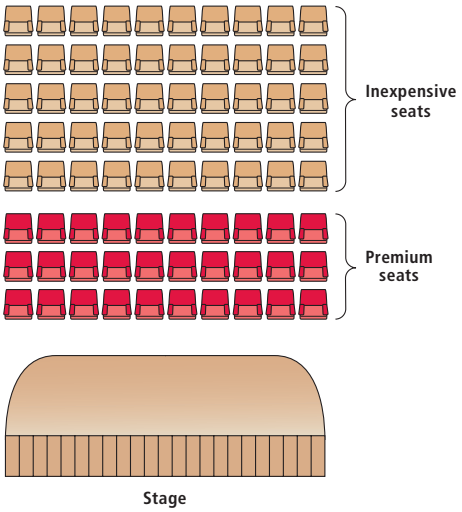
Lesson 2-1

The Distributive Property and Removing Parentheses

BIG IDEA By applying the Distributive Property, you can rewrite the product $a(b + c)$ as the sum $ab + ac$.

Suppose an auditorium has 8 rows of seats with 10 seats to a row. The tickets for the first 3 rows cost more than the tickets for the other 5 rows.



Two ways to count the number of seats in the auditorium illustrate a useful pattern. One way is to treat all the seats alike. Multiply the number of seats in each row, 10, by the total number of rows, 5 + 3 or 8.

$$10(5 + 3) = 10 \cdot 8 = 80$$

The second way is to count the number of inexpensive seats and premium seats separately, then add the results.

$$10 \cdot 5 + 10 \cdot 3 = 50 + 30 = 80$$

These two ways of counting the number of seats yield the same result.

$$10(5 + 3) = 10 \cdot 5 + 10 \cdot 3$$

This is an example of the basic property that involves both addition and multiplication. It is called the *Distributive Property of Multiplication over Addition* since the multiplication by 10 in $10(5 + 3)$ is "distributed" to both terms in the parentheses.

Mental Math

a. Order from least to greatest: $\frac{5}{32}, \frac{1}{4}, \frac{1}{8}, \frac{3}{16}$.

b. Order from least to greatest: $-\frac{5}{32}, -\frac{1}{4}, -\frac{1}{8}, -\frac{3}{16}$.

The Distributive Property of Multiplication over Addition

For all real numbers a, b, and c, $c(a + b) = ca + cb$.

The name of this property is very long, so we just call it the *Distributive Property*. The Distributive Property can be used to rewrite the product $c(a + b)$ as the sum of terms $ca + cb$. This is called expanding the expression. Expanding a product has the effect of removing parentheses.

STOP QY1

QY1

Use the Distributive Property to expand $6(x + 5)$.

Example 1

Jason earns money by mowing the lawns of three houses in his neighborhood each week. Since the lawns are of different sizes, he charges the neighbors different prices: *a*, *b*, and *c*. He mows the second neighbor's lawn twice a week, so each week Jason earns $a + 2b + c$ dollars. The neighbors pay monthly. Assuming 4 weeks per month, Jason earns $4(a + 2b + c)$ dollars each month. Use the Distributive Property to give an expression that is equivalent to $4(a + 2b + c)$.

Solution Distribute 4 over each term. Then simplify.

$$4(a + 2b + c) = 4 \cdot a + 4 \cdot (2b) + 4 \cdot c$$
$$= 4a + 8b + 4c$$

Check The expression $4a + 8b + 4c$ indicates that Jason has mowed the first lawn 4 times, the second lawn 8 times, and the third lawn 4 times during the month.

Recycling grass clippings into lawns lowers soil temperature, reduces water loss, and reduces yard waste going into landfills.

Source: Servicemaster

Explaining a Multiplication Shortcut

Suppose a motel room costs \$59 a day, and a person will stay in that room for 8 days. To find the total cost, you can multiply $8 \cdot 60$, and then subtract $8 \cdot 1$. (Calculate as if the price for each of the eight days was \$60. Then subtract \$1 per day for 8 days.) The Distributive Property explains why this works.

$$8 \cdot 59 = 8(60 - 1) = 8 \cdot 60 - 8 \cdot 1, \text{ or } \$472$$

Here we have distributed the multiplication over a *subtraction*. Since $60 - 1 = 60 + (-1)$, the subtraction can be thought of adding the opposite. Some people like to think of this variant of the Distributive Property as a separate property.

The Distributive Property of Multiplication over Subtraction

For all real numbers a, b, and c, $c(a - b) = ca - cb$.

Thus, there are two forms of the Distributive Property used to expand expressions. You can use either of these versions to expand a subtraction expression, as shown in Example 2 on page 68.

Example 2

Expand $-11(5 - 6w)$.

Solution Begin by rewriting the subtraction expression as an addition expression.

$$\begin{aligned} -11(5 - 6w) &= -11(5 + (-6w)) \\ &= -11 \cdot 5 + -11 \cdot -6w \\ &= -55 + 66w \end{aligned}$$

Check Substitute the same value for w in both the given expression and the expanded expression. We use $w = 3$. Remember to follow the order of operations.

When $w = 3$, $-11(5 - 6w) = -11(5 - 6 \cdot 3) = -11(-13) = 143$.
When $w = 3$, $-55 + 66w = -55 + 66 \cdot 3 = -55 + 198 = 143$.
It checks.

GUIDED

Example 3

Expand $2x(5x - 3)$.

Solution

$2x(5x - 3) = 2x \cdot \underline{\ ?\ } - 2x \cdot \underline{\ ?\ }$ Distributive Property

$= \underline{\ ?\ } - \underline{\ ?\ }$ Multiplication

The Distributive Property also works in the cases where the multiplier is on the right, as in $(a + b)c$, because multiplication is commutative. So, $(a + b)c = c(a + b) = ca + cb$.

Expanding a Fraction

Because every division can be converted to multiplication, the Distributive Property can also be used to rewrite expressions involving division. Suppose the sum $(a + b)$ is to be divided by c.

$\frac{a+b}{c} = \frac{1}{c}(a + b)$ Dividing by c is the same as multiplying by $\frac{1}{c}$.

$= \frac{1}{c} \cdot a + \frac{1}{c} \cdot b$ Distributive Property of Multiplication over Addition

$= \frac{a}{c} + \frac{b}{c}$ Algebraic definition of division

In this way, a fraction with a sum in its numerator can be rewritten using $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$. This step may allow you to simplify an expression, as shown in Example 4.

Example 4

Use the Distributive Property to write $\frac{36 + 3x}{18}$ as a sum of two fractions.

Solution

$$\frac{36 + 3x}{18} = \frac{36}{18} + \frac{3x}{18} = 2 + \frac{x}{6}$$

Check Let $x = 6$. (Do you see why we chose 6?)

Then $\frac{36 + 3x}{18} = \frac{36 + 3 \cdot 6}{18} = \frac{36 + 18}{18} = \frac{54}{18} = 3$.

Also $2 + \frac{x}{6} = 2 + \frac{6}{6} = 3$. It checks.

STOP QY2

▶ **QY2**

Multiple Choice For all x, $\frac{20 - 8x}{4}$ equals which of the following?

A $5 - 8x$

B $3x$

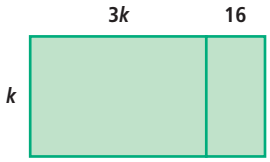
C $5 - 2x$

D $3 - 8x$

Questions

COVERING THE IDEAS

1. Use the Distributive Property to find equivalent expressions.
   a. $n(k + w)$ b. $g(d - e)$ c. $\frac{n + p}{r}$
2. Write an expression for the area of the largest rectangle below in two ways.



   a. as length times width b. as the sum of areas
3. A person buys 45 envelopes at \$1.03 each. Explain how you can use the Distributive Property to calculate the total cost in your head.
4. Calculate in your head the total width of 5 windows that are each 39 inches wide.

In 5–10, expand the expression.

5. $(m + 4)5$
6. $-30x(x + 2 + 4n)$
7. $12(k - \frac{1}{6})$
8. $(2b + c)10b$
9. $6(3v - 8w + 9z^3)$
10. $-7a(a - b)$
11. Suppose the cost of a cell phone call is \$0.12 per minute. Two calls are made. One lasts 17 minutes and the other lasts 6 minutes. Find the total cost of the calls in two different ways.
12. Rewrite $\frac{24 + 6x}{8}$ as the sum of two fractions. Check your answer.

APPLYING THE MATHEMATICS

In 13 and 14, complete each sentence to show examples of the Distributive Property.

13. $24(k + m) = 24\_\_?\_\_ + 24\_\_?\_\_$

14. $10a + 80 = 10(\_\_?\_\_ + \_\_?\_\_)$

15. For each hour of television, there is an average of $13\frac{1}{2}$ minutes of commercials. If you watch 9 hours of television in a week, how many minutes of commercials will you see? Explain how you can find the answer in your head.

16. You and your friend decide to start a dog-walking business and plan to charge \$12 for each dog walked.
    a. You are scheduled to walk 13 dogs this week and your friend has scheduled 17 more appointments. Determine the total amount of money you will earn using two different methods.
    b. You are scheduled to walk 13 dogs this week. If your friend schedules x more appointments for the week, how much money will you earn?

20% of owned dogs were adopted from an animal shelter.

Source: www.thepetprofessor.com

REVIEW

17. The table below gives the number of films (up to 2006) in which the ten top-ranked actresses have starred. **(Lesson 1-6)**

Actress	Number of Films
Cate Blanchett	34
Patricia Clarkson	52
Toni Collette	32
Kirsten Dunst	52
Scarlett Johansson	27
Nicole Kidman	46
Julianne Moore	52
Samantha Morton	32
Michelle Pfeiffer	46
Kate Winslet	30

Source: Internet Movie Database

    a. Find the mean number of films.
    b. Find the mean absolute deviation. Explain what the mean absolute deviation means in this case.

18. Graph $y = (0.65x)^3$ and $y = 2^x$ on a graphing calculator with the following window settings: $0 \le x \le 60$ and $5 \le y \le 45$. **(Lessons 1-5, 1-3)**

a. Create a table of values for both graphs with $x = -10, -5, 0, 5, 10$, and 15.

b. Are the expressions equivalent? Why or why not?

In 19 and 20, use the Associative and Commutative Properties of Multiplication to compute in your head. (Lesson 1-1)

19. $2 \cdot 11 \cdot 3 \cdot 1.5$

20. $8 \cdot 2 \cdot 7 \cdot 5$

21. A raffle to raise money for charity sells tickets for \$4 each. The winner of the raffle will receive half of all the money raised from selling the tickets, and the other half will be given to the charity. If 721 tickets are sold, how much money will be given to the charity? **(Lesson 1-1)**

In 22 and 23, which of the numbers –2, 5, and 8 makes the sentence true? (Lesson 1-1)

22. $12 + 3n - 4 < 7$

23. $11 \ge -3y + \frac{15}{3}$

24. According to a study by the Kaiser Family Foundation, typical American teenagers spent an average of 12.5% of their days watching television in 2004. **(Previous Course)**

a. How many hours does this represent in a day?

b. How many hours does this represent in one week?

c. How many days does this represent in one year?

Teens spend the majority of their TV-watching time between the hours of 8 P.M. and 11 P.M.

Source: Nielsen Media Research

EXPLORATION

25. Some people overgeneralize the Distributive Property. They think that because $6x + 3x = 9x$, both of the following must be true. Find a counterexample to each equation to show that it is false.

a. $6x \cdot 3x = 18x$

b. $\frac{6x}{3x} = 2x$

QY ANSWERS

1. $6x + 30$

2. C