Chapter 8 Summary and Vocabulary

- In Chapter 7, you saw many applications of the operation of powering to situations of exponential growth and decay. Another important application of powering is to the counting of objects: If there are n ways to select each object in a sequence of length L, then n^L different sequences are possible. **Powers** are also convenient for writing very large and very small numbers and are essential in **scientific notation.**

- The operation of taking a number to a power has many properties that connect it with multiplication and division. If two powers of the same number are multiplied or divided, the result is another power of that number: $x^m \cdot x^n = x^{m+n}$ and $\frac{x^m}{x^n} = x^{m-n}$. From these results, we can verify again that when $x \neq 0$, $x^0 = 1$. We can also deduce that $x^{-n} = \frac{1}{x^n}$.

- The terms **square root** and **cube root** come from the historical origins of these ideas in geometry. If a square has side s, its area is s^2, "s squared." If its area is A, the length of its side is $\sqrt{A}$, or $A^{\frac{1}{2}}$. If a cube has edge e, then its volume is e^3 or, "e cubed." If a cube has volume V, then each of its edges has length $\sqrt[3]{V}$.

- The ***n*th power** of a product xy is the product of the nth powers: $(xy)^n = x^n \cdot y^n$. Closely related to squares are square roots, and so we have $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$. Similarly, the nth power of a quotient is the quotient of the nth powers: $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ and so $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$. These properties help to simplify and rewrite expressions involving radicals.

- Two important applications of squares and square roots are the **Pythagorean Theorem** and the **distance formula** between two points in a coordinate plane, which is derived from that theorem.

Vocabulary

8-1
scientific notation

8-6
square
squared
square root
radical sign ($\sqrt{}$)
cube
cubed
cube root

8-7
radicand

Theorems and Properties

Multiplication Counting Principle (p. 458)
Arrangements Theorem (p. 459)
Product of Powers Property (p. 465)
Power of a Power Property (p. 466)
Quotient of Powers Property (p. 469)
Negative Exponent Property (p. 474)
Negative Exponent Property for Fractions (p. 475)
Power of a Product Property (p. 481)
Power of a Quotient Property (p. 482)
Square of the Square Root Property (p. 490)
Pythagorean Theorem (p. 492)
Cube of the Cube Root Property (p. 493)
Product of Square Roots Property (p. 498)
Quotient of Square Roots Property (p. 499)

Chapter 8 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

1. **Multiple Choice** $x^4 \cdot x^7 =$

   A x^{11} B x^{28} C $2x^{11}$ D $2x^{28}$

2. Rewrite 5^{-3} as a simple fraction.

3. Order from least to greatest: $(-4)(-3)$, $(-3)^4$, $(-4)^{-3}$

In 4–6, simplify the expression.

4. $\sqrt{600}$ 5. $\sqrt{25x}$ 6. $2^{\frac{1}{2}} \cdot 50^{\frac{1}{2}}$

In 7–12, simplify the expression.

7. $y^4 \cdot y^2$ 8. $(10m^2)^3$ 9. $\frac{a^{15}}{a^3}$

10. $\left(\frac{m}{6}\right)^3$ 11. $g^4 \cdot g \cdot g^0$ 12. $\frac{6n^2}{4n^3 \cdot 2n}$

13. Rewrite $4y^{-3}w^2$ without a negative exponent and justify your answer.

14. Rewrite $\frac{2}{x^2} \cdot \frac{5}{x^5}$ as a single fraction without negative exponents. Justify your steps.

15. The prime factorization of 288 is $2^5 \cdot 3^2$. Use this information to find the prime factorization of $10(288)^2$.

16. Rewrite $\left(\frac{3}{y^2}\right)^{-3}$ without parentheses or negative exponents.

17. Evaluate $\sqrt[3]{30}$ to the nearest thousandth.

18. If $f(x) = 1{,}000(1.06)^x$, estimate $f(-3)$ to the nearest integer.

19. State the general property that justifies each step.

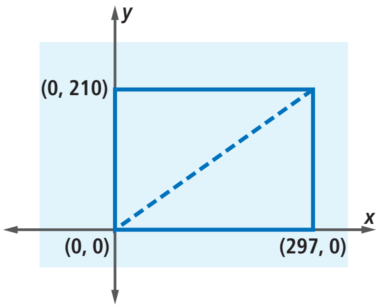
    a. $\left(\frac{x^{-9}}{x^{-5}}\right)^3 = \frac{(x^{-9})^3}{(x^{-5})^3}$ b. $= \frac{x^{-27}}{x^{-15}}$

    c. $= x^{-12}$ d. $= \frac{1}{x^{12}}$

20. Find a value of m for which $m^2 \neq m^{-2}$.

21. What is the distance between the points (9, 5) and (1, −10)?

22. Common notebook paper outside the United States is called A4, with dimensions 210 mm by 297 mm. A piece of A4 paper is placed on a grid as shown below. If the paper is cut along a diagonal, how long is the cut line?



23. The volume of a cube is 30 cubic inches. What is the length of an edge of the cube?

24. A square has a diagonal with length 12 meters. What is the area of the square?

25. In some states, a license plate consists of 2 letters followed by 4 digits. How many different license plates are possible that fit this description? (Note: There are ten digits: 0, 1, 2, . . ., 9.)

Chapter 8 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers.

OBJECTIVE A Simplify products, quotients, and powers of powers. (Lessons 8-2, 8-3, 8-4, 8-5)

In 1–7, simplify the expression.

1. $2m^2 \cdot 3m^3 \cdot 4m^4$
2. $\frac{6y^8}{12y^4}$
3. $\frac{7xy^2}{3x^2y^{-2}}$
4. $b^{-2}(4b^5)$
5. $\left(\frac{v^4}{v^8}\right)^6$
6. $\frac{7.28 \cdot 10^{115}}{8.3 \cdot 10^{72}}$
7. $a^{10}(3a^3) + 5a^{13}(a^7 - 2)$

In 8 and 9, rewrite the expression a. without fractions and b. without negative exponents.

8. $\frac{a^{-1}b^4}{b^{-2}c^3}$
9. $\frac{60x^2}{45x^{-2}}$
10. Rewrite $-ab^{-1}$ without a negative exponent.
11. Rewrite $(7x^{-7})(6y^{-3})$ without a negative exponent.

OBJECTIVE B Evaluate negative integer powers of real numbers. (Lessons 8-4, 8-5)

In 12–15, rewrite the expression as a decimal or fraction without an exponent.

12. 4^{-3}
13. 6^{-2}
14. $\left(\frac{1}{10}\right)^{-2}$
15. $\left(\frac{2}{3}\right)^{-5}$
16. If $f(x) = 2x^{-4}$, what is $f(-3)$?
17. The value of a house today is estimated at \$150,000 and has been growing at 3% a year. Its value x years from now will be $150{,}000(1.03)^x$. What was its value 5 years ago? Round your answer to the nearest thousand dollars.
18. Write $603.8 \cdot 10^{-4}$ in decimal form.
19. If $0.0051 = 5.1 \cdot 10^n$, what is n?

In 20 and 21, tell whether each expression names a positive number, a negative number, or zero.

20. a. $5^3 + 3^{-5}$
    b. $3^5 - 3^{-5}$
21. a. $\left(-\frac{1}{2}\right)^{-3}$
    b. $-\left(\frac{1}{2}\right)^{-3}$

OBJECTIVE C Rewrite powers of products and quotients. (Lessons 8-5, 8-9)

In 22–29, rewrite the expression without parentheses.

22. $(xy)^5$
23. $(60m^2n^3)^2$
24. $30\left(\frac{1}{3}u^4v\right)^3$
25. $\left(\frac{3}{4}\right)^{11} \cdot \left(\frac{6}{8}\right)^{-4}$
26. $\frac{1}{2}\left(\frac{1}{v}\right)^3 - (2v)^{-3}$
27. $\left(\frac{-8s}{t^2}\right)^{-4}$
28. $(m^3n^{-2})(m^2n^{-3})^3$
29. $\left(\frac{2y^2z^{-3}}{6y^{-3}z^4}\right)^{-2}$

OBJECTIVE D Simplify square roots. (Lessons 8-6, 8-7)

In 30–36, simplify the expression. Assume the variables stand for positive numbers.

30. $\sqrt{6} \cdot \sqrt{24}$
31. $(4^3 + 4^3)^{\frac{1}{2}}$
32. $\sqrt{3^2 + 4^2}$
33. $5\sqrt{7} \cdot 2\sqrt{3}$
34. $\sqrt{17m} \cdot \sqrt{17m}$
35. $\sqrt{\frac{4x^2}{y^2}}$
36. $\sqrt{36a^{36}b^4}$

OBJECTIVE E Evaluate cube roots. (Lesson 8-6)

In 37–40, give the exact cube root of the number, or the cube root rounded to the nearest thousandth.

37. –8
38. 1
39. 200
40. 1,330

In 41 and 42, estimate the number to the nearest thousandth and check your answer by an appropriate multiplication.

41. $\sqrt[3]{0.05}$

42. $\sqrt[3]{10}$

PROPERTIES The principles behind the mathematics

OBJECTIVE F **Test a special case to determine whether a pattern is true.** (Lesson 8-9)

43. Tell whether the pattern $x^4 = x^3$ is true for the given value of x.

a. $x = 1$

b. $x = 0$

c. $x = -1$

d. Based on your answers to Parts a–c, do you have evidence that the pattern is true, or are you sure it is not always true? Explain your reasoning.

44. Consider the pattern $(xy)^{-2} = \frac{1}{x^2y^2}$, where x and y are not zero.

a. Is the pattern true when $x = 5$ and $y = 3$?

b. Is the pattern true when $x = -4$ and $y = 0.5$?

c. Do you have evidence that the pattern is true for all nonzero real number values of x and y? Explain your reasoning.

In 45 and 46, find a counterexample to the pattern.

45. $-4a = a^{-4}$

46. $(x + y)^2 = x^2 + y^2$

OBJECTIVE G **Identify properties of powers that justify a simplification, from the following list: Zero Exponent Property (Chapter 7); Negative Exponent Property; Power of a Product Property; Power of a Quotient Property; Product of Powers Property; Quotient of Powers Property; Power of a Power Property.** (Lessons 8-2, 8-3, 8-4, 8-5)

In 47–52, identify the property or properties that justify the simplification. Assume all variables represent positive numbers.

47. $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

48. $a^{10} = a^8 \cdot a^2$

49. $(b^4)^0 = 1$

50. $2^{-n} = \frac{1}{2^n}$

51. $4^{-2} \cdot 4^3 = 4$

52. $\frac{w^2a^2}{w^2h^{-1}} = aah$

53. Show and justify two different ways to simplify $\left(\frac{12}{13}\right)^{-4}$.

54. Show and justify two different ways to simplify $\frac{m^{-1}}{n^{-1}}$.

USES Applications of mathematics in real-world situations

OBJECTIVE H **Use powers to count the number of sequences possible for repeated choices.** (Lesson 8-1)

55. A test contains 5 questions where the choices are *always*, *sometimes but not always*, or *never*.

a. How many different answer sheets are possible?

b. If you guess, what is the probability that you will answer all 5 questions correctly?

c. If you guess, what is the probability that you will answer all 5 questions incorrectly?

56. Excluding the five vowels A, E, I, O, and U in the English language, how many 4-letter acronyms are possible?

57. A restaurant serves two different types of pizza (thin crust and deep dish), three sizes, and nine toppings. How many different pizzas with one topping are possible?

58. Imagine that a fair coin is tossed 12 times. The result of each toss is recorded as H or T.

a. How many different sequences of H and T are possible?

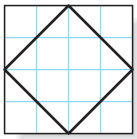
b. What is the probability that all the tosses are heads?

c. What is the probability of getting the sequence HTHHTTTHTTTH?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE I Represent squares, cubes, square roots, and cube roots geometrically. (Lesson 8-8)

59. The area of the tilted square below is 8 square units. What is the length of a side of the tilted square?



60. A square has area 1,000 square units.

a. Give the exact length of a side.

b. Estimate the length of a side to the nearest hundredth.

61. If a square has side xy, what is its area?

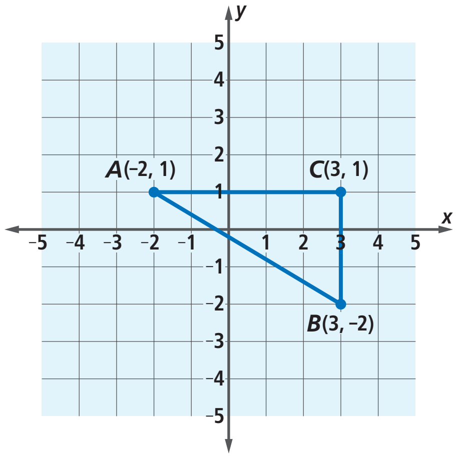
62. If a cube has an edge of length 2.3 cm, what is its volume?

63. A cube has a volume of 1 cubic meter. What is the length of an edge?

64. A cube has a volume of 2 cubic meters. What is the length of an edge?

OBJECTIVE J Calculate distances on the *x*-*y* coordinate plane. (Lesson 8-8)

In 65 and 66, use the graph below.



65. Find each length.

a. AC **b.** BC **c.** AB

66. If O is the point (0, 0), find OB and OC.

In 67–70, find the distance between the points.

67. (2, 11) and (11, 2) **68.** (6, 5) and (3, –5)

69. (a, b) and (–1, 4) **70.** (x, y) and (h, k)

71. Stanton, Nebraska is about 8 miles east and 3 miles south of the center of Norfolk, Nebraska.

a. On a straight line distance, is it true that Stanton is less than 9 miles from the center of Norfolk?

b. Draw a picture to justify your answer to Part a.