Lesson 2-3

Explaining Number Puzzles

BIG IDEA Algebra explains why many number puzzles work and how to invent them.

The number puzzle at the right was sent by e-mail. Try it yourself.

>From: <joe>
>Re: I can guess your age
>
>Follow the steps below, and I
>bet I can guess your age.
>
>1.) Write down your age.
>2.) Add 3.
>3.) Multiply by 5.
>4.) Subtract your age.
>5.) Add 9.
>6.) Divide by 4.
>7.) Subtract 6.
>8.) The number you got
> is your current age!

Mental Math

a. In $\triangle ABC$, $m\angle A = 35°$ and $m\angle B = 105°$. What is $m\angle C$?

b. In $\triangle LMN$, $m\angle L = x$ and $m\angle M = 74°$. What is $m\angle N$?

c. In $\triangle WYZ$, $m\angle W = r$ and $m\angle Y = s$. What is $m\angle Z$?

The number puzzle raises some questions. Does it work for other ages? Will it work if you count your age in months? Will it work for your great-grandfather Odell? Will it work for the baby a mother is expecting in six months, who the family fondly refers to as $-\frac{1}{2}$ year old because she is half a year before her birth? Will it work for any age?

Explaining the Puzzle

The steps shown below are for a 16-year-old student.

Step 1 Write your age.	16
Step 2 Add 3.	$16 + 3 = 19$
Step 3 Multiply by 5.	$19(5) = 95$
Step 4 Subtract your age.	$95 - 16 = 79$
Step 5 Add 9.	$79 + 9 = 88$
Step 6 Divide by 4.	$88 \div 4 = 22$
Step 7 Subtract 6.	$22 - 6 = 16$
The final result is the age.	**16**

Using algebra to explain the puzzle on page 79, we let the variable A stand for the age used in the first step. Now we follow the given directions, simplifying as we go by using the Distributive Property.

Step 1 Write your age. A

Step 2 Add 3. $A + 3$

Step 3 Multiply by 5. $5(A + 3) = 5A + 5 \cdot 3 = 5A + 15$

Step 4 Subtract your age. $5A + 15 - A = 4A + 15$

Step 5 Add 9. $4A + 15 + 9 = 4A + 24$

Step 6 Divide by 4. $\frac{4A + 24}{4} = \frac{4A}{4} + \frac{24}{4} = A + 6$

Step 7 Subtract 6. $A + 6 - 6 = A$

The algebra shows that any age works. The "trick" is really just an algebraic process. No matter what the original age is, the process will end with the same age from which you started.

The "Seven Is Heaven" Puzzle

In the next puzzle, the result is not the original number, but it is surprising in a different way.

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Example 1

Work the puzzle on the right with a few numbers. Then use a variable to create an expression to explain why the puzzle works.

Solution Follow the steps in the puzzle at the right.

Seven Is Heaven Puzzle	
Step 1	Pick a number.
Step 2	Add 1.
Step 3	Multiply by 2.
Step 4	Multiply by 3.
Step 5	Subtract 4.
Step 6	Add 5.
Step 7	Subtract 6 times your original number.
Your answer should be 7.	

Step 1 ___?___

Step 2 ___?___

Step 3 ___?___

Step 4 ___?___

Step 5 ___?___

Step 6 ___?___

Step 7 ___?___

Your answer should be 7.

We show the process beginning with any number n.

Step 1 Begin with a number.	Any number n
Step 2 Add 1.	$n + 1$
Step 3 Multiply by 2.	$2(n + 1) = 2n + 2$
Step 4 Multiply by 3.	$3(2n + 2) = 6n + 6$
Step 5 Subtract 4.	$6n + 6 - 4 = 6n + 2$
Step 6 Add 5.	$6n + 2 + 5 = 6n + 7$
Step 7 Subtract 6 times your original number.	$6n + 7 - 6n = 7$
Your answer should be 7.	7

Using Algebra to Create Number Puzzles

In Examples 2 and 3, we show how algebra can be used to create a number puzzle. This number puzzle will begin and end with the same number. Begin by choosing a variable to represent the starting number. A new expression is formed by performing an arithmetic operation on the existing expression. After several steps, carefully choose operations to return the expression to the variable.

Example 2

Create a number puzzle that begins and ends with the same number.

Solution We show the process and create one puzzle.

Changing

Begin with n.	n
Add 6.	$n + 6$
Multiply by 4.	$4(n + 6) = 4n + 24$
Divide by 2.	$\frac{4n + 24}{2} = \frac{4n}{2} + \frac{24}{2} = 2n + 12$
Subtract 19.	$2n + 12 - 19 = 2n - 7$

Returning

Add 7.	$2n - 7 + 7 = 2n$
Divide by 2.	$\frac{2n}{2} = n$
The answer equals n.	n

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Example 3

Create a number puzzle so that the answer always equals 6.

Solution

Step 1	Begin with *n*.	n
Step 2	Subtract 4.	?
Step 3	Multiply by 8.	$8(n - 4) =$?
Step 4	Add 8*n*	? = ?
Step 5	Divide by 16.	? = ?
Step 6	Add 8.	? = ?
Step 7	Subtract *n*.	? = ?

One of algebra's greatest strengths is the use of a variable to create algebraic expressions that can represent *all* the possibilities of a number puzzle, or even a real-life puzzle. This helps to explain mysteries and many other things in mathematics and the world.

Questions

COVERING THE IDEAS

1. Why would you use a variable in a number puzzle?

In 2–5, complete the number puzzle at the beginning of this lesson for the individual.

2. a person who is 50 years old
3. great-grandfather Odell, who is 97 years old
4. a baby expected to be born in half a year
5. a person who is 192 months old

In 6 and 7, complete the "Seven Is Heaven" puzzle on page 80 for a number *x* in the given range.

6. $250 < x < 500$
7. $0 < x < 1$
8. Will the "Seven Is Heaven" puzzle work for negative numbers? Explain why or why not.
9. Complete the "We're Number One!" puzzle at the right by starting with each number shown.
   a. 7 b. –2.9 c. *n*

We're Number One! Puzzle	
Step 1	Pick a number.
Step 2	Subtract 8.
Step 3	Add 7.
Step 4	Multiply by 6.
Step 5	Add 5.
Step 6	Add 4.
Step 7	Divide by 3.
Step 8	Subtract 2 times your original number.
Step 9	Your answer should be 1.

10. Complete the "Double Trouble" puzzle at the right by starting with each number shown.
    a. 17 b. 0.4 c. n

Double Trouble	
Step 1	Pick a number.
Step 2	Subtract 11.
Step 3	Multiply by 3.
Step 4	Add 5 times your original number.
Step 5	Add 1.
Step 6	Divide by 4.
Step 7	Add 8.
Step 8	Your answer should be twice your original number.

APPLYING THE MATHEMATICS

11. Create a number puzzle that begins and ends with the same number.

12. The "Seven Is Heaven" puzzle in Example 1 always ended with 7. Create an "Eight Is Great" puzzle that always ends with 8.

In 13 and 14, use the "magic square" at the right. In a magic square, the sums of any row, column, or diagonal are equal. In this magic square the sums are all 15.

6	7	2
1	5	9
8	3	4

13. Add 13 to every number in the magic square.
    a. Is the result still a magic square?
    b. By how much did the sum of the rows, columns, and diagonals change?
    c. Add k to each of the numbers in the magic square. Is the result still a magic square? Explain your answer using algebra.

14. Multiply every number in the magic square by 7.
    a. Is the result still a magic square?
    b. By how much did the sum of the rows, columns, and diagonals change?
    c. If you multiply each number in the magic square by m, will it still be a magic square? Explain your answer using algebra.

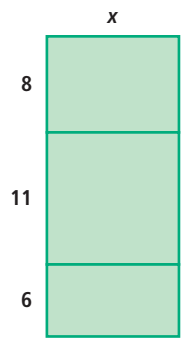
15. Refer to the card trick described on page 64. Make a table with 5 rows. In each row, give a number that the spectator might choose and the resulting number of cards in the small pile after Step 3 of the trick.

REVIEW

In 16–19, determine whether the terms are like or unlike. (Lesson 2-2)

16. $k, \frac{k}{6}$ 17. $-y, \frac{26}{y}$ 18. $22n, 22n^2$ 19. $5d^2, -d^{-2}$

20. The figure at the right consists of a big rectangle split into three rectangular parts. **(Lesson 2-2)**
    a. Write the area as length times width.
    b. Write the area as the sum of three areas.



21. Write each expression as a single fraction. **(Lesson 2-2)**

a. $\frac{3}{5} + \frac{4}{3}$ **b.** $\frac{x}{2} + \frac{2x}{7}$ **c.** $\frac{2}{5y} + \frac{4}{5y}$

In 22 and 23, use the Distributive Property to simplify the expression. Check your answer by substituting 5 for the variable in both the original expression and your answer. (Lesson 2-1)

22. $4(3(2x + 7))$ **23.** $n(11(3(n - 4)))$

24. A local coffee shop sells "Coffee Club" cups for \$1.99. With the cup, each additional refill of coffee only costs \$1.25. Let n = the number of refills purchased. **(Lesson 1-4)**

a. Make a table for 0, 1, 2, 3, 4, 5, and 6 refills purchased.

b. Graph your results from Part a.

c. Write an expression describing the cost of n refills.

25. Write an expression for the amount of money each person has or owes after w weeks. **(Lesson 1-1)**

a. Eddie is given \$100 and spends \$4 per week.

b. Liseta owes \$350 on a stereo and is paying it off at \$5 per week.

In 26–30, evaluate the expression. (Previous Course)

26. $-23 + 41$ **27.** $-42 - (-87)$ **28.** $(-8)(12)$

29. $(-13)(-6)$ **30.** $\frac{-24}{6}$

In the United States, it is estimated that as many as 70% of 18–24 year olds drink coffee in the morning.

Source: National Coffee Association of U.S.A., Inc.

EXPLORATION

31. The puzzle at the right involves a square array of numbers. Pick a number in the first row, then pick a number from the second row that is in a different column than the first number. Then choose the number in the third row that is not in either column as the first or second number. One example is 4, –24, and –15.

8	4	–20
–24	–12	60
6	3	–15

a. Multiply the three numbers together. What value do you get for the product?

b. Repeat the process. What do you notice?

c. The numbers in this puzzle at the right were chosen carefully. To see how, begin by replacing 3 with a and 4 with b. Then the other seven numbers can be expressed in terms of a and b. This has already been done for some cells in the diagram at the right. Fill in the remaining cells, and explain why the puzzle works.

?	b	$-5a$
?	$-ab$	?
$2a$	a	?