

Lesson

1-5

Using Graphs and
Tables of Functions

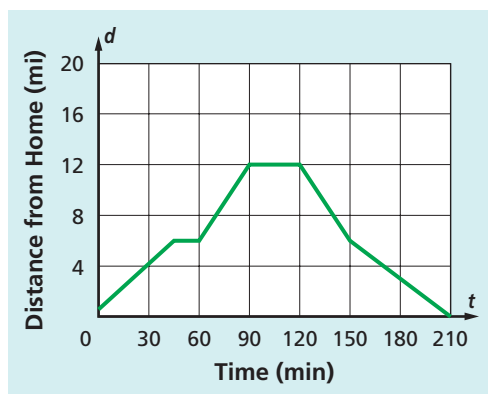
► **BIG IDEA** The graph of a function can tell a story and from it you can learn great deal about the function.

When a graph is plotted in the (x, y) coordinate plane, x is the independent variable, and y is the dependent variable. Time is often an independent variable, as in Example 1.

GUIDED

Example 1

Frank Furter rode his bicycle to Chuck Roast's house to plan a back-to-school picnic. The boys then rode a short distance to the park and then back to Frank's house. The graph below models Frank's trip where t is time in minutes and d is Frank's distance from home in miles.



Use the graph of Frank's bicycle trip to answer the following questions.

- Which variable is a function of the other variable? Express this relationship in $f(x)$ notation. (Give the function the name f .)
- State the domain and range of the function in this situation.
- Estimate Frank's distance from home after 60 minutes, and write your answer using $f(x)$ notation.
- Estimate how much time it took Frank to first reach 12 miles from home. Write your answer using $f(x)$ notation.
- What is the significance of the left horizontal line segment? Estimate its endpoints, and describe it using $f(x)$ notation.

(continued on next page)

Mental Math

Suppose Berta drove 280 miles in four hours.

- What was her average speed?
- Berta drove for another 30 minutes at the same speed. How far has she traveled in all?
- Berta stopped for a half-hour for lunch, then drove another 210 miles at the same speed. How long did she drive after lunch?
- If Berta started her trip at 9:30 A.M., at what time did she arrive at her destination?

Solution

- a. The time t is on the horizontal axis, and the distance d is on the vertical axis. So, the distance from home d is a function of the length of time t since Frank left home.

In $f(x)$ notation, $\underline{\quad} = f(\underline{\quad})$.

- b. The domain is the set of possible values of the independent variable.

The domain is $\{t \mid \underline{\quad} \leq t \leq \underline{\quad}\}$ minutes.

The range is the set of values attained by the dependent variable.

The range is $\{d \mid \underline{\quad} \leq d \leq \underline{\quad}\}$ miles.

- c. When $t = 60$, $d \approx \underline{\quad}$. The distance is about $\underline{\quad}$ miles.

In $f(x)$ notation, $f(\underline{\quad}) \approx \underline{\quad}$ miles.

- d. The leftmost point on the graph at $d = 12$ is $(90, 12)$. It took about $\underline{\quad}$ minutes for Frank to ride 12 miles away from home.

In $f(x)$ notation, $f(\underline{\quad}) = 12$ miles.

- e. From about $t = \underline{\quad}$ to about $t = \underline{\quad}$ minutes, Frank's distance remained constant at about 6 miles. This shows that Frank stayed at Chuck's house for about 15 minutes. One explanation is that it might take time for Chuck to get ready to leave for the park.

In $f(x)$ notation, for $\underline{\quad} \leq t \leq \underline{\quad}$, $f(\underline{\quad}) = 6$ miles.

Creating and Reading Tables on a Grapher

Sometimes it is helpful to represent the ordered pairs of a function in a table. In previous courses, you may have found values for tables by substituting values into a formula. For example, if $y = 15 - 4x$ and $x = -2$, then $y = 15 - 4(-2) = 23$, and the ordered pair is $(-2, 23)$. In the previous lesson, you used your grapher's TRACE feature to find ordered pairs. Your grapher can also generate a table of values automatically.

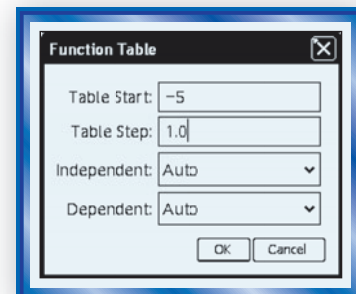
Activity

MATERIALS CAS or graphing calculator

Suppose a puddle of water is evaporating and its depth $D(t)$ in inches after t days is given by the formula $D(t) = 6 - \frac{1}{16}t^2$.

Step 1 Enter the equation into your grapher.

Step 2 Set up a table to start at $x = -5$ and to generate values in increments of 1. If your grapher needs an end value, use $x = 5$.



Step 3 Your table should be similar to the one to the right. If necessary, you can see more values in the table by scrolling with the up and down arrows on your grapher.

x	f1(x):...
	$6 - 1/16$
-5.	4.4375
-4.	5.
-3.	5.4375
-2.	5.75
-1.	5.9375
0.	6.
5.	

Step 4 The values in the x column represent which variable in this problem?

The values in the $f(x)$ column represent which variable in this problem?

Do all of the ordered pairs in the table realistically model the puddle of water situation? Why or why not?

Step 5 Determine the water depth after 6 days. Write your answer in $D(t)$ notation.

Step 6 When is the water depth 5.75 inches? Write your answer in $D(t)$ notation.

Recall that a function *can* have two or more ordered pairs with the same second coordinate. For instance, in the table from the Activity, when $f(x) = 5$, then $x = -4$ or 4 . However, only $x = 4$ models a real number of days.

Unlike graphs in which smooth or *continuous* curves show the ordered pairs at *all* values of the domain and range, tables give a *discrete* view of a function that shows only some of the ordered pairs. For example, your table in the Activity only shows the ordered pairs for integer values of x . To find more pairs, you can change the table increment in the table setup.

STOP See Quiz Yourself at the right.

A function does not need to be defined by a formula in order to create a table. A function can be defined simply as a set of ordered pairs. A function defined by a set of ordered pairs has a domain and range limited to the values in the ordered pairs.

▶ QUIZ YOURSELF

Find two other ordered pairs for the puddle function D by changing the table increment to 0.5.

Example 2

In 2000, the Centers for Disease Control and Prevention (CDC) found the mean height h in centimeters for girls in the U.S. at various ages a where $h = f(a)$. Some of these data are given in the table below.

a	2	4	6	8	10
h	85	101	115	127	138

- What is the domain of the function defined by this table?
- What is the range of this function?
- Find $f(6)$. What does this represent?
- For what value of a does $f(a) = 138$?

Solution

- The domain is the set of values the independent variable a can have.
The domain is $\{2, 4, 6, 8, 10\}$.
- The range is the set of possible values of the dependent variable h .
The range is $\{85, 101, 115, 127, 138\}$.
- $f(6) = 115$. At age 6, girls in the U.S. have an average height of 115 cm.
- $f(a) = 138$ when $a = 10$.



The average height of adult women in the United States in 2002 was 5' 3.75".

Questions

COVERING THE IDEAS

In 1–3, refer to the graph in Example 1.

- How far from home was Frank when he ended his trip? How does the graph show this?
- Estimate $f(75)$ and explain what it means.

In 11–13, a set of ordered pairs is given in a table. Is the set a function? If it is, give its domain and range. If it is not, explain why not.

11.

x	y
-3	-0.1
-2	-0.06
-1	-0.03
0	0
1	0.03
2	0.06
3	0.1

12.

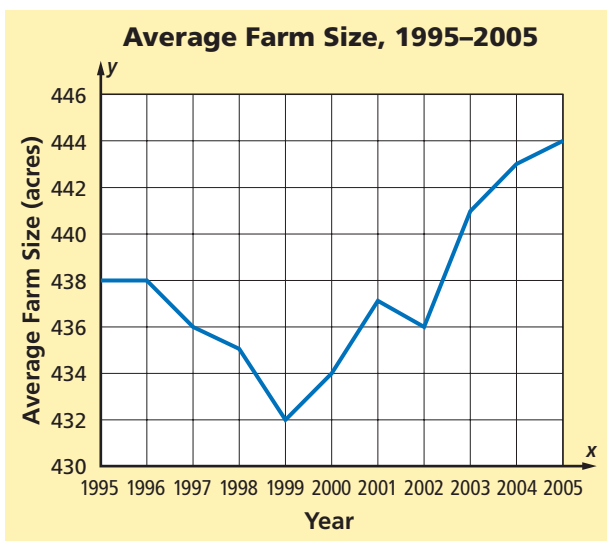
x	y
0	-6
1	-4
2	-2
3	0
2	2
1	4
0	6

13.

x	y
-6	0
-4	1
-2	2
0	3
2	2
4	1
6	0

In 14–16, refer to the table and graph below. They give information about the number of farms and the average size (in acres) of these farms across the United States from 1995 to 2005.

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005
Number of Farms (thousands)	2,200	2,190	2,190	2,190	2,190	2,170	2,160	2,160	2,130	2,110	2,100



14. If the average size function is called A , what is $A(2000)$?
15. If the function giving the number of farms is called N , find $N(2002)$.
16. a. Calculate $A(2004) \cdot N(2004) - A(1996) \cdot N(1996)$.
 b. Explain in words what the computation in Part a represents.

17. For the equation $s = 4 - r^2$:

a. Find the values of s which correspond to the given values of r .

r	-3	-2	-1	0	1	2	3
s	?	?	?	?	?	?	?

b. Plot (r, s) on a coordinate graph for each of the values in the table. Is s a function of r ? Explain.

c. Plot (s, r) on a coordinate graph for each of the values in the table. Is r a function of s ? Explain.

REVIEW

In 18 and 19, use this information. In the United States many teachers grade tests on a scale of 1–100. In France it is common to grade tests on a scale of 1–20 (with 20 being the highest). Suppose G is a function that converts a grade in the American system to a grade $G(x)$ in the French system. (Lesson 1-4)

18. Write an expression for $G(x)$.

19. Give a reasonable domain and range for the function G .

20. Write out the elements of the set $\{S \mid S \text{ is a state in the United States, and } S \text{ does not share a border with any other state}\}$. (Lesson 1-4)

21. Consider all ordered pairs of the form $(x, 2x - \pi x^2)$. Do these ordered pairs describe a function? If they do, describe this function using mapping notation. If they do not, explain why not. (Lessons 1-3, 1-2)

22. The formula for the surface area of a sphere with radius R is $A = 4\pi R^2$.

a. Write the function mapping R onto A in $f(x)$ notation.

b. Which is the dependent variable and which is the independent variable? (Lessons 1-3, 1-2)

23. The radius of the planet Neptune is approximately 24,750 kilometers. Estimate its surface area to the nearest million square kilometers. (Previous Course)

EXPLORATION

24. Think about your trips to and from school yesterday. Draw a graph similar to the graph in Example 1 to model your trip. What is your independent variable? What is your dependent variable? Are there any horizontal lines on your graph? If so, what do they represent? If not, why not?



In 2007, the high school graduation rate in France was 85%.

QUIZ YOURSELF ANSWER

Answers vary. Samples:
(1.5, 5.8594); (2.5, 5.6094).