Lesson

1-2 Relations and Functions

BIG IDEA Functions are the mathematical models of relationships between two variables.

Vocabulary

relation
function
dependent variable
independent variable
input, output
is a function of
maps
mathematical model

Mental Math

Rewrite the expression as the power of a single variable, if possible.

a. $x^7 \cdot x^2$

b. $\frac{a^7}{a^2}$

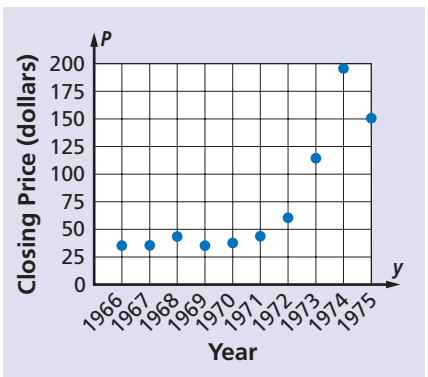
c. $d^8 + d^4$

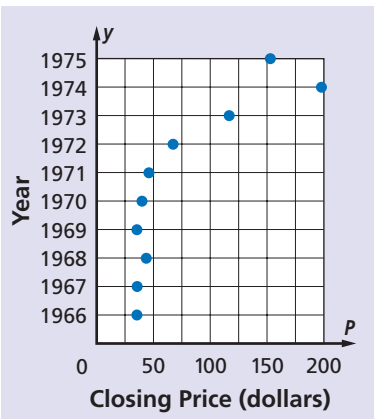
d. $n^{16} \cdot \frac{n^3}{n^5}$

You saw a graph of the yearly closing prices of gold at the beginning of this chapter. The closing prices for 1966 to 1975 are given in the following table.

Year y	Closing Price p (in dollars)
1966	35.4
1967	35.5
1968	43.5
1969	35.4
1970	37.6
1971	43.8
1972	65.2
1973	114.5
1974	195.2
1975	150.8

Two graphs of these data are shown below.





The graph on the left on the previous page shows the ordered pairs (y, p) in the set {(1966, 35.4), (1967, 35.5), (1968, 43.5), (1969, 35.4), (1970, 37.6), (1971, 43.8), (1972, 65.2), (1973, 114.5), (1974, 195.2), (1975, 150.8)}. The graph on the right shows the ordered pairs (p, y) in the set {(35.4, 1966), (35.5, 1967), (43.5, 1968), (35.4, 1969), (37.6, 1970), (43.8, 1971), (65.2, 1972), (114.5, 1973), (195.2, 1974), (150.8, 1975)}.

Each of these sets of ordered pairs and their graphs shows the relationship between the variables y and p. In general, a **relation** is any set of ordered pairs. Any correspondence or pairing between two variables can be written as a set of ordered pairs. In the relation (y, p) graphed on the left, no two ordered pairs have the same first coordinate. Each value of y relates to *one and only one* value of p. A relation having this property describes a *function*.

Definition of Function

A **function** is a set of ordered pairs (x, y) in which each first component x of the pair is paired with exactly one second component y.

In the relation (y, p), the variable p is the **dependent variable** because finding its value depends on knowing the value of y. The variable y is the **independent variable**. Sometimes the independent variable values of a function are called **inputs** and the dependent variable values are called **outputs**.

When the relationship between two quantities is a function, we say that the dependent variable (the second component) **is a function of** the independent variable (the first component). In our gold example, the price of gold at the end of the year is a function of the year.

We also say that the function **maps** each value of the independent variable onto the corresponding value of the dependent variable. Because of this property, every function is a relation, and we can reword the definition of function given earlier.

Definition of Function (reworded)

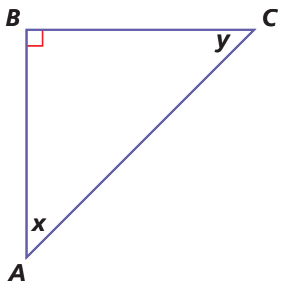
A **function** is a relation in which no two ordered pairs have the same first component x.

Not every relation is a function. Consider the relation with the ordered pairs (p, y). Both (35.4, 1966) and (35.4, 1969) are in this relation. That is, if the closing price of gold is 35.4, the year could be either 1966 or 1969. Because of this ambiguity, y is not a function of p.

The table and list of ordered pairs in the situation about gold prices are examples of *mathematical models* of the yearly gold prices. A **mathematical model** for a real situation is a description of that situation using the language and concepts of mathematics. Often the situation has to be simplified for the mathematical model. In the case of gold, since the price of gold is always changing, we used the closing price of the end of the year. Sometimes models are created to describe properties of mathematical objects.

Example 1

In a right triangle, the sum of the measures of the two acute angles is 90 degrees. Suppose x is the degree measure of one acute angle, and y is the degree measure of the other, as shown in the drawing at the right.



You can write an algebraic sentence to describe this situation.

$$x + y = 90$$

Now you can subtract x from both sides of this equation to get y alone on one side.

$$y = 90 - x$$

In this question, which of x or y is the dependent variable and which is the independent variable? Explain your answer.

Solution In this formula, y is the dependent variable because finding its value depends on knowing the value of x. This means that the independent variable is x.

A function can be described in words, by a formula, or in a graph or table.

Example 2

The percentage P of the adult U.S. population aged 25 and over who had earned a 4-year college degree by the year Y is given in this table.

Year (Y)	1986	1988	1990	1992	1994	1996	1998	2000	2002	2004	2006
Percent (P)	19	20	21	21	22	24	24	26	27	28	28

a. Is Y a function of P? Why or why not?

b. Is P a function of Y? Why or why not?

Solution

a. Think: If Y is a function of P, then for each value of P there should be only one value of Y. So Y is not a function of P because P = 21 is paired with two different years, Y = 1990 and Y = 1992.

b. P is a function of Y because each year is paired with only one value of P. That is, there was a unique percentage of graduates each of these years.

STOP **See Quiz Yourself at the right.**

In Example 2, P is a function of Y, so Y is the independent variable and P is the dependent variable.

▶ QUIZ YOURSELF

Are there other ordered pairs that show Y is not a function of P? If so, what are they?

Questions

COVERING THE IDEAS

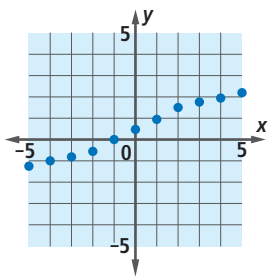
1. In your own words, what is a function?
2. a. What is the difference between a function and a relation?
   b. Give an example of a relation that is not a function.
3. Give the definition and an example of a mathematical model.
4. Consider the table below.

s	–10	–5	0	5	10
t	100	25	0	25	100

   a. Is t a function of s? Explain your answer.
   b. Is s a function of t? Explain your answer.
5. Recall the formula for the circumference of a circle, $C = 2\pi r$. Is C a function of r? Justify your answer.

In 6–8, a relation is graphed. Is the relation a function? How can you tell?

6.



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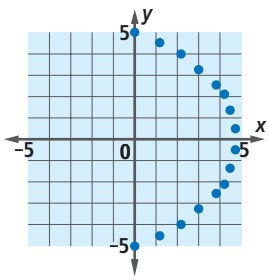

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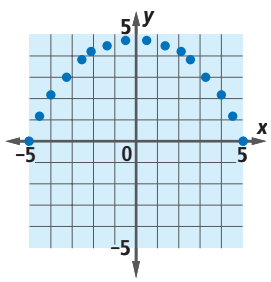


APPLYING THE MATHEMATICS

9. Let D be the degree measure of each interior vertex angle in a regular polygon with n sides. The formula $D = \frac{180(n-2)}{n}$ gives D as a function of n.
   a. Identify the dependent variable. Explain your answer.
   b. Find the number of degrees in one of the interior angles of a regular hexagon.
   c. Draw a regular hexagon. Check that your answer to Part b is correct either by using a protractor, or by giving a logical argument based on the Triangle-Sum Theorem, which states that the sum of the measures of the angles of a triangle is 180°.

10. The table below shows data about some of the costs of running a vehicle. Y = year, T = the average cost (cents) per mile of owning and operating the vehicle, and G = the average cost (cents) per mile of gasoline for the vehicle. (The data assume an average of 15,000 vehicle-miles per year.)
    a. Is T a function of Y? Explain your answer.
    b. Is Y a function of G? Again, explain your answer.

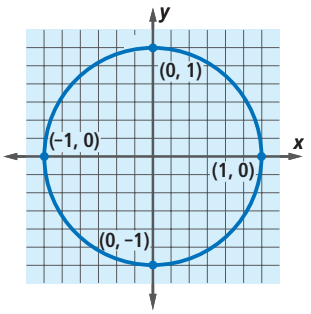
Y = Year	T = Average Cost per Mile to Own and Operate (in cents)	G = Average Cost per Mile for Gasoline (in cents)
1985	23.2	5.6
1990	33.0	5.4
1991	37.3	6.6
1992	38.8	5.9
1993	38.7	5.9
1994	39.4	5.6
1995	41.2	5.8
1996	42.6	5.6
1997	44.8	6.6
1998	46.1	6.2
1999	47.0	5.6
2000	49.1	6.9
2001	51.0	7.9
2002	50.2	5.9
2003	51.7	7.2
2004	56.2	6.5
2005	52.2	9.5
2006	52.2	8.9

Source: American Automobile Association

In 11 and 12, find the value of the dependent variable for the given value of the independent variable.

11. $p = 3x^2, x = -5$

12. $d = -6b^2 + 2b - 8, b = 5.6$



13. Recall from geometry that the *unit circle* is the circle with center (0, 0) and radius 1. A graph of the unit circle is given at the right. Is the unit circle a graph of a function? Explain your answer.

REVIEW

A lesson reference following a review question indicates a place where the idea of the question is discussed.

14. **Multiple Choice** Which of the following equations are formulas? There may be more tha``n one correct answer. **(Lesson 1-1)**

A $A = \pi r^2$ **B** $Ax + By = C$

C $E = mc^2$ **D** $(x - a)(x + a) = x^2 - a^2$

In 15–20, state the answer in terms of the variables given. (Previous Course)

15. Farid bought m blank CDs for a total of b dollars. What was his cost per CD?

16. Butter normally costs n dollars per pound. If butter is on sale for s dollars per pound, how much will you save?

17. How many different outfits can be made from K skirts and L shirts?

18. A bank pays you I interest on P dollars of savings. How much do you have altogether?

19. If a recipe for m people requires g eggs, how many eggs will a recipe for n people require?

20. If a website has h hits now and is getting m hits an hour, how many hits will it have in all, t hours from now?

In 2007, the average per capita egg consumption in the United States was 253.8 eggs

EXPLORATION

21. The word *function* has several meanings beside the definition given in this lesson. Discuss the similarities and differences between the mathematical definition of function and its nonmathematical definitions.

QUIZ YOURSELF ANSWER

Yes; (24, 1996) and (24, 1998)