Lesson 2-2 The Distributive Property and Adding Like Terms

Vocabulary

like terms
coefficient
factoring

BIG IDEA By applying the Distributive Property, you can add or subtract like terms.

In Lesson 2-1, you learned that the Distributive Property can be used to remove parentheses by changing $c(a + b)$ into $ca + cb$. Now we will reverse the direction.

Adding Like Terms: From $ac + bc$ to $(a + b)c$

Algebraic expressions such as $x^2 + 3x + 5$ or $4a - 9b$ are made up of terms. The terms of $x^2 + 3x + 5$ are x^2, $3x$, and 5. Recall that a term is either a single number or a variable, or a product of numbers and variables. In an expression, addition separates terms. For instance, the terms of $4m^3 - 2m + 9.2m$ are $4m^3$, $-2m$, and $9.2m$. The terms of $-8k^2n + \frac{1}{3}k - 77$ are $-8k^2n$, $\frac{1}{3}k$, and -77.

The terms $-2m$ and $9.2m$ are called **like terms** because they contain the same variables raised to the same powers.

Like Terms	Unlike Terms
$3t$ and $40t$	$5t$ and $16t^2$ (different powers)
y^2 and $-19y^2$	$37x^3$ and y^3 (different variables)
$200u^5c^3$ and $8u^5c^3$	$9u^5c^3$ and $4u^{10}c$ (different powers)

Reversing the sides of the Distributive Property in Lesson 2-1 shows how to add or subtract like terms. For any real numbers a, b, and c, $ac + bc = (a + b)c$ and $ac - bc = (a - b)c$.

Here are the sums of two of the three pairs of like terms from the table above.

$$\begin{aligned} 3t + 40t &= (3 + 40)t \\ &= 43t \end{aligned} \qquad \begin{aligned} y^2 + (-19y^2) &= 1y^2 + -19y^2 \\ &= (1 + -19)y^2 \\ &= -18y^2 \end{aligned}$$

STOP QY

Mental Math

a. $-1(1 + -1)$

b. $-1(1 + -1(1 + -1))$

c. $-1(1 + -1(1 + -1(1 + -1)))$

QY

Find the sum of $200u^5c^3$ and $8u^5c^3$.

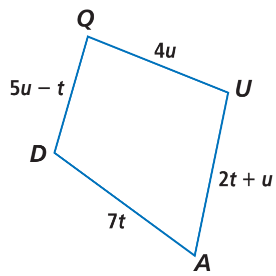
Notice that in combining $y^2 + -19y^2$, the first step was to rewrite y^2 as $1y^2$. Multiplying a number by 1 does not change its value. The following expression can be combined in a similar way.

$$\begin{aligned} 5n - n &= 5n - 1n \\ &= (5 - 1)n \\ &= 4n \end{aligned}$$

When there are two or more collections of like terms in an expression, you can group the like terms together using the Commutative and Associative Properties of Addition.

Example 1

Write a simplified expression for the perimeter of the quadrilateral *QUAD.*



Solution The perimeter is the sum of the lengths of the sides.

$4u + (2t + u) + 7t + (5u - t)$

Group like terms, changing subtraction to addition.

$= (2t + 7t + -t) + (4u + u + 5u)$

Combine like terms using the Distributive Property. Notice that u is the same as $1 \cdot u$ and $-t$ is the same as $-1 \cdot t$.

$= (2t + 7t + -1t) + (4u + 1u + 5u)$

Add like terms.

$= 8t + 10u$

Check Pick values for t and u. We pick 10 for t and 25 for u because they are easy numbers to multiply. The values of the original and simplified expressions must be equal. Substitute 10 for t and 25 for u into the initial expression.

$$\begin{aligned} 4u + 2t + u + 7t + 5u - t &= 4 \cdot 25 + 2 \cdot 10 + 25 + 7 \cdot 10 + 5 \cdot 25 - 10 \\ &= 100 + 20 + 25 + 70 + 125 - 10 \\ &= 330 \end{aligned}$$

Substitute 10 for t and 25 for u into the simplified expression.

$8t + 10u = 8 \cdot 10 + 10 \cdot 25 = 80 + 250 = 330$

The values of the original and simplified expressions are equal, so it checks.

A number that is a factor in a term is called a **coefficient** of the other variables in the term. For instance in $-8k^2n$, -8 is the coefficient of k^2n. You can see from Example 1 that *like terms are combined by adding the coefficients.*

It is very important to distinguish coefficients from exponents. For example, the Distributive Property applies to $3x + 11x$, but not to $x^3 + x^{11}$. To give a numerical example, $2 \cdot 10 + 3 \cdot 10 = (2 + 3)10 = 5 \cdot 10 = 50$, but $10^2 + 10^3 = 100 + 1{,}000$ or 1,100 is not equal to 10^5 or 100,000. *Different powers of the same number are not like terms.*

Sometimes the Distributive Property can be used twice to simplify an expression, first to remove parentheses and then to combine like terms.

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Example 2

Combine like terms for $(4f^2 + f + 9) + 10(12f - f^2 - 3)$.

Solution First remove parentheses.

$= 4f^2 + f + 9 + \underline{\ ?\ }f - \underline{\ ?\ }f^2 - \underline{\ ?\ }$

Group like terms, changing subtractions to additions.

$= [\underline{\ ?\ }f^2 + (\underline{\ ?\ }f^2)] + (\underline{\ ?\ }f + \underline{\ ?\ }f) + [\underline{\ ?\ } + (\underline{\ ?\ })]$

Add like terms.

$= \underline{\ ?\ }f^2 + \underline{\ ?\ }f + \underline{\ ?\ }$

Be sure to check that the simplified expression is equivalent to the original expression by substituting a number for f into both expressions and making sure they are equal.

Explaining the Addition of Fractions

Adding fractions with the same denominator is an example of adding like terms. Below are two sums to consider: one is numeric and the other is algebraic. Dividing by a number (5 or m) is the same as multiplying by its reciprocal $\left(\frac{1}{5} \text{ or } \frac{1}{m}\right)$.

$$\begin{aligned} \frac{3}{5} + \frac{4}{5} &= 3 \cdot \frac{1}{5} + 4 \cdot \frac{1}{5} \\ &= (3 + 4) \cdot \frac{1}{5} \\ &= 7 \cdot \frac{1}{5} \\ &= \frac{7}{5} \end{aligned} \qquad \begin{aligned} \frac{k}{m} + \frac{2}{m} &= k \cdot \frac{1}{m} + 2 \cdot \frac{1}{m} \\ &= (k + 2) \cdot \frac{1}{m} \\ &= \frac{k + 2}{m} \end{aligned}$$

If the fractions do not have a common denominator, they must be changed to equivalent fractions that have a common denominator before they can be considered as like terms.

Factoring

When $ac + bc$ is rewritten as $(a + b)c$, the original addition has been changed into a multiplication in which $(a + b)$ and c are factors. This process is called **factoring.** We say that c has been "factored out" of the expression. Notice that in this process, instead of removing parentheses, factoring introduces parentheses. For this reason, some people say that factoring "undoes" expansion.

Example 3

Factor 8 out of the expression $8x + 8y$.

Solution First write $8x + 8y = 8(\underline{\ ?\ } + \underline{\ ?\ })$. To get $8x$, there must be an x. To get $8y$, there must be a y.

$8x + 8y = 8(x + y)$

GUIDED

Example 4

Factor $3z^2$ out of the expression $3mz^2 - 6pz^2$.

Solution

$3mz^2 - 6pz^2 = 3z^2(\underline{\ ?\ } - \underline{\ ?\ })$

(*Hint:* Work backwards to see what $3z^2$ could be multiplied by to get each original term.)

Questions

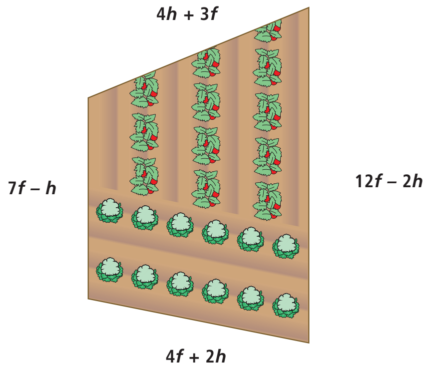
COVERING THE IDEAS

1. Determine whether the term and $8m^2$ are like terms.
   a. $8m$ b. $8x^2$ c. m^2 d. $3m^2$

In 2–7, combine like terms.

2. $\frac{7}{11} + \frac{5}{4}$
3. $19x + -7x$
4. $8y^3 + y^3 + 5y$
5. $(6x + 2y) + (2x + y)$
6. $(9m^2p + 5mp^2 + 8) + (-4m^2 + 7mp^2)$
7. $\frac{x + 1}{3m} + \frac{4}{3m}$
8. Group the six terms below into three pairs of like terms.
   $5m^3n^2, -8.9m, 68m^2n, -m^2n, m, \frac{4}{9}m^3n^2$

9. Gregory measured his garden using the lengths h and f of his hands and feet. Write a simplified expression for the perimeter of his garden below.



Fill in the Blanks In 10 and 11, factor the expression.

10. $16k - 16m = 16(\underline{\ ?\ } - \underline{\ ?\ })$

11. $2a^2 + 8b = 2(\underline{\ ?\ } + \underline{\ ?\ })$

In 12–14, simplify the expression.

12. $27 + 10(z + 3) + 8z$

13. $p(3p + 1) + 5p^2$

14. $2x + 5(x - 4 + 3x) + (2 - 2x)$

15. Factor 5 out of $15ab + 40c - 10$.

16. Factor $3y$ out of $9y^2 - 24xy$.

APPLYING THE MATHEMATICS

17. Frank, Susan, and Kazu collect stamps. Frank has f stamps, and Susan and Kazu each have 4 times as many stamps as Frank. Write an expression for the number of stamps they have altogether.

18. Some taxicab companies allow their drivers to keep $\frac{3}{10}$ of all the money they collect. The rest goes to the company. If a driver collects F dollars from the fares, write an expression for the company's share.

19. Find a counterexample to show that $6^x + 3^x = 9^x$ is not always true.

20. Does $6\sqrt{x} + 3\sqrt{x} = 9\sqrt{x}$ for all values of x? Why or why not?

Cabs contain a meter that indicates the fare based on the distance covered and other factors.

REVIEW

21. Timothy was asked to simplify $-4(-5 + 3n + -4m)$. His answer was $20 + 3n + -4m$. Write a note to Timothy explaining what he did wrong. **(Lesson 2-1)**

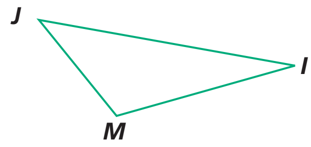
22. Malia went out to dinner with two friends. They lost track of the number of sodas ordered. But before the bill came, they figured they owed $3x + 48$ dollars, where x represented the number of drinks. If they split the bill evenly, what expression represents how much Malia owes? **(Lesson 2-1)**

In 23 and 24, explain how to use the Distributive Property to calculate the cost in your head. (Lesson 2-1)

23. 8 footballs at $39.95 each

24. 20 bottles of water at $0.99 each

25. Suppose JI is 5 centimeters more than JM, and IM is 7 centimeters less than twice JM. Recall that JI represents the distance from J to I. **(Lesson 1-2)**



a. Write an expression for the length of each side of the triangle if $x = JM$.

b. Write a simplified expression for the perimeter of $\triangle JIM$.

26. **a.** Describe the pattern below with one variable.

$$7^2 < 7^3$$

$$6.2^2 < 6.2^3$$

$$\left(\frac{11}{8}\right)^2 < \left(\frac{11}{8}\right)^3$$

b. Find an integer that is an instance of this pattern.

c. Find an integer that is a counterexample to the pattern.

d. Find a noninteger that is a counterexample to the pattern. **(Lesson 1-2)**

Matching In 27–30, match each algebraic expression to its English expression. (Lesson 1-1)

27. $2(x + 5)$	**a.** six less than a number
28. $2x + 5$	**b.** double the sum of five and a number
29. $x - 6$	**c.** five more than double a number
30. $6 - x$	**d.** take away a number from six

31. Suppose an apartment rents for $775 per month. Find the rent for the given time. **(Previous Course)**

a. 8 months

b. 3 years

c. 4.5 years

d. y years

e. m months

In 32 and 33, use the following information. The cost c of painting the four walls of a room is given by $c = \frac{p(4\ell w)}{300}$, where p is the price per gallon of paint and ℓ and w are the length and width of the room in feet. (Previous Course)

32. Find the cost of painting a bedroom with walls that are 16 feet by 8 feet and using paint that costs \$29.95 per gallon.

33. At \$18.99 per gallon, what is the cost of painting a living room with walls that are 24 feet by 10 feet?

In May 2004, the median hourly wage for house painters was \$14.55.

Source: Bureau of Labor Statistics

EXPLORATION

34. Consider this sequence of sums of increasingly large multiples of x whose coefficients are alternately positive and negative.

Step 1	x
Step 2	$x + -2x$
Step 3	$x + -2x + 3x$
Step 4	$x + -2x + 3x + -4x$
Step 5	$x + -2x + 3x + -4x + 5x$
⋮	

a. Simplify the five lines that are shown.

b. What will be the simplified form of the 25th line?

c. What will be the simplified form of the 100th line?

d. What is the simplified form of the nth line when n is even?

e. What is the simplified form of the nth line when n is odd?

QY ANSWER

$208u^5c^3$