Chapter 1 Summary and Vocabulary

In the columns at the right are the most important terms and phrases for this chapter. You should be able to give a general description and a specific example of each and a precise definition for those marked with an asterisk (*).

- The language of algebra is based on numbers and **variables**. These are combined in **expressions**, and two or more expressions connected by a verb make up an **algebraic sentence**.

- **Formulas** are equations stating that a single variable is equal to an expression with one or more variables on the other side. Formulas are evaluated and can be rewritten using the rules for the **order of operations** and properties of equality.

- A **function** is defined as a set of ordered pairs in which each first coordinate can be paired with exactly one second coordinate. The first coordinate of the ordered pair is the **independent variable**; the second coordinate is the **dependent variable**. Thus, a function pairs two variables in such a way that each value of the independent variable corresponds to exactly one value of the dependent variable. The **domain** of a function is the set of possible values for the independent variable, while the **range** is the set of values obtained for the dependent variable. Many functions are **mathematical models** of real situations or other mathematical properties.

- Functions can be used to represent situations where one value of an ordered pair is known and the other unknown. They can also be used to describe situations in which objects are transformed or otherwise mapped onto other objects. Each of these types of situations may arise in everyday contexts and in all branches of mathematics.

- Functions are often named with a single letter. Euler's **$f(x)$ notation** $f(x) = x + 20$, and the **mapping notation** $f: x \rightarrow x + 20$, both describe the same function. Graphing utilities use $f(x)$ notation and often name functions y_1, y_2, y_3, and so on. A CAS defines functions with $f(x)$ notation, but also accepts equations and formulas that are not solved for one variable.

Vocabulary

1-1
*variable
algebraic expression
expression
algebraic sentence
evaluating an expression
order of operations
vinculum
*equation
*formula

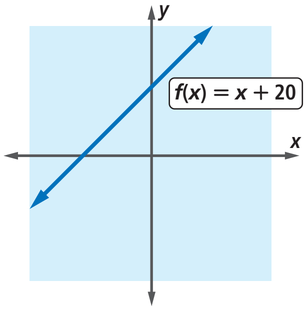
1-2
relation
*function
*independent variable
*dependent variable
input, output
is a function of
maps
mathematical model

1-3
*$f(x)$ notation
argument of a function
value of a function
mapping notation

1-4
real function
*domain of a function
*range of a function
set builder notation
trace
standard window
*natural numbers
*counting numbers
*whole numbers
*integers
rational numbers
real numbers
irrational numbers

- Along with words and formulas, functions are also represented by tables and graphs.

x	y
–2	18
–1	19
0	20
1	21
2	22



- A **sequence** is a function whose domain is the set of all positive integers or the set of positive integers from a to b. Sequences may be defined with an **explicit formula** in which the nth **term** can be calculated directly from n. In a sequence S, the nth term is identified as S_n.

Theorems and Properties

Distributive Property p. 41
Opposite of a Sum Theorem p. 44

Vocabulary

1-7
equivalent formulas

1-8
*sequence
term of a sequence
subscript
index
*explicit formula
discrete function

Chapter 1 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

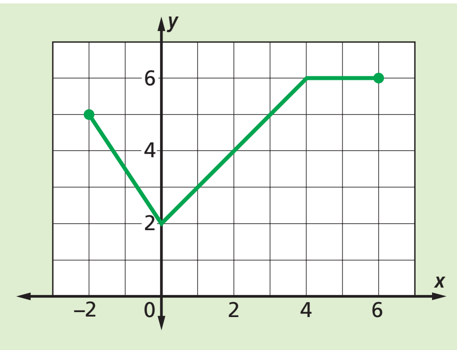
1. If $f(x) = x^2 - 4^x$, find $f(-1.5)$ to the nearest hundredth.

2. Suppose that f is a function and a is a real number. Describe the difference in the statements "$f(a) = 7$" and "$f(7) = a$."

3. The table below defines the function g. Find $g(-2)$ and $g(2)$.

r	–1	–2	–1	0	1	2	3
$g(r)$	6	17	15	2	–4	3	–2

In 4 and 5, let f be the function graphed below.



4. Find the domain and range of f.

5. Use the graph to estimate each of the following:
   a. $f(3)$
   b. All values of x such that $f(x) = 4$.

6. Suppose $S: n \rightarrow \frac{n}{2}(3(n - 1))$. Evaluate $S(10)$.

7. Which formula below is solved for c?
   A $A = \frac{1}{2}c_1c_2$
   B $c = 2\pi r$
   C $a^2 + b^2 = c^2$

8. Suppose you travel m miles in 4 hours. Write a formula that gives your speed s in miles per hour.

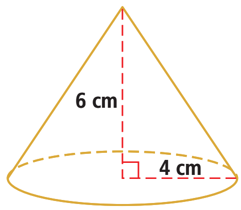
In 9–11, solve for the variable.

9. $5(7x - 4) = 50$

10. $3.2a = 0.75 + 1.2a$

11. $p + 0.2(1200 - p) = 244.4p$

In 12–14, use the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.



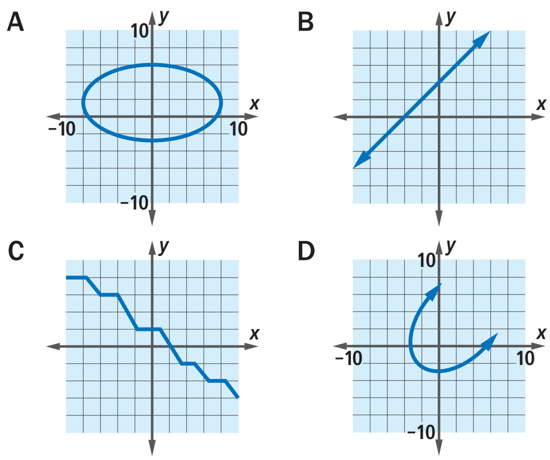
12. Find the volume of the cone at right to the nearest cubic centimeter.

13. Suppose $h = 6$ and the domain of the volume function is $0 < r \leq 10$. What is the range of the function?

14. a. Solve this formula for h in terms of V and r.
    b. Solve this formula for r in terms of V and h.

15. A function f contains only the points (0, 7), (4, 8), (8, 9) and (16, 11). Give its domain and range.

16. **Multiple Choice** Which of the following are graphs of functions?



In 17 and 18, determine whether the relation is a function. Justify your answer.

17. $\{(9, 7), (7, 5), (18, 12), (9, 4)\}$

18.

x	0	1	2	4
y	0	2	3	3

(Consider x to be the independent variable.)

19. a. Write the first five terms of the sequence $a_n = -4^n + 2$.

b. What is a_7?

20. Use a graphing utility to graph the function $f(t) = 6 - t^2$ with a domain $\{t | -5 \le t \le 5\}$.

a. What is the range of this function?

b. To the nearest hundredth, estimate where $f(t) = 0$.

21. Create a table for $y = -3 \cdot \left(\frac{1}{2}\right)^x$ with a start value of $x = -6$, a table increment of 2 and, if necessary, an end value of 2.

a. List the first five ordered pairs in the table.

b. For what value of x does $y = -0.75$?

22. The following table of values represents the wind chill index for various temperatures when there is a 10 mph wind. Use the table to answer the following questions.

A = Actual temperature (°F)	30	20	10	0	–10	–20	–30
$W(A)$ = Wind-chill index (°F)	21	9	–4	–16	–28	–41	–53

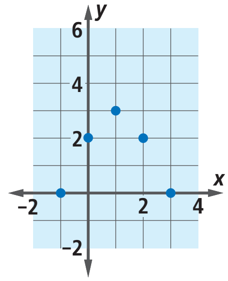
Source: National Oceanic and Atmospheric Administration

Let A = the actual temperature and let W be the name of the function that maps A onto the wind-chill index.

a. Evaluate $W(10)$ and explain what it means in terms of the wind chill.

b. For what value of A does $W(A) = -41$?

23. Refer to the graph at the right.



a. List the ordered pairs in the graph.

b. Is this a function? Explain why or why not.

c. State the domain and range.

24. The population, $P(y)$ in millions, of a certain city is modeled by the function with equation $P(y) = 2 \cdot 1.005^y$, where y is the year minus 2000 (for example, in 2006, $y = 6$). How many more people will be living in the town in the year 2020 than in 2006?

25. Multiple Choice What CAS command would you use to rewrite the expression $(a^2 + 1)(a + 1)(a - 1)$?

A `SOLVE()`　　B `FUNCTION()`

C `EXPAND()`　　D `DEFINE()`

26. The probability of tossing a six-sided die and rolling a 3 n times in a row can be represented by the sequence $p_n = \left(\frac{1}{6}\right)^n$. What is the probability of rolling a 3 four times in a row?

27. On a recent flight between St. Louis and Los Angeles, $\frac{1}{4}$ of the tickets sold were for seats in business class, $\frac{1}{10}$ in first class, and the remaining 195 tickets were in coach class. How many tickets were sold altogether?

28. The Pythagorean Theorem states that if a and b are the legs of a right triangle and c is its hypotenuse, then $a^2 + b^2 = c^2$. What happens to the value of c if you switch the values of a and b?

Chapter 1 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SPUR stands for Skills, Properties, Uses, and Representations. The Chapter Review Questions are grouped according to the SPUR Objectives in this chapter.

SKILLS Procedures used to get answers

OBJECTIVE A **Evaluate expressions and formulas, including correct units in answers.** (Lesson 1-1)

1. Evaluate $100 \cdot (1 + r)^t$ when $r = 0.06$ and $t = 3$.
2. If $m = \frac{4y(y+2)}{9}$, find m when $y = 16$.
3. If $d = \frac{1}{2}gt^2$, find d when $g = 9.8\frac{\text{m}}{\text{sec}^2}$ and $t = 3$ sec.
4. Evaluate $\frac{n}{2^{2(n-1)}}$ when $n = 4$.

OBJECTIVE B **Use mapping and $f(x)$ notation for functions.** (Lesson 1-3)

5. If $f(x) = -5x + 10$, what is $f(5)$?
6. **Fill in the Blank** Suppose $g: t \rightarrow t - 2t^3$. Then $g: -1 \rightarrow$ __?__.

In 7 and 8, a function is described by an equation.

a. Rewrite the function in mapping notation.

b. Evaluate the function at $x = 9$.

7. $h(x) = 12x - 2\sqrt{x}$
8. $j(x) = 3 - 4 \cdot 2^x$

In 9 and 10, a function is given in mapping notation.

a. Rewrite the function in $f(x)$ notation.

b. Evaluate the function at $x = 8$.

9. $f: x \rightarrow 4 - 27x$
10. $c: x \rightarrow \frac{\sqrt{2x}}{5}$

OBJECTIVE C **Solve and check linear equations.** (Lesson 1-6)

In 11–18, solve and check.

11. $12x = \frac{4}{5}$
12. $\frac{3}{8}(b + 5) = 9$
13. $-\frac{5}{3} = \frac{4}{3} - v$
14. $L = 2L - (6 - 6L)$
15. $-s + 5s = 4(2s + 3)$
16. $0.02(800z + 50) = -500 - 24z$
17. $\frac{m}{8} + \frac{m}{6} - 2 = -3m$
18. $y - 8(y + 5) = 13y$

OBJECTIVE D **Solve formulas for their variables.** (Lesson 1-7)

19. Solve for t in the formula $x = 12 - 6t$.
20. Solve for q in the formula $E = \frac{kq}{r^2}$.
21. Solve for h in the formula $A = \frac{1}{3}(\pi r_1^2 h - \pi r_2^2 h.)$
22. Solve for n in the formula $t = a + (n - 1)d$.
23. Explain why the equation $s = \frac{4\pi}{\sqrt{2bds}}$ is not solved for s.
24. **Multiple Choice** Which of the following is not a formula solved for the volume, radius, or height of a cylindrical barrel?

A $V = hr^2 \cdot \pi$ B $\frac{V}{\pi r^2} = h$

C $r^2 = \frac{V}{h\pi}$ D $\sqrt{\frac{V}{\pi h}} = r$

OBJECTIVE E **Find terms of sequences.** (Lesson 1-8)

In 25 and 26, write the first five terms of the sequence. These sequences are defined for all integers $n \geq 1$.

25. $a_n = -3 - 6n$
26. $b_n = -5 \cdot 2^n$
27. If $c_n = n^2 - n$, find $c_4 + c_3$.
28. When $d_n = (-0.5)^n$, find $3d_2 + d_1$.

OBJECTIVE F **Use a CAS to solve equations or expand expressions.** (Lesson 1-7)

29. Use the SOLVE() function on your CAS to solve the formula for the volume of a cone, $V = \frac{1}{3}\pi r^2 h$, for r. What two solutions does it give? Which one is the correct formula for r?

In 30 and 31, suppose that while solving an equation for the variable c on a CAS, you encounter the intermediate equation $5 + 5r + ab = 5c$.

30. What should you enter next? Finish solving this equation on your CAS.

31. **Multiple Choice** A student solving the equation obtains the CAS result $c = \frac{1}{5}(ab + 5r + 5)$. What CAS command will yield a simplified solution?

A ANS*5 B EXPAND(ANS)
C ANS/5 D SOLVE(ANS, c)

PROPERTIES Principles behind the mathematics

OBJECTIVE G **Determine whether a given relation is a function.** (Lessons 1-2, 1-5)

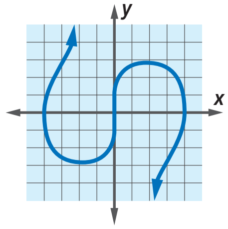
In 32–33, determine whether the given table or set of ordered pairs describes y as a function of x.

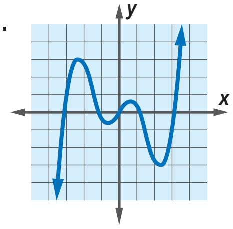
32.

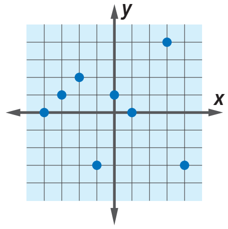
x	1	3	5	7	9
y	3	5	7	9	1

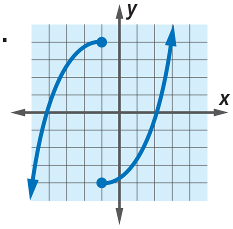
33. $\{(0, 3), (1, -2), (2, -1), (1, 2), (-1, 5)\}$

In 34–37, determine whether the given graph represents the graph of a function. Justify your answer.

34.


35.


36.


37.


OBJECTIVE H **Determine the domain and range of a function.** (Lessons 1-4, 1-5)

In 38–43, find the domain and range of the function described by the given table, set of ordered pairs, equation, or graph.

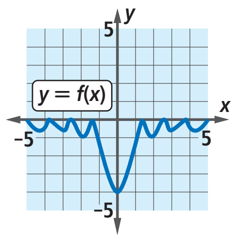
38.

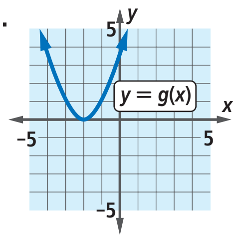
t	2	3	5	7	11
$e(t)$	1	2	3	4	5

39. $\{(0, -1), (1, -1), (2, -1), (-1, -1), (-2, -1)\}$

40. $f(x) = x^2$

41. $g(y) = 3y^5$

42.


43.


44. From the graph in Question 42, find the approximate values of x for which $f(x) = 0$.

45. What is $g(-3)$ in Question 43?

46. Graph the function $h(x) = (x - 2)^2$ on a grapher using a standard window, and use the graph to determine the function's domain and range.

OBJECTIVE I **Describe relationships between variables in a formula.** (Lesson 1-7)

47. Isaac Newton's Law of Universal Gravitation states that every object exerts a gravitational force on every other object. The formula $F = \frac{Gm_1m_2}{r^2}$ gives the magnitude of the gravitational force F that a body of mass m_1 exerts on a body of mass m_2 if they are a distance r apart. G is the gravitational constant. What happens to the value of F if m_1 and m_2 are switched? What does this imply about the force that the body of mass m_2 exerts on the body of mass m_1?

48. Rewrite the formula $V = \ell wh$, for the volume of a rectangular box with length ℓ, width w, and height h, in terms of ℓ when $w = \ell = h$. What situation does this formula represent?

49. **Multiple Choice** Given $x \neq 0$, $y \neq 0$, and $k \neq 0$, which of the following formulas is equivalent to the formula $y = \frac{k}{x}$?

A $k = \frac{y}{x}$ B $x = ky$ C $y = \frac{x}{k}$

D $x = \frac{k}{y}$ E $k = \frac{x}{y}$

USES Applications of mathematics in real-world situations

OBJECTIVE J **Evaluate and interpret values of functions in real-world situations.** (Lessons 1-3, 1-4)

50. Michelle's annual salary is $43,000. She gets an increase of 8% at the end of each year. The sequence $a_n = 43{,}000 \cdot 1.08^n$ gives Michelle's salary in the nth year. At this growth rate, what will Michelle's salary be in the 5th, 10th, 15th, and 20th years?

51. The formula $S(x) = x + \frac{x^2}{20}$ relates a car's stopping distance $S(x)$ in feet to the car's speed x in miles per hour. Determine how many more feet it takes a car traveling at 85 mph to stop than a car traveling at 55 mph.

52. The distance in feet $d(t)$ a dropped object falls in t seconds is given by the function $d(t) = 16t^2$. Suppose you drop a ball from a height of 60 feet.

a. About how far will the ball fall in 1.3 seconds?

b. Find $d(2)$. Explain why this result does not make sense in this situation.

53. Janelle opened a savings account with $200. Each month she adds $35 to her account.

a. Create a table with the amounts in her account for the first six months.

b. Write a formula for the sequence giving her savings account balance at the end of each month.

In 54–56, let $M(x)$ and $W(x)$ be the populations of Minnesota and Wisconsin, respectively, in the year x.

Year x	Population of Minnesota $M(x)$	Population of Wisconsin $W(x)$
1970	3,806,103	4,417,821
1980	4,075,970	4,705,642
1990	4,375,099	4,891,769
2000	4,919,479	5,363,675

Source: U.S. Census Bureau

54. What does $M(1970)$ represent?

55. Calculate $W(1980) - M(1980)$. What does this represent?

56. Calculate $W(2000) - W(1990)$. What does this represent?

OBJECTIVE K **Use linear equations to solve real-world problems.** (Lesson 1-6)

57. Suppose a baby blue whale weighs 4,000 lb at birth and gains 200 lb a day while nursing. A formula that gives its weight W after d days of nursing is $W = 4000 + 200d$.
    a. Write an equation that can be used to find the number of days d a young blue whale has been nursing if it weighs W lb.
    b. Use your answer to Part a to find d for $W = 14{,}000$.

58. At George Washington High School, all students are in grade 10, 11, or 12. This year $\frac{3}{10}$ of the students are in grade 10, 400 students are in grade 11, and $\frac{1}{5}$ of all students are in grade 12. How many students are at George Washington High this year?

59. Cara wants to park her car in a parking lot that charges \$6 for the first hour and \$1.50 for each additional hour. So the cost c in dollars to park for h hours is $c = 6 + 1.5(h - 1)$. How long can Cara park if she has \$20? (Parking time is always rounded up to the next hour, so if she parks for 70 minutes, she must pay for 2 hours.)

60. Frederick is paying off a \$3,000 loan in installments of \$160 per month. So the amount left to pay in dollars after m months will be $3000 - 160m$. After how many full months will Frederick be out of debt?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE L **Use a graphing utility to graph functions and generate tables for functions.** (Lessons 1-4, 1-5, 1-8)

In 61 and 62, a function is given.

a. Use a graphing utility to graph f in the standard window.

b. Use your graph to find all values of x when $f(x) = 0$.

61. $f(x) = x^3 - 8x^2 + 19x - 12$

62. $f(x) = -2 + 2^2 - x^2$

63. Use a graphing utility to graph A when $A(t) = 50 \cdot 1.04^t$, with the domain restricted to $\{t |\ t \geq 0\}$. What is the range of this function?

64. Use a graphing utility to graph B when $B(t) = 50 \cdot 0.96^t$, with the domain restricted to $\{t |\ t \geq 0\}$. What is the range of this function?

In 65 and 66, a function f is given.

a. Create a table for f with a start value of 1 and a table increment of 0.2. List the first five ordered pairs in the table.

b. Give a value of x for which f(x) is within 0.05 of -0.4.

c. **Multiple Choice** What appears to be true of $f(x)$?

A $f(x)$ is always positive.

B $f(x)$ is always negative.

C $f(x)$ starts off positive and becomes negative.

D none of the above

65. $f(x) = 2^x - 4^x + 3^x$

66. $f: x \rightarrow \frac{2x - 1}{6 - 5x}$