Lesson

1-6

Solving Equations

▶ **BIG IDEA** By solving $y = f(x)$ for x you can find the value of x that leads to a particular value of y.

Suppose you are studying a function f. Two key questions often arise.

1. Given a value of x, what is the value of $f(x)$?
2. Given a value of $f(x)$, what is the value of x?

In the previous lesson, you learned how to answer these questions using graphs and tables. If the function is described with an equation, then answering the first question usually involves evaluating an expression. Answering the second question usually involves solving an equation. Example 1 illustrates these two situations. The expressions and equations are easy enough that you should be able to work with them by hand, but we also show how to use a CAS on the same questions since a CAS is useful for more complicated problems.

Mental Math

a. How many outfits can you make with 4 t-shirts and 5 pairs of pants, if one of the pairs of pants cannot be worn with two of the shirts?

b. How many outfits can you make with 4 blouses, 5 skirts, and 6 pairs of shoes?

c. How many outfits can you make with 4 shirts, x pairs of pants, y pairs of shoes, and 7 ties?

Example 1

In the board game Monopoly®, Boardwalk is the most expensive property. The one-time cost of the property plus a hotel is $1,400, while the rent earned from another player landing on it is $2,000. If $n =$ the number of players landing on Boardwalk, then $f(n) = 2000n - 1400$ is the owner's total profit after n players have landed on the property.

a. What will be the owner's profit after 4 people land on Boardwalk?

b. Is it possible for the owner to make exactly $10,000 in profit?

Solution 1

a. The profit is given by $f(4)$.
   $f(n) = 2000n - 1400$
   So, $f(4) = 2000 \cdot 4 - 1400 = 6600$.
   The owner's profit will be $6,600.

b. The question asks you to find n when $f(n) = 10{,}000$. Substitute for $f(n)$ and solve the equation.

$$2000n - 1400 = 10{,}000$$
$$2000n - 1400 + 1400 = 10{,}000 + 1400 \quad \text{Add 1400 to both sides.}$$
$$2000n = 11{,}400$$
$$n = 5.7 \qquad \text{Divide both sides by 2000.}$$

The property owner needs 5.7 people to land on Boardwalk. So it is not possible to earn exactly \$10,000 in profit.

Solution 2 Use a CAS.

a.

Define $f(n) = 2000 \cdot n - 1400$	*Done*
$f(4)$	6600

b.

Define $f(n) = 2000 \cdot n - 1400$	*Done*
$f(n) = 10000$	$2000 \cdot n - 1400 = 10000$
$(2000 \cdot n - 1400 = 10000) + 1400$	
	$2000 \cdot n = 11400$
$\dfrac{2000 \cdot n = 11400}{2000}$	$n = \dfrac{57}{10}$

$\frac{57}{10}$ is not an integer, so it is not possible for an owner to earn exactly \$10,000 in profit.

STOP See Quiz Yourself 1 at the right.

Recall that when you subtract (or add the opposite of) a number from each side of an equation, you are applying the Addition Property of Equality to find an *equivalent,* but simpler, equation. There are many other properties that are frequently applied when solving equations. The properties of real numbers used in algebra are listed in Appendix A. Skim this Appendix now, and refer to it whenever you need to check the meaning or name of a particular property. The next three examples explore how one of these properties, the Distributive Property, can be used to solve equations.

> **QUIZ YOURSELF 1**
>
> Can the owner make exactly \$12,000 in profit? Explain.

Distributive Property

For all real numbers a, b, and c,
$$c(a + b) = ca + cb.$$

Using the Distributive Property

In Example 2, the Distributive Property is used to solve an equation containing parentheses.

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Example 2

Darius had $500 to spend at the baseball card convention. He decided to spend all his money buying the rookie cards of his two favorite players. At the end of the season, Darius found that the first card increased in value by 21%, while the second card decreased in value by 7%. He spent d dollars on the first card, so he spent $(500 - d)$ dollars on the second card.

So,

$$T(d) = 1.21d + 0.93(500 - d)$$

gives the total value of the two cards. If Darius's cards were worth $550 at the end of the season, how much did he spend on each card?

Solution 1 You are given that $T(d) = 550$, and asked to find d. Solve an equation.

$$1.21d + 0.93(500 - d) = 550$$

$1.21d + \underline{\ ?\ } - \underline{\ ?\ } = 550$	Distribute the 0.93.
$\underline{\ ?\ } + \underline{\ ?\ } = 550$	Add like terms.
$\underline{\ ?\ } = \underline{\ ?\ }$	Subtract 465 from each side.
$d \approx \underline{\ ?\ }$	Divide each side by 0.28.
$500 - \underline{\ ?\ } = \underline{\ ?\ }$	Find the cost of the other card.

For the cards to be worth __?__ at the end of the season, Darius needed to spend about __?__ on the first card and about __?__ on the second card.

Solution 2 Enter the equation in a CAS. The CAS automatically simplifies the equation to get one of the steps in Solution 1. You can then solve this equation.

$1.21 \cdot d + .93 \cdot (500 - d) = 550$

$.28 \cdot d + 465. = 550$

The 1909 T206 Honus Wagner sold for $2.8 million in 2007, the most any baseball card has been sold for.

Clearing Fractions in Equations

Often, an equation modeling a real situation will involve fractions. If you want to clear an equation of fractions for simplicity, multiply each side of the equation by a common multiple of the denominators. If there is more than one term on either side of the equation, you will then need to apply the Distributive Property. Example 3 illustrates this procedure.

Example 3

In game 5 of the 2006 NBA Finals between Miami and Dallas, Dirk Nowitzki scored $\frac{1}{5}$ of Dallas's points and Josh Howard scored $\frac{1}{4}$ of Dallas's points. The rest of the Dallas Mavericks scored a total of 55 points. How many points did Dallas score?

Solution Write an equation to model the situation, then solve it.

If P = the total number of points scored by Dallas in the game, then Dirk scored $\frac{1}{5}$ of P, Josh scored $\frac{1}{4}$ of P, and the rest of the team scored 55 points. So, $P = \frac{1}{5}P + \frac{1}{4}P + 55$.

To clear the fractions in this equation, multiply both sides by a common multiple of the denominators 4 and 5. One common multiple is 20.

$$20P = 20\left(\tfrac{1}{5}P + \tfrac{1}{4}P + 55\right)$$

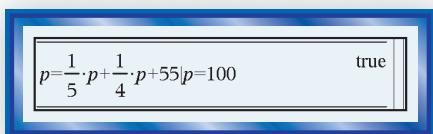|	
$20P = 4P + 5P + 1100$	Distribute the 20.
$20P = 9P + 1100$	Combine like terms.
$11P = 1100$	Subtract $9P$ from each side.
$P = 100$	Divide each side by 11.

So, the Dallas Mavericks scored 100 points, with Dirk scoring $\frac{1}{5}(100) = 20$ points and Josh scoring $\frac{1}{4}(100) = 25$ points.

Check Use the SUCH THAT command on your CAS to verify that the computed solution is true. On the CAS pictured here this function is represented by a vertical bar |.

$$p = \frac{1}{5} \cdot p + \frac{1}{4} \cdot p + 55 \, | \, p = 100 \qquad \text{true}$$

In words, this display reads "*p* equals one-fifth *p* plus one-fourth *p* plus 55 such that *p* equals 100 is true", so it checks. Sometimes "such that" is read "with."

Dirk Nowitzki, of the Dallas Mavericks.

Opposite of a Sum Theorem

From the Distributive Property and the fact that $-1 \cdot x = -x$ for all x, you can deduce that

$-(a + b) = -1 \cdot (a + b) = -1 \cdot a + -1 \cdot b = -a + -b = -a - b.$

This result is the Opposite of a Sum Theorem.

Opposite of a Sum Theorem

For all real numbers a and b,
$$-(a + b) = -a + -b = -a - b.$$

The next example applies this theorem in solving an equation.

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Example 4

Suppose $g(a) = 3a - (7 - 5a)$. For what value of a is $g(a) = 12$?

Solution

$12 = 3a - (7 + \underline{\ ?\ })$	Write the subtraction in parentheses as a sum.
$12 = 3a - \underline{\ ?\ } + \underline{\ ?\ }$	Opposite of a Sum Theorem
$12 = \underline{\ ?\ } - 7$	Add like terms.
$\underline{\ ?\ } = \underline{\ ?\ }$	Add 7 to each side.
$\underline{\ ?\ } = a$	Divide each side by the coefficient of a.

Check To check your answer, calculate the value $g(a)$ using the value you calculated for a. The result should be 12.

🛑 **See Quiz Yourself 2 at the right.**

Questions

COVERING THE IDEAS

In 1 and 2, suppose a bathtub contains 60,000 cubic inches of water. If water can be drained from the tub at a rate of 800 cubic inches per second, then $w(t) = 60{,}000 - 800t$ represents the volume of water left in the tub after draining for t seconds.

1. What volume of water will be in the tub after 18 seconds?

2. After how many seconds will the tub be empty?

In 3 and 4, assume Jamila has a collection of 200 nickels and dimes. If n is the number of nickels in her collection, then $c(n) = 0.05n + 0.10(200 - n)$ represents the face value amount of money her collection is worth.

3. Evaluate $c(122)$. What does the answer mean in the context of this problem?

4. Solve $c(n) = 19.15$. What does the answer mean in the context of this problem?

In 5–7, an equation is given. Solve the equation and check your solution using the SUCH THAT command on a CAS.

5. $4y + 50 = 5y + 42$

6. $2z + 2 = 2 - 6z$

7. $4 = \frac{6}{x}$

In 8 and 9, identify a common multiple of the denominators and solve the equation.

8. $\frac{h}{6} + \frac{h}{10} = 1$ **9.** $\frac{1}{8}x + \frac{2}{3}x = 5$

10. Fill in the Blank According to the Opposite of a Sum Theorem, $-(3 - 9y) = $ ___?___.

In 11 and 12, an equation is given.

a. Solve each equation.

b. Check your work.

11. $3x - (x + 1) = 7$ **12.** $5n - (9 - 5n) = 9$

APPLYING THE MATHEMATICS

In 13–15, use this information. A bowler's *handicap* is a bonus given to some bowlers in a league. The handicap h is a function of A, the bowler's average score, and is sometimes determined by the formula $h(A) = 0.8(200 - A)$, when $0 < A < 200$.

13. What is the handicap for a bowler whose average is 135?

14. If a bowler has a handicap of 28, what is the bowler's average?

15. What is the domain of the function h?

In 16–19, a. solve the equation by hand, and
b. enter the equation into a CAS and solve the resulting equation.

16. $0.05x + 0.12(50000 - x) = 5{,}995.8$

17. $\frac{1}{3}t + \frac{1}{4}t + 6 = t$

18. $\frac{2}{3}p + \frac{1}{2} = p - 3$

19. $0.05z + 0.1(2z) + 0.25(100 - 3z) = 20$

20. Gloria owns a video rental store. Two-fifths of the store items are comedies, one-eighth are horror films, one-fourth are dramas, and the remaining 270 are video games.

a. How many items are there in total?

b. How many comedies are there?

c. How many horror films are there?

21. Suppose $f(n) = \frac{n - 1}{n + 2}$.

a. What is $f(18)$?   b. If $f(n) = \frac{46}{49}$, find n.

In September 1895, the first meeting of the American Bowling Congress met in New York City. The ABC standardized dimensions for bowling balls, pins, and lanes.

Source: www bowlingmuseum.com

22. **Multiple Choice** Which of the following sentences are equivalent to $3(x - 7) = \frac{5}{2}$? There may be more than one correct answer.

   A $3x - 21 = \frac{5}{2}$ B $3x - 21 = \frac{5}{21}$ C $x - 7 = \frac{5}{6}$

REVIEW

In 23–26, refer to the graph below. In the graph, $x =$ the year, $i(x) =$ the value in millions of dollars of imports into the United States, and $E(x) =$ the value of exports from the United States, also in millions of dollars. (**Lessons 1-3 and 1-4**)

U.S. Trade in Goods and Services
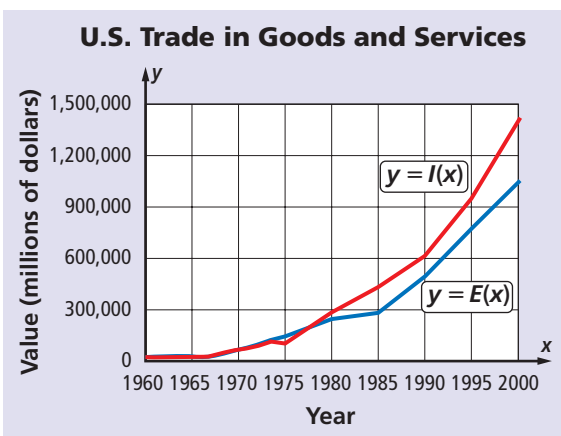
23. Estimate $I(1995)$.
24. In what unit is the dependent variable of function E measured?
25. In what year(s) was $E(x) > \$500,000$ million?
26. A negative *balance of trade* exists when a country's imports are greater than its exports. Write a few sentences comparing the balance of trade for the period 1960–1980 to the balance of trade for the period 1980–2000.
27. A cylindrical soft drink can has a lateral area of about 241 cm². If the radius of the can is 3.2 cm, approximate the can's height to the nearest tenth of a cm. (**Previous Course**)

EXPLORATION

28. Veronica was asked to find the product of two given numbers. By mistake, she added instead of multiplying. Yet she got the right answer! What two different numbers might have been given?
29. The graph for Questions 23–26 only displays the values of U.S. imports and exports through the year 2000. Research on the Internet to find more recent import and export data. What can you say about trends in imports, exports, or balance of trade by looking at the more recent data?

QUIZ YOURSELF ANSWERS

1. No; the owner would need 6.7 people to land on Boardwalk to get exactly $12,000 profit.

2. false