Lesson 1-7 Rewriting Formulas

Vocabulary

equivalent formulas

BIG IDEA Every formula defines one variable in terms of other variables; by using equation-solving properties you can manipulate a formula so that any one of the variables is defined in terms of the rest of the variables.

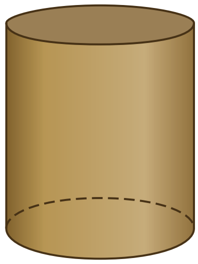
Mental Math

The 180-member marching band is going on a field trip. Suppose that one bus can transport 50 students.

a. How many buses are needed to transport the band members?

b. One chaperone for every 10 students is going on the trip. Now how many buses are needed?

c. The instruments take up 15 seats on one of the buses. Now how many buses are needed?



The volume V of a cylindrical barrel is given by the formula $V = \pi r^2 h,$ where r is the barrel's radius and h is the barrel's height. This formula gives the volume V in terms of r and h. If you have values for r and h, you can use this formula to calculate V. For instance, if a barrel has radius 30 cm and height 100 cm, then

$$\begin{aligned} V &= \pi(30 \text{ cm})^2(100 \text{ cm}) \\ &= 90{,}000\pi \text{ cm}^3 \\ &\approx 283{,}000 \text{ cm}^3. \end{aligned}$$

In some situations it is useful to solve a formula for one of the other variables. Example 1 illustrates how to convert a formula to a more useful form.

Example 1

Justin Case is an engineer for a barrel manufacturing company. He wants to design a cylindrical barrel to contain 500,000 cm^3 of liquid and is considering several different radii and heights. To do this, it is useful to solve the volume formula for h in terms of r.

a. Solve the formula $V = \pi r^2 h$ for h in terms of V and r.

b. Find the height of a barrel with radius 30 cm that will contain 500,000 cm^3 of liquid.

(continued on next page)

Solution 1

a. Solve for h in terms if V and r.

$V = \pi r^2 h$ Original formula

$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$ Divide both sides by πr^2. (You can do this because $r \neq 0$.)

$\frac{V}{\pi r^2} = h$ Simplify.

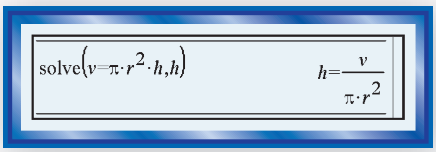
b. Substitute the given values for V and r.

$h = \frac{V}{\pi r^2} = \frac{500{,}000 \text{ cm}^3}{\pi(30 \text{ cm})^2} \approx 176.8 \text{ cm}$

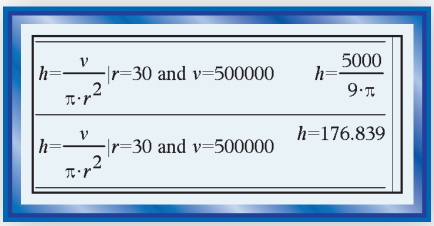
The barrel needs to be about 180 cm high to contain 500,000 cm^3 of liquid.

Solution 2

a. Use the SOLVE( ) command on a CAS to solve for h. One CAS display is shown below. Entering ",h" after the formula in the command line tells the CAS to solve for h.



b. Substitute the given values for V and r to find h. We use the SUCH THAT command and the AND connector to enter both variable values at once. The display below shows both an exact answer and an approximate answer.



See Quiz Yourself 1 at the right.

The formulas $V = \pi r^2 h$ and $h = \frac{V}{\pi r^2}$ are *equivalent formulas* because any V, r, and h that satisfy one of them also satisfies the other. Two formulas are **equivalent formulas** when the values of the variables that satisfy them are the same.

QUIZ YOURSELF 1

Find h when $V = 500{,}000 \text{ cm}^3$ and $r = 32$ cm.

Example 2

The formula $c = \frac{65t}{r - 65}$ computes the number c of hours needed for one car driving r miles per hour to catch up to another car that is t hours ahead and driving at 65 mph. Solve the formula for t in terms of c and r.

Solution Solve the formula for t by hand. Compare these steps to the computations on a CAS.

By hand		On a CAS
$c = \frac{65t}{r - 65}$	Write down the formula (at left) and enter it on a CAS (at right).	$c=\frac{65\cdot t}{r-65}$ $\quad$ $c=\frac{65\cdot t}{r-65}$
$c \cdot (r - 65) = 65t$	Multiply each side by $(r - 65)$. You may see the word ANS on your screen before you push ENTER. ANS refers to the last thing displayed on the CAS. In this case, it is the whole formula for c.	$\left(c=\frac{65\cdot t}{r-65}\right)\cdot(r-65)$ $\quad$ $c\cdot(r-65)=65\cdot t$
$cr - 65c = 65t$	Use the Distributive Property. The CAS shown here does this by using the EXPAND( ) command.	$\text{expand}(c\cdot(r-65)=65\cdot t)$ $\quad$ $c\cdot r-65\cdot c=65\cdot t$
$\frac{cr - 65c}{65} = t$	Divide each side by 65.	$\frac{c\cdot r-65\cdot c=65\cdot t}{65}$ $\quad$ $\frac{c\cdot(r-65)}{65}=t$

The formula is solved for t in terms of c and r. The result is the same, whether done by hand or with CAS.

STOP **See Quiz Yourself 2 at the right.**

Using a CAS as an aid when solving equations has some advantages. It can be faster and more efficient than computing by hand. Using a CAS can also simplify the process for solving for a variable in an equation, especially when that variable's exponent is greater than 1, or when there is more than one variable, as Example 3 illustrates. The process can also be simplified by substituting values for some of the variables to *reduce* the problem to a function of a single variable.

QUIZ YOURSELF 2

a. Why is $c = \frac{65t}{r - 65}$ undefined when $r = 65$?

b. Find t if $r = 70$ and $c = 10$. Explain your result in the context of the problem.

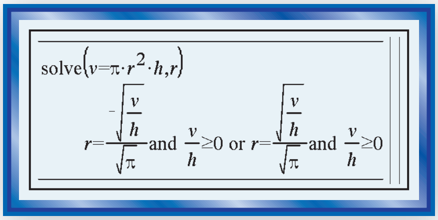
Example 3

a. Solve $V = \pi r^2 h$ for r.

b. Assume a fixed volume of 500,000 cm^3. Write the formula for r in terms of h only.

Solution

a. Solve the formula for r on a CAS. The CAS result at the right shows two different values for r.



So either $r = \frac{-\sqrt{\frac{V}{h}}}{\sqrt{\pi}}$ or $r = \frac{\sqrt{\frac{V}{h}}}{\sqrt{\pi}}$.

The first expression is negative, which is impossible in this situation, since $r > 0$.

So, $r = \frac{\sqrt{\frac{V}{h}}}{\sqrt{\pi}}$. You could also write this as $r = \frac{\sqrt{V}}{\sqrt{h\pi}}$.

b. Substitute 500,000 for V. This can be done by hand, or by using the SUCH THAT command on a CAS.

Notice that the CAS simplified $\sqrt{500000}$ but left the answer in exact form. So $r = 500 \cdot \frac{\sqrt{2}}{\sqrt{h \cdot \pi}}$ is a formula for r in terms of h.

The volume formula solved for r is substantially more complicated than the formula solved for h. The added complexity is not surprising because in the original formula, r was squared, and h was not.

Questions

COVERING THE IDEAS

In 1–2, refer to the formula $V = \pi r^2 h$ in the lesson.

1. What is the volume of a cylindrical barrel of radius 5 cm and height 25 cm?
2. The formula $\frac{V}{\pi r^2} = h$ is solved for __?__.
3. Graph the function from Example 3b giving r in terms of h when V is assumed to be 500,000. Use a window that shows x and y values as large as 100.

In 4 and 5, complete the sentence "__?__ is written in terms of __?__" for the given formula.

4. $A = s^2$ (area of a square)
5. $V = \frac{1}{3}Bh$ (volume of a pyramid)

6. Refer to the formula $c = \frac{65t}{r - 65}$ from Example 2. Assume the first car has a 2-hour head start over the second car. Find the second car's catch up time for each speed. Explain what your answers mean in the context of the problem.
   a. 75 mph b. 85 mph c. 45 mph
7. Refer to Example 2. Solve the formula for r.

APPLYING THE MATHEMATICS

8. Imagine you are riding in the second car in the situation of Example 2. Let t be the number of hours head start the other car had at a speed of $65\ \frac{\text{mi}}{\text{hr}}$. Let c be the number of hours it will take you to catch up at a rate $r\ \frac{\text{mi}}{\text{hr}}$. So, in time c, you will have traveled rc miles, while the other car will have traveled $65(t + c)$ miles. Thus, when you catch up, $rc = 65(t + c)$. Solve for c.
9. **Multiple Choice** Which formula is easiest to use if you want to find W and you are given G and k?
   A $G = 17Wk$ B $W = \frac{G}{17k}$ C $k = \frac{G}{17W}$
10. **Multiple Choice** In chemistry, a given mass and volume of gas will satisfy the equation $\frac{T_1}{P_1} = \frac{T_2}{P_2}$, where P_1 and T_1 are the pressure and temperature at one time, and P_2 and T_2 are the pressure and temperature at another time. Which is an equivalent formula solved for P_2?
   A $P_2 = \frac{T_2 T_1}{P_1}$ B $P_2 = \frac{T_1}{P_1 T_2}$ C $P_1 = \frac{P_2 T_1}{T_2}$ D $P_2 = \frac{P_1 T_2}{T_1}$

In 11 and 12, the *pitch P* of a gabled roof is a measure of the steepness of the slant of the roof. Pitch is the ratio of the vertical rise *R* to half the *span S* of the roof: $P = \frac{R}{0.5\,S}$. (The span is the longest horizontal distance from one side of the roof to the other.)

11. a. Solve the pitch formula for S.
    b. If a builder wants a roof to have a pitch of $\frac{6}{15}$ and a rise of 8 feet, what must be the span of the roof?
12. The photograph at the right shows the roof of the Throne Hall of the Cambodian Royal Palace in Phnom Penh. Using the interior triangle, measure the rise and span of the roof and estimate its pitch.

13. If $y = kx^2$ and $y = 12$ when $x = 4$, solve for k.

14. The formula $C = \pi d$ gives the circumference of a circle in terms of its diameter d.
    a. Solve this formula for π.
    b. Use your result in Part a to write an English sentence that gives the definition of π.

In 15 and 16, use the formula $A = \frac{1}{2}h(b_1 + b_2)$ for the area A of a trapezoid with bases of length b_1 and b_2 and height h.

15. Solve the formula for b_1.

16. a. In the original formula, how does the value of A change if the values of b_1 and b_2 are switched? To what geometrical situation does this correspond?
    b. How does the original formula change if $b_1 = b_2$? To what geometrical situation does this correspond?
    c. How does the original formula change if $b_2 = 0$? To what geometrical situation does this correspond?

REVIEW

17. Solve the equation. **(Lesson 1-6)**
    a. $\frac{5}{8}x = 10$
    b. $\frac{5}{8}x + 30 = 10$
    c. $\frac{5}{8}x + 30 = 10 + \frac{1}{3}x$

18. Describe how you would use your calculator to generate a table for the function with equation $y = x^2 - 2x$ and with values starting at -5 in increments of 0.1 **(Lesson 1-5)**

19. a. Graph the function f when $f(x) = 3x - \frac{x^2}{9}$ for all real numbers x.
    b. What are its domain and range?
    c. What is its range if its domain is restricted to $\{x| \ 2 \le x \le 6\}$? **(Lesson 1-4)**

20. The velocity v of an object that starts from rest and accelerates at rate a over a distance d is given by the formula $v = \sqrt{2ad}$. What is the velocity of an object which started at rest and accelerated at 6 $\frac{\text{m}}{\text{s}^2}$ over a distance of 6.75 m? **(Lesson 1-1)**

EXPLORATION

21. Formulas with three or more variables are common in science and financial applications. Find such a formula and describe what its variables represent and what it computes.

QUIZ YOURSELF ANSWERS

1. $h \approx 155.4$ cm

2a. The denominator of the fraction would be zero, and division by zero is not defined.

b. $t = \frac{10}{13}$. A car driving 70 $\frac{\text{mi}}{\text{hr}}$ will need 10 hours to catch up with a car driving 65 $\frac{\text{mi}}{\text{hr}}$ that had a $\frac{10}{13}$-hour head start.