Lesson

1-3 Function Notations

Vocabulary

$f(x)$ notation
argument of a function
value of a function
mapping notation

BIG IDEA A function f may be written as a set of ordered pairs $(x, f(x))$, described in $f(x)$ notation by an equation $y = f(x)$, or defined by the mapping f: $x \rightarrow y$.

Consider these three equations:

$$y = 90 - x,$$
$$y = 1.8x + 32,$$
$$y = \sqrt{25 - x^2}.$$

The first equation relates the degree measures x and y of the two acute angles in a right triangle. The second equation converts a temperature x in degrees Celsius into a temperature y in degrees Fahrenheit. The third equation gives the length y of one leg of a right triangle with another leg of length x and hypotenuse of length 5. Each formula is easy to use, but when they are all together on the page, all the x's and y's can be confusing. To avoid this confusion, you can use ***f*(*x*) notation.** If A names the angle function, T names the temperature conversion function, and L names the formula for leg length, you can write:

$A(x) = 90 - x$	$A(x)$ is read "A of x."
$T(x) = 1.8x + 32$	$T(x)$ is read "T of x."
$L(x) = \sqrt{25 - x^2}$	$L(x)$ is read "T of x."

Now each function can be simply referred to by its name: A, T, or L. Any letter or string of letters (such as ABS or $SQRT$) or letters and numbers (such as $T1$) can name a function.

f(*x*) notation

The symbol $f(x)$ is read "f of x". It does *not* mean f times x. This notation is attributed to Leonhard Euler (pronounced "oiler"), an extraordinary Swiss mathematician. In 1770, Euler wrote what is considered the most influential algebra book of all time, *Vollständige Anleitung zur Algebra (Complete Introduction to Algebra)*. In honor of him, $f(x)$ notation is also called "Euler's notation."

Mental Math

a. How much would 15 cans of tuna cost if each can costs 59 cents?

b. How much would 15 frozen dinners cost if each dinner costs $5.90?

c. How much would 1.5 pounds of mixed nuts cost at a price of $5.90 per pound?

d. How much would 4.5 pounds of mixed nuts cost at a price of $5.90 per pound?

Leonhard Euler (1707–1783)

You can combine $f(x)$ notation with a descriptive name for the independent variable. If you replace x with C for degrees Celsius, function T above is described by

$$T(C) = 1.8C + 32.$$

The letter C is the independent variable, or input, and $T(C)$ names the dependent variable, or output. T is the name of the function which multiplies the input C by 1.8 and adds 32.

You may also think of the function as

$$T(\ ) = 1.8(\ ) + 32.$$

The same variable, number, or expression is placed in the ( ) wherever the ( ) appears. What is written in the parentheses is called the **argument of the function**; the result is called the **value of the function**.

Example 1

If $T(C) = 1.8C + 32$, evaluate $T(20)$.

Solution Substitute 20 for C in the equation $T(C) = 1.8C + 32$.

$$T(20) = 1.8(20) + 32$$
$$T(20) = 36 + 32$$
$$T(20) = 68$$

Check Look at a thermometer with both Celsius and Fahrenheit scales to see that 20°C = 68°F.

STOP See Quiz Yourself 1 at the right.

> **QUIZ YOURSELF 1**
> In the function A on page 20, calculate $A(32)$.

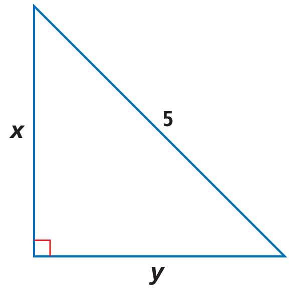
Computer Algebra Systems (CAS)

In this course, you will often use a *computer algebra system* (CAS). Computer algebra systems have many uses. They allow you to define and find values of functions and expressions, simplify and solve algebraic equations and inequalities, and perform operations with algebraic expressions. Computer algebra systems use $f(x)$ notation to define functions and $f1$, $f2$, $f3$, and so on to name the functions to be graphed. Once defined, you can use these names on the home screen to find values of the function such as $f1(17)$ or $f3(\pi)$. The following Activity shows how to define and evaluate functions on a CAS.

Activity

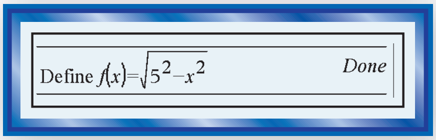
MATERIALS CAS

Let *f* be the function that gives the length of one leg of a right triangle with a hypotenuse of 5 in terms of *x*, the length of the other leg.

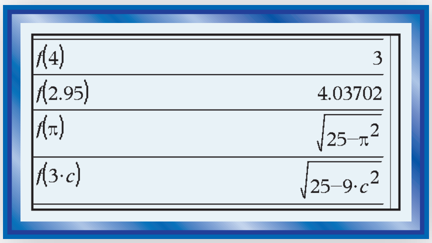


Step 1 Find an equation for *y* in terms of *x*.

Step 2 Put your calculator in REAL (not complex) mode. Define the function *f* on your CAS using the equation you found in Step 1. The entry is made on the command line and the CAS may write the expression in a different form for easier reading. Here is what one calculator shows.



Step 3 Use your CAS to evaluate $f(4)$, $f(2.95)$, $f(\pi)$, and $f(3 \cdot c)$. One CAS shows the following results.



Step 4 Use your CAS to evaluate $f(6)$. Write a brief explanation of the result.

STOP **See Quiz Yourself 2 at the right.**

QUIZ YOURSELF 2

Use your CAS to find the values of $f(2)$ and $f(-2)$.

Mapping Notation for Functions

In the Activity, the function *f* is described by $f(x) = \sqrt{5^2 - x^2}$. Another notation, called **mapping notation**, is sometimes used for functions. Mapping notation uses a colon (:) and an arrow (→). The function from the Activity would be written in mapping notation as follows:

$$f: x \to \sqrt{5^2 - x^2}.$$

This is read, "the function *f* maps *x* onto $\sqrt{5^2 - x^2}$." For $f(4) = 3$, in mapping notation we write

$$f: 4 \to 3.$$

Example 2

Let $f: x \to \sqrt{5^2 - x^2}$. Evaluate $f(3)$ and $f(r) - f(3)$.

Solution 1 Substitute 3 for *x*.

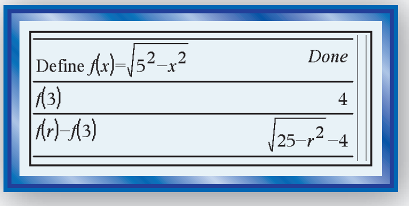
$f(3) = \sqrt{5^2 - 3^2} = 4$

Substitute *r* for *x*. $\sqrt{5^2 - x^2} = \sqrt{5^2 - r^2} = \sqrt{25 - r^2}$

$f(r) = \sqrt{5^2 - r^2} = \sqrt{25 - r^2}$

So $f(3) = 4$ and $f(r) - f(3) = \sqrt{25 - r^2} - 4$.

Solution 2 Use a CAS. Define $f(x)$. Then evaluate $f(3)$ and $f(r) - f(3)$, as shown at the right.



Questions

COVERING THE IDEAS

In 1–3, how is each read?

1. $f(x)$
2. $T: a \to a^2 + 3a + 4$
3. $A(x) = \frac{1}{2}x(5 - x)$

In 4–8, suppose $f(x) = \sqrt{25 - x^2}$. Evaluate the expression. If necessary, round to the nearest hundredth.

4. $f(0)$
5. $f(\sqrt{2})$
6. $f(-5)$
7. $6 \cdot f\left(-\frac{1}{2}\right)$
8. $f(a) - f(b)$

In 9–11, use the function with equation $T(C) = 1.8C + 32$.

9. Evaluate $T(86)$.
10. Write this function in mapping notation.
11. What would you enter into your CAS command line to define the function T?
12. Is the value of a function the dependent or the independent variable?

In 13–14, let $g: x \to 12 - 2x$.

13. **Fill in the Blank**
    a. $g: 12 \to$ _?_
    b. $g: -3 \to$ _?_
14. Rewrite the function g using $f(x)$ notation.

APPLYING THE MATHEMATICS

In 15 and 16, let $V(r) = \frac{4}{3}\pi r^3$.

15. a. What is the argument of the function V?
    b. What is the value of the function V when $r = 6$?
    c. What is the value of the function V when the argument is $3R$?
16. What argument of the function V produces a value of $\frac{4}{3}\pi$?
17. Define $g(x) = 2^x$ and $h(x) = x^2$ on your CAS.
    a. Complete the following table. (When necessary, round to the nearest hundredth.)

X	$h(x)$	$g(x)$	$h(x) - g(x)$
–4	16	0.06	15.94
–2	?	?	?
–0.5	?	?	?
0.5	?	?	?
2.5	?	?	?
3	?	?	?
3.5	?	?	?
4.5	?	?	?
10	100	?	?

    b. Is $h(x) > g(x)$ for all values of x? Explain based on the results of your table.
    c. The g and h functions in your CAS remain stored in memory until you either replace them with new definitions or delete them. Find out how to check the variable and function contents of your CAS memory.
    d. Are g and h stored as functions in your CAS memory? How can you tell?
    e. Delete the h function from your CAS memory. What happens now when you use your CAS to evaluate $h(8)$?

In 18–20, use the table below, in which x is the year and $p(x)$ is the average price (in cents) of one gallon of unleaded gasoline during the year x.

x	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
$p(x)$	123.4	105.9	116.5	151.0	146.1	135.8	159.1	188.0	229.5	258.9

Source: http://www.economagic.com

18. **a.** Evaluate $p(2006) - p(2005)$.

b. Explain in words what you have just calculated.

c. What does the answer to Part a mean about the price of gasoline?

19. For what values of x is $p(x) > 150$?

20. Solve $p(x) = 116.5$ for x.

21. Given that $g: x \rightarrow |-x| + (x - 2)^3$, evaluate.

a. $g(4)$ **b.** $g(-4)$

c. $g(2.376)$ **d.** $g(4b)$

REVIEW

22. Is y a function of x? Why or why not? **(Lesson 1-2)**

x	0	1	–1	2	–2
y	0	1	1	8	8

23. Amy has three sisters and two brothers. Suppose she writes out the relation of all pairs (x, y) where x and y are siblings of hers and x is y's brother. Is this relation a function? **(Lesson 1-2)**

24. Berto is saving money for a car. He has \$657 in his account. He plans on saving \$50 each week. How much will he have **(Lesson 1-1)**

a. after 11 weeks? **b.** after n weeks?

25. **a.** Give an equation for the horizontal line through $(-4, 6)$.

b. Give an equation for the vertical line through $(-4, 6)$.

c. Write equations for the horizontal line and the vertical line that intersect at (h, k). **(Previous Course)**

EXPLORATION

26. Leonhard Euler contributed to all branches of mathematics. Various theorems, formulas, and functions are named after him. Research Euler's *totient function* and write a paragraph about it.

QUIZ YOURSELF ANSWERS

1. $A(32) = 58$

2. $f(2) \approx 4.58$; $f(-2) \approx 4.58$