Lesson

1-4

Graphs of Functions

> ▶ **BIG IDEA**   Because a *real function* is a set of ordered pairs of real numbers, it can be graphed on the coordinate plane.

A **real function** is a function whose independent and dependent variables stand only for real numbers. You can graph a real function on a rectangular coordinate graph. If $y = f(x)$, then the coordinates of points on the graph have the form (x, y) or $(x, f(x))$.

An Example from Driver's Education

When a driver attempts to stop a car, the distance d the vehicle travels before stopping is a function of the car's speed x and can be modeled by the equation $d = x + \frac{x^2}{20}$. If we give the name C to the function giving the stopping distance, then $C(x) = x + \frac{x^2}{20}$. Several ordered pairs of this function C are shown in the table below.

Speed (mph) $= x$	0	10	20	30	40	50	60	70
Car's Stopping Distance (ft) $= C(x)$	0	15	40	75	120	175	240	315

On the following page are two graphs relating the data above. The graph at the left is a graph of the function C for values of x from 0 to 70. All the points on it are of the form $(x, C(x))$. To find the value of $C(60)$ from the graph, start at 60 on the x-axis. Read up to the curve and then across to find the value on the y-axis. So, $C(60) = 240$. Notice that this agrees with the value in the table above.

The graph at the right includes the graph of a second function S. S maps the speed x of an SUV (sport utility vehicle) onto its stopping distance $S(x)$ at that speed. Using function notation helps distinguish the y-coordinates of the graphs when more than one function is displayed.
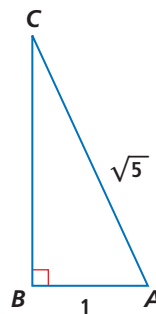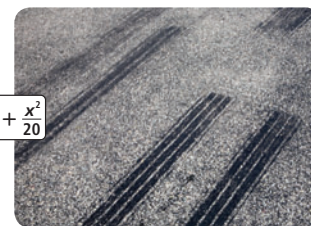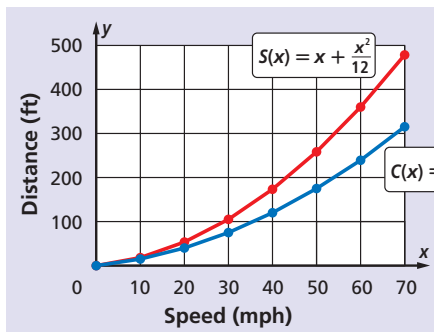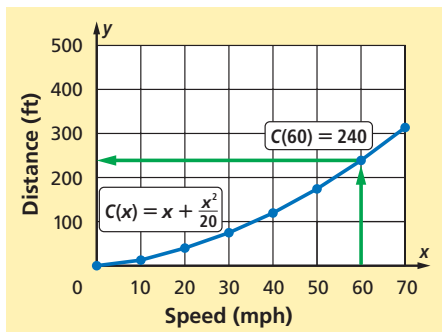
Mental Math

Refer to △ABC below. Estimate to the nearest integer.

a. BC

b. the area of △ABC

c. the perimeter of △ABC

Skid marks on a road caused by heavy braking.

Domain and Range of a Function

For the functions C and S on the previous page, the situation and table determine what values of the independent and dependent variables to include in the graph. The set of allowable values for the independent variable is called the **domain of the function**. Allowable speeds for both C and S are from 0 to 70 mph, so the domain for each is $\{x| \, 0 \leq x \leq 70\}$. This notation, called **set-builder notation**, is read "the set of all x such that 0 is less than or equal to x and x is less than or equal to 70." Some people write $\{x: 0 \leq x \leq 70\}$ for set-builder notation.

The **range of a function** is the set of values for the dependent variable that can result from all possible substitutions for the independent variable. According to the table, when x has values from 0 to 70, the values of $C(x)$ range from 0 to 315. So the range of C is $\{y| \, 0 \leq y \leq 315\}$. For these values of x, the graph shows that values of $S(x)$ range from 0 to about 480. So the range of S is $\{y| \, 0 \leq y \leq 480\}$.

When graphing a function on a graphing utility, you need to consider its domain and range in order to get an appropriate picture. The *window* of the graph must be large enough for you to see what you want to see, but not so large that the graph is too tiny. In this Activity you will learn how to set your grapher's window and use its TRACE feature to further examine the functions C and S.
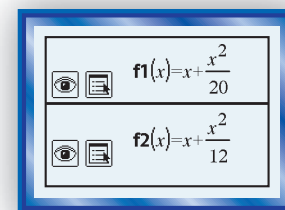
Activity

MATERIALS CAS or graphing calculator

Step 1 Locate your grapher's place for entering equations to be graphed.

Step 2 Enter $x + \dfrac{x^2}{20}$ for the first function and $x + \dfrac{x^2}{12}$ for the second function. A possible display is shown at the right.

(continued on next page)

$$f1(x)=x+\frac{x^2}{20}$$

$$f2(x)=x+\frac{x^2}{12}$$

Step 3 Set the viewing window large enough to show the graphs of both C and S. Because the range of S is larger, use its domain and range for the window. Your graphs should now look similar to the ones shown at the right.
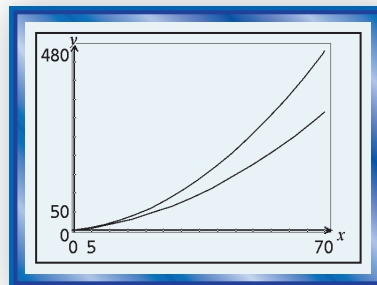
Step 4 The **TRACE** feature on your grapher allows you to estimate input and output values, much as you would on a graph drawn by hand. Trace along the graphs within the window, as shown. The numbers on the screen are the approximate x- and y-coordinates of the point on the graph where the TRACE marker is currently placed. Which ordered pair on the graph of function S appears at the right edge of the window?
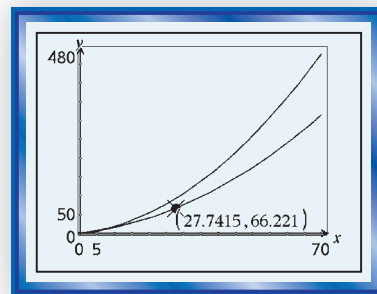
Step 5 The upper limit of allowable speeds in the table for C is 70 because that is the maximum speed limit in many parts of the U.S. However, some states have limits up to 75 mph. Continue tracing the C and S functions off the right side of the viewing window to estimate $C(75)$ and $S(75)$. Then define a new window which allows you to see allowable values of $C(x)$ and $S(x)$ over the domain $\{x|\ 0 \le x \le 75\}$.

Step 6 Your grapher has a default window, often called the **STANDARD WINDOW**, that shows all four quadrants at a reasonably close scale. On many calculators, the default window is where $\{x|\ -10 \le x \le 10\}$, $\{y|\ -10 \le y \le 10\}$; and the x-scale and the y-scale are both 1. This means that both the horizontal (x) axis and the vertical (y) axis are viewable from -10 to 10, with 1 unit between tick marks. Find and use the standard window to graph C and S, as shown at the right.
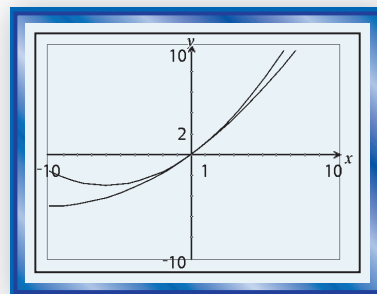
Step 7 Do all the ordered pairs displayed in this window of the graphs make sense in the situations of car and SUV speeds? Explain your answer.

Step 8 Trace along the graphs of the *C* and *S* functions beyond the left edge of the standard window. You will see that the graphs of these functions are in Quadrants I, II, and III. Set the viewing window as specified below.

$$-40 \le x \le 40 \quad x \text{ scale} = 5$$
$$-40 \le y \le 40 \quad y \text{ scale} = 5$$
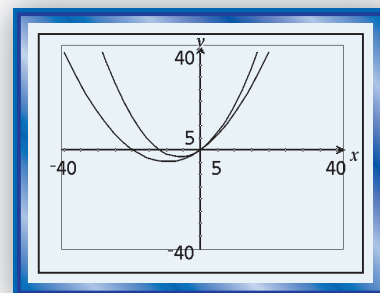
Step 9 Trace along the graphs of *C* and *S* again to estimate the minimum value of each function.

Negative speeds and negative distances are not reasonable measures. However, when you examine the graphs of the *C* and *S* functions without regard to stopping distances, you see that the functions are defined for negative values of *x* and *y*. When examining functions outside of real situations, it is common practice to identify the largest possible domain and its associated range.

Since there exist minimum values for $C(x)$ and $S(x)$, the biggest range for *C* is about $\{y|\ y \ge -5\}$ and for *S* it is about $\{y|\ y \ge -3\}$. But even if you imagine the graphs extending forever to the right and left, there do not seem to be minimum or maximum values for *x*. So *x* can be any real number, and we say that the domain of both *C* and *S* is the set of all *real numbers*, or the set of all reals.

Some sets of numbers that you are probably familiar with are frequently used as domains and often appear as ranges.

The set of **natural numbers** or **counting numbers** {1, 2, 3, 4, 5, ...}
The set of **whole numbers** {0, 1, 2, 3, 4, 5, ...}
The set of **integers** {..., –3, –2, –1, 0, 1, 2, 3, ...}
The set of **real numbers** (those numbers that can be represented by decimals)

  Samples: 0, 1, –7, 35 million, 2.34, π, $\sqrt{5}$

The set of **rational numbers** (those numbers that can be represented as ratios of the form $\frac{a}{b}$, where *a* and *b* are integers and $b \ne 0$)

  Samples: 0, 1, –7, $\frac{2}{3}$, $1\frac{9}{11}$, $-\frac{34}{10}$, 0.0004, $-9.6\overline{18}$, $\sqrt{16}$

The set of **irrational numbers** (real numbers that are not rational)

  Samples: π, $\sqrt{5}$, $\sqrt{10}$, e

The diagram at the right shows how each set of numbers relates to the others. A number in any set is also in any set in the path above it. For example, an integer is also a rational number and a real number, but not all integers are whole numbers or natural numbers.

🛑 **See Quiz Yourself at the right.**
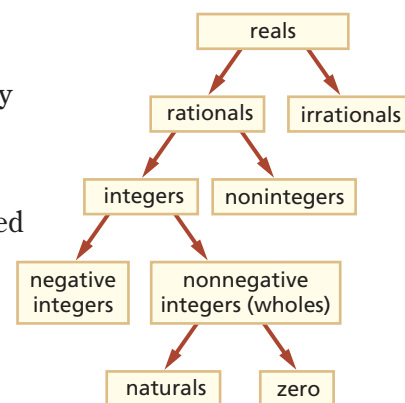
▶ **QUIZ YOURSELF**

Give an example of a negative rational number that is not an integer.

Questions

COVERING THE IDEAS

In 1–3, refer to the graphs of functions C and S at the beginning of this lesson.

1. Give their common domain.

2. Give the range of S.

3. Explain how to estimate $C(45)$ from the graph.

In 4–6, refer to the Activity.

4. Use your grapher's TRACE feature to estimate $C(55)$.

5. Assume you are in a state with a maximum speed limit of 55 mph. Define a window which shows all allowable values of $C(x)$ and $S(x)$ over the domain $\{x \mid 0 \le x \le 55\}$.

6. What is the standard window on your grapher?

In 7 and 8, the graph of a function is given. From the graph, determine the function's domain and range. Use set-builder notation.

7.

8.
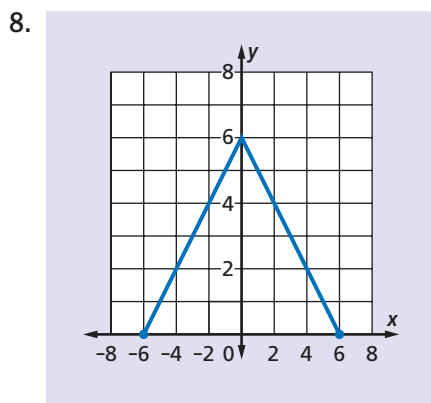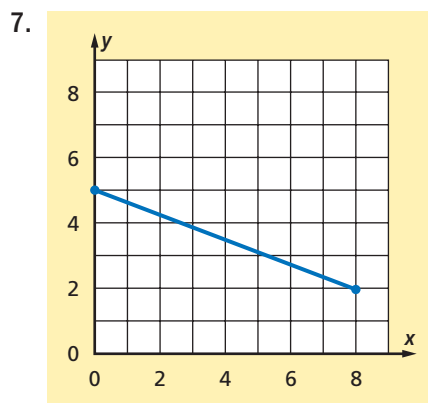
In 9–11, identify each number as an integer, a rational number, an irrational number, or a real number. A number may belong to more than one set.

9. –97

10. $\frac{23}{47}$

11. $-\sqrt{18}$

12. Name a rational number that is not an integer.

13. Name a real number that is an integer and not positive.

14. Name a real number that is an irrational number between 0 and 1.

APPLYING THE MATHEMATICS

15. The graph at the right shows the relationship between the age of a grapefruit tree and the diameter of its trunk. Let $A(d)$ be the age of a grapefruit tree whose diameter is d inches.

    a. Estimate the range of A.

    b. Estimate $A(10)$ from the graph.

    c. What diameters correspond to a tree aged between 10 and 20 years?

    d. Sketch a graph of this relation with age on the x-axis and diameter on the y-axis. Is this the graph of a function?

Trunk Diameter vs. Tree Age
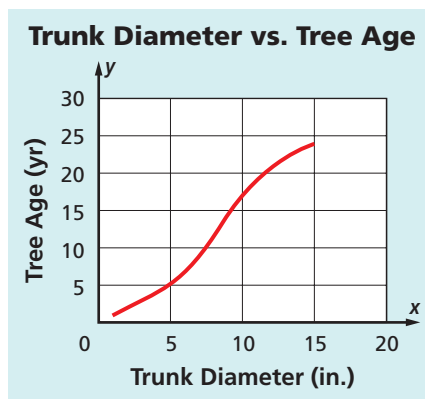
16. The volume V of a sphere with radius of length r is given by the formula $V = \frac{4}{3}\pi r^3$.

    a. State the domain for r. (*Hint*: Are there any numbers that do not make sense for the value of the radius?)

    b. Find the volume of a beach ball with a 15 cm radius. Round your answer to the nearest tenth.
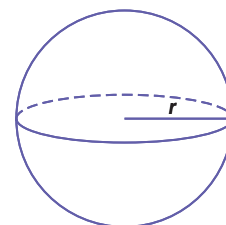
In 17 and 18, graph the function in a standard window on your grapher, then answer the questions.

17. $f(x) = 5$

    a. What is $f(15)$?

    b. State the domain and range of f.

18. $g(x) = -2x^2 - 7$

    a. What is the maximum value of g?

    b. State the domain and range of g.

19. On your grapher or CAS, graph the function $f(x) = \frac{4}{1+x}$.

    a. Use your grapher to estimate $f(0)$. Check your answer using algebra.

    b. For what value of x is $f(x) = \frac{1}{2}$? Use your grapher and check your answer using algebra.

20. Refer to Question 17 in Lesson 1-3, where you were asked to compare functions with equations $g(x) = 2^x$ and $h(x) = x^2$.

a. Graph g and h together on your grapher in the window defined below.

| xmin	xmax	xscale	ymin	ymax	yscale
−5	5	1	−2	15	1

b. In this window, sometimes $h(x) > g(x)$ and sometimes $g(x) > h(x)$. Your grapher has a ZOOM feature which allows you to get a magnified view of certain regions of your graph. Use ZOOM and TRACE to approximate values of x for which $h(x) > g(x)$.

21. A formula for the height h of an object in free fall t seconds after its release from a height of 25 feet is $h = -16t^2 + 25$.

a. Graph this equation on your grapher. Sketch the graph. (Most graphers use x for the independent variable, and $y1$, $y2$, etc. for the dependent variables, so you must substitute x for t and y for h when entering this equation.)

b. Use the TRACE feature on your grapher to estimate the height of the object after 1.1 seconds.

In an egg drop, contestants build contraptions to protect an egg to be dropped from a certain height. The winning contraption will protect the egg from breaking.

REVIEW

22. Suppose that for all x, $d(x) = 2x^3 - x + 3$. Evaluate. (**Lesson 1-3**)

a. $d(2)$ **b.** $d(-2)$ **c.** $d(\pi r)$

23. Use a CAS to find $S(100)$ when $S(n) = \dfrac{n(n+1)}{2}$. (**Lesson 1-3**)

24. Let $n = $ the number of sides of a polygon and $f(n) = $ the number of its diagonals. The figures below show that $f(4) = 2$ and $f(5) = 5$. Find $f(3)$ and $f(6)$. (**Lesson 1-3**)
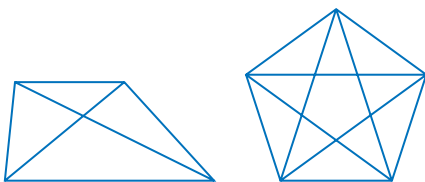
In 25 and 26, solve and check. (**Previous Course**)

25. $\frac{1}{3}r - 27 = 60$ **26.** $2x = \frac{3}{2}x + 4$

EXPLORATION

27. Find a coordinate graph in a newspaper or magazine. Does it represent a function? Why or why not? If it is a function, identify the domain and range of the function.

QUIZ YOURSELF ANSWER

Answers vary. Sample: $-\frac{4}{5}$