

Lesson

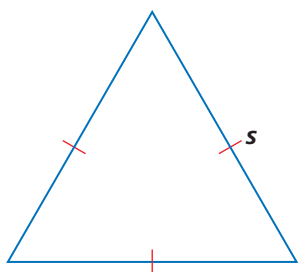
1-1

The Language of Algebra

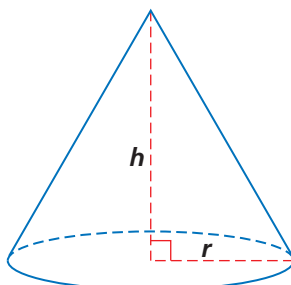
► **BIG IDEA** Algebra is a language with expressions and sentences. There are precise rules for evaluating algebraic expressions so that the meaning and values of expressions are unambiguous.

The language of algebra uses numbers and variables. It lets you describe patterns and relationships between quantities. A **variable** is a symbol that can be replaced by any one of a set of numbers or other objects. When variables stand for numbers, and numbers and variables are combined using the operations of arithmetic, the result is called an **algebraic expression**, or simply an **expression**.

For instance, the expression $\frac{s^2\sqrt{3}}{4}$ for the area of an equilateral triangle with side length s uses the variable s and the number $\frac{\sqrt{3}}{4}$. The expression for the volume of a cone with radius r and height h is $\frac{1}{3}\pi r^2 h$. That expression involves the variables r and h and the numbers π and $\frac{1}{3}$. Both expressions involve the operations of multiplication and powering.



$$A = \frac{s^2\sqrt{3}}{4}$$



$$V = \frac{1}{3}\pi r^2 h$$

An **algebraic sentence** consists of expressions related with a verb in symbolic form.

Common verbs are shown in the table at the right:

Examples of algebraic sentences are $A = \pi r^2$, $A \approx 3.14 \cdot r^2$, $a + b = b + a$, and $3x + 9 < 22$.

Vocabulary

variable
algebraic expression
expression
algebraic sentence
evaluating an expression
order of operations
vinculum
equation
formula

Mental Math

- What is the area of a square with perimeter 28 inches?
- What is the area of a rectangle with length 6 cm and perimeter 20 cm?
- What is the width of a rectangle with length 12 ft and area 108 ft²?
- What is the length of a rectangle with width 25 mm and perimeter 130 mm?

Symbol	Meaning
=	is equal to
<	is less than
>	is greater than
≤	is less than or equal to
≥	is greater than or equal to
≠	is not equal to
≈	is approximately equal to

Writing Expressions and Sentences

From your earlier study of algebra, you have gained experience writing expressions and sentences, modeling real situations, and evaluating expressions or sentences. In Example 1 below, part of the solution is written using this **typestyle**. This style is used to indicate what you might write on your homework paper as the solution to the problem.

Example 1

Joseph has a collection of 1,380 comic books and buys 17 new titles every month. If Joseph continues collecting comic books in this way, how many comic books will he have after m months?

Solution Make a table. Beginning with 1,380 comic books, in each month there will be an increase to Joseph's collection of 17 issues.

Months from Now	Number of Comics
1	$1380 + 1 \cdot 17$
2	$1380 + 2 \cdot 17$
3	$1380 + 3 \cdot 17$
4	$1380 + 4 \cdot 17$

Notice in this table that the arithmetic in the right column is not carried out. This makes the pattern easier to see. The number in the left column, which gives the number of months, is always in a particular place in the expression in the right column. You should see the following pattern.

$$m \qquad 1380 + m \cdot 17$$

Because of the Commutative Property of Multiplication, $m \cdot 17 = 17m$. So, after m months, Joseph will have $1380 + 17m$ comic books.

Check Pick a value for m not in the table and substitute it in $1380 + 17m$. We pick $m = 5$, indicating 5 months from now, and get $1380 + 17m = 1380 + 17 \cdot 5 = 1,465$ comic books. Then calculate the number of comics 5 months from now using the table. The table shows that in 4 months, Joseph would have $1380 + 4 \cdot 17 = 1,448$ comics. Add 17 for the fifth month to get $1448 + 17 = 1,465$. It checks.

You could also describe the situation in Example 1 with the sentence $C = 1380 + 17m$, where C is the number of comic books after m months.



The first American comic book was *The Adventures of Obadiah Oldbuck*, by Rudolphe Töpffer, published in 1842.

STOP See Quiz Yourself 1 at the right.

Quiz Yourself (QY) questions are designed to help you follow the reading. You should try to answer each Quiz Yourself question before reading on. The answer to the Quiz Yourself is found at the end of the lesson.

▶ QUIZ YOURSELF 1

In the situation of Example 1, how many comic books would Joseph have after 6 years?

Evaluating Expressions and Formulas

Substituting numbers for the variables in an expression and calculating a result is called **evaluating the expression**. In the expression $1380 + 17m$ in Example 1, we multiplied the value of m by 17 and then added 1380. We were using the standard rules for **order of operations** to evaluate the expression.

Rules for Order of Operations

1. Perform operations within parentheses or other grouping symbols from the innermost group out.
2. Within grouping symbols, or if there are no grouping symbols:
 - a. Take powers from left to right.
 - b. Multiply and divide in order from left to right.
 - c. Add and subtract in order from left to right.

GUIDED

Example 2

A *Guided Example* is an example in which some, but not all of the work is shown. You should try to complete the example before reading on. Answers to Guided Examples are in the Selected Answers section at the back of this book.

Find the value of $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$ when $a = 3$, $b = -1$, and $c = -4$.

Solution

Step 1 Substitute: $\frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{? - \sqrt{?^2 - 4 \cdot ? \cdot ?}}{2 \cdot ?}$

Step 2 In both the fraction and the radical symbol ($\sqrt{\quad}$), the bar (—) is a grouping symbol. (The bar is called a **vinculum**.) The square root vinculum is inside the fraction vinculum, so work inside the square root first.

Compute the power and then do the multiplications followed by the subtraction. (Watch the sign!)

$$= \frac{? - \sqrt{?}}{2 \cdot ?}$$

Step 3 Now compute the square root and subtract in the fraction's numerator. Then multiply in the fraction's denominator.

$$= \frac{?}{?}$$

Step 4 You may wish to rewrite the fraction in lowest terms.

$$= \underline{?}$$

STOP See Quiz Yourself 2 at the right.

An **equation** is a sentence stating that two expressions are equal. A **formula** is an equation stating that a single variable is equal to an expression with one or more different variables on the other side. The single variable on one side of a formula is said to be written *in terms of* the other variables. Below are some examples.

$x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$	both an equation and the Quadratic Formula
$A = \pi r^2$	both an equation and a formula
$y = 3x + 4$	both an equation and a formula
$a + b = b + a$	an equation that is not a formula
$x = 15$	an equation that is not a formula

Formulas are useful because they express important ideas with very few symbols and can be easily applied to many situations.

Example 3 shows how to evaluate an expression with more than one variable taken from a real situation. It also illustrates how to work with units in evaluating expressions.

GUIDED

Example 3

A kilowatt-hour is one kilowatt of power used for one hour.

- What does it cost to have a 60-watt bulb turned on for 33 hours at a cost of 9.53¢ per kilowatt-hour?
- Give a formula for the cost in terms of the three numbers in the problem.

Solution

$$\text{a. Cost} = 60 \text{ watts} \cdot \underline{?} \text{ hours} \cdot \underline{?} \frac{\text{cents}}{\text{kilowatt-hour}}$$

You need to change 60 watts to kilowatts. Since 1 kilowatt = 1000 watts,

$$1 = \frac{1 \text{ kilowatt}}{1000 \text{ watts}}. \text{ This fraction is a } \textit{conversion factor}.$$

$$\text{Thus, } 60 \text{ watts} = 60 \text{ watts} \cdot \frac{1 \text{ kilowatt}}{1000 \text{ watts}} = \underline{?} \text{ kilowatts.}$$

$$\text{Cost} = \underline{?} \text{ kilowatts} \cdot \underline{?} \text{ hours} \cdot \underline{?} \frac{\text{cents}}{\text{kilowatt-hour}}$$

(continued on next page)

▶ QUIZ YOURSELF 2

Evaluate

$$88 - 16 \div 2 \cdot 3^{6-4}.$$

Notice that the units are multiplied and divided as if they were numbers.

$$= \frac{? \text{ kilowatts} \cdot \text{hours} \cdot \text{cents}}{\text{kilowatt} \cdot \text{hour}} = ? \text{ cents}$$

- b. Let p = the number of watts in the bulb. Let t = the number of hours. Let u = the (unit) cost in cents per kilowatt-hour. Then Cost = $?$.

Questions

COVERING THE IDEAS

These questions cover the content of the lesson. If you cannot answer a Covering the Ideas question, you should go back to the reading for help in obtaining an answer.

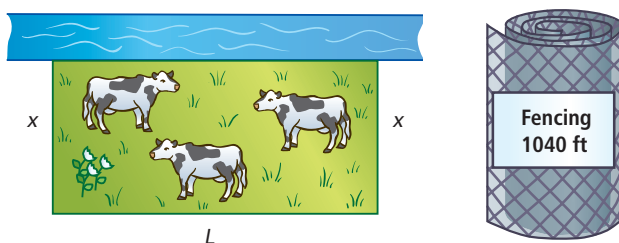
- Refer to page 5. The price of shares of stock in gold-mining companies tends to go up as the price of gold goes up. In what year between 1975 and 2000 would it have been best to put your money in gold stocks for five years?
- In your own words, describe the difference between an equation and a formula.
- In your own words, describe the difference between an expression and an equation.
- Name all the variables in $\pi\left(\frac{d}{2}\right)^2$.
 - Classify $\pi\left(\frac{d}{2}\right)^2$ as an equation, formula, expression, or sentence. Explain your answer.
- Give an example of an algebraic expression not found in the reading.
- Give an example of an algebraic sentence not found in the reading.
- Consider the sentence $c^2 = a^2 + b^2$.
 - Is this sentence an equation? Why or why not?
 - Is this sentence a formula? Why or why not?
- Evaluate $\frac{-b + \sqrt{b^2 - 4ac}}{2}$ when $a = -5$, $b = 30$, and $c = -25$.
- Paula has collected 6 years of back issues of the magazine *Nature Today and Tomorrow*. *Nature Today and Tomorrow* prints 51 issues per year. If Paula reads two issues per day, how many issues will Paula have left to read after m months? Consider that the average month has 30 days.

10. a. Evaluate $12 \div 3 \cdot 5^{(4-2)} + 76$.
 b. Indicate the order in which you applied the five operations in Part a.
11. Refer to Example 3. Find the cost of operation of a central air conditioner that uses 3.5 kilowatts and runs for 12 hours a day for one week at a cost of 9.53¢ per kilowatt-hour.

APPLYING THE MATHEMATICS

These questions extend the content of the lesson. You should study the examples and explanations if you cannot answer the question. For some questions, you can check your answers with the ones in the Selected Answers section at the back of this book.

12. a. The expression $\frac{6^2-7}{5}$ is not equivalent to the expression $6 \cdot 6 - 7 \div 5 \cdot 5$. Why not? Use the order of operations to justify your answer.
 b. Insert parentheses into the second expression to make it equivalent to the first expression.
13. a. If a person owns C comic books and buys 31 new comics every month, how many comic books will this person have after t years?
 b. If a person owns C comic books and buys b new comics every month, how many comic books will this person have after t years?
14. Yuma used 1,040 ft of fence to enclose the rectangular pasture shown below. One side borders a river where there is already a thick hedge. That side needed no fencing.



- a. Let x be the width of the pasture as labeled. Write an expression for L , the length of the pasture, in terms of x .
 b. Write an expression for the area of the pasture in terms of L and x .
 c. Write an expression for the area of the pasture in terms of x only.
 d. Suppose Yuma wants the pasture to enclose at least 60,000 square feet. Write a sentence relating your answer in Part c to the area the fence must enclose.

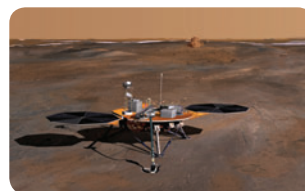
In 15–17, evaluate each expression to the nearest tenth when $x = 7.2$, $y = \sqrt{3}$, and $z = -2$.

15. $\frac{10x}{y^4 - z^3}$

16. $\frac{x}{y^2} + z$

17. $y - x^2 - z$

18. The formula $d = \frac{1}{2}gt^2$ tells how to find d , the distance an object has fallen during time t , when it is dropped in free fall from near the Earth's surface. The variable g represents the acceleration due to gravity. Near the Earth's surface, $g = 9.8 \frac{\text{m}}{\text{sec}^2}$.
- About how far will a rock fall in 5 seconds if it is dropped close to the Earth's surface?
 - About how far will a rock fall in 5 seconds if it is dropped near the surface of the moon, where $g = 1.6 \frac{\text{m}}{\text{sec}^2}$?
 - Looking at the results of Parts a and b, notice that the smaller g -value on the moon resulted in the rock falling a shorter distance. What conjecture might you make about the g -value on Mars in relation to Earth, if a rock dropped close to the surface of Mars fell 46.25 m in 5 seconds?



In May 2008, NASA sent Phoenix to Mars, looking for evidence of life.

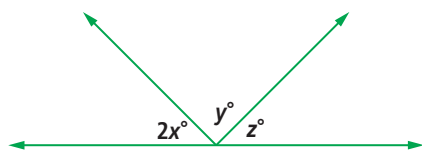
REVIEW

Every lesson contains review questions to practice ideas you have studied earlier.

In 19–23, tell which expression, (a) $x + y$, (b) $x - y$, (c) $y - x$, (d) xy , (e) $\frac{x}{y}$, or (f) $\frac{y}{x}$, correctly answers the given question. (Previous Course)

- You download x files in y minutes. What is your rate of download in files per minute?
- Toy cars are made x times the size of the actual car they represent. If the original car is y feet long, what is the length of the related toy car?
- You had x dollars, but after paying for lunch you have y dollars left. How much did lunch cost?
- Mindy gave Jian x marbles. Then Jian lost some of his marbles. If he has y marbles left, how many of his marbles did Jian lose?
- Destinee walked x mph for y hours. How many miles did she walk?
- Write examples of situations different from those in this lesson that lead to each expression you did not use as an answer in Questions 19–23. (Previous Course)

25. **Multiple Choice** Which sentence correctly relates the angle measures in the figure below? (Previous Course)



- A $x + y + z = 180$ B $x = 90 - y - z$
 C $2x + y + z = 180$ D none of these
26. What name is given to the polygon that is the shape of a stop sign? (Previous Course)
27. a. Solve $2x = 4x + 18$ for x .
 b. Check your work. (Previous Course)

EXPLORATION

These questions ask you to explore topics related to the lesson. Sometimes you will need to use references found in a library or on the Internet.

28. The graph of gold prices on page 5 looks distinctly different before and after 1971. Describe this difference in your own words. Do research on the Internet or at a library to find out what event happened to affect gold prices, and explain why this event caused the pattern of the graph to change.
29. Silver is another economically important metal. Using the information from the Internet or a library, make a graph of silver prices since 1960 similar to the graph of gold prices on page 5. Does your graph of silver prices follow the same patterns as the graph of gold prices?

QUIZ YOURSELF ANSWERS

1. 2604
 2. 16