 - b + c)$
16. $(-4a)(-a)$

APPLYING THE MATHEMATICS

17. Theo and Rafael completed the toothpick activity from Lesson 1-3. The expression Theo wrote for the number of toothpicks in the nth term was $2n + n + 1$, and the expression Rafael wrote was $4n - (n - 1)$.
    a. Graph $y = 2n + n + 1$ and $y = 4n - (n - 1)$ on your calculator. Do the expressions appear to be equivalent?
    b. Use the Opposite of a Difference Property to show the expressions are equivalent.
18. Evaluate each of the following expressions.
    a. $(-1)^6$ b. $(-1)^8$ c. $(-1)^7$ d. $(-1)^9$
19. **True or False** Justify your answer.
    a. $(-5)^3 = -5^3$ b. $(-5)^4 = -5^4$ c. $-(5^4) = -5^4$

20. a. Which powers of –1 are positive?

b. Which powers of –1 are negative?

c. If –1 was changed to –3, would this change your answers to Parts a and b? Explain why or why not.

21. The command "about-face" in the military signals a soldier to rotate 180 degrees. Two commands of "about-face" result in the soldier facing in the original direction again. How does the table at the right relate to $(-1)^n$, where n is the number of about-faces?

Number of About-Faces	Facing Direction
1	Reverse
2	Forward
3	Reverse
4	Forward
.	.
.	.
.	.

22. Determine whether the number is positive, negative, or zero.

a. $(-5)^{10}$ b. $(-1)(-5)^{10}$ c. $(-1)^{10}(-5)^{10}$

d. $(5)^{10}(-5)^{10}$ e. $[5 + (-5)]^{10}$ f. $(-1)^{10}(-5)$

23. ***Skill Sequence*** Evaluate each expression.

a. $-\frac{1}{2} \cdot -\frac{2}{3}$

b. $-\frac{1}{2} \cdot -\frac{2}{3} \cdot -\frac{3}{4}$

c. $-\frac{1}{2} \cdot -\frac{2}{3} \cdot -\frac{3}{4} \cdot -\frac{4}{5}$

d. $-\frac{1}{2} \cdot -\frac{2}{3} \cdot -\frac{3}{4} \cdot \ldots \cdot -\frac{9}{10}$

REVIEW

24. a. Pick a number and complete the number puzzle below.

Step 1 Subtract 1 from your number.

Step 2 Multiply this by 8.

Step 3 Add 20.

Step 4 Divide by 4.

Step 5 Subtract 5.

Step 6 Divide by 2.

Step 7 Add 1.

The result will be your original number.

b. Let n be your original number. Develop an algebraic expression for each step of the puzzle. (**Lesson 2-3**)

In 25–27, simplify the expression. (Lessons 2-2, 2-1)

25. $3(t + 5) + (4t - 7) + (-7t + 8)$

26. $2(2v - 11w) + 3(v + 7w)$

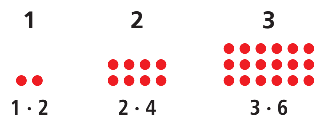
27. $\frac{7}{y} - \frac{4}{3y}$

In 28 and 29, use the following situation. A golf caddie who works at Pine Oaks Country Club makes $15 per golfer, but he has to pay the country club a $12 equipment fee each day.

28. Write an expression for the amount a golf caddie makes if he caddies for g golfers each day. (Lesson 1-1)

29. Write an expression for the amount a golf caddie makes after working 6 days, if he caddies for g golfers each day. (Lesson 2-2)

30. Consider the sequence made with dots below. Each term is 1 row and 2 columns larger than the previous term. (Lesson 1-2)



a. Complete the table describing the number of dots for the next three terms.

n	1	2	3	4	5	6
dots	$1 \cdot 2 = 2$	$2 \cdot 4 = 8$	$3 \cdot 6 = 18$	?	?	?

b. Write an expression for the number of dots required to make the *n*th term.

31. **a.** Find two numbers, each greater than 1, with a product of 728.

b. Find two numbers, each less than zero, with a product of 72.8.

c. Find two numbers, each greater than 1, with a product of 7.28. (Previous Course)

EXPLORATION

32. The difference of two numbers is subtracted from their sum. What can be said about the answer? Explain how you explored this problem.

Caddies carry clubs, replace divots, rake sand traps or bunkers, look for lost balls, and clean the player's club after each time it is used.

Source: www.teachingkidbusiness.com

QY ANSWERS

1. $-3.5, \frac{17}{8}$

2. $-a^2 - 2b$

3. yes