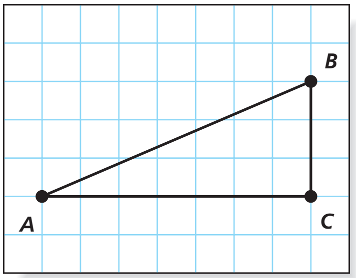
Lesson

8-8

Distance in a Plane

▶ **BIG IDEA** Using the Pythagorean Theorem, the distance between any points in a plane can be found if you know their coordinates.

Competitions involving small robots (sometimes called Robot Wars™ or BotBashes™) began in the late 1990s as engineering school projects but quickly spread to competitions open to the public. Even television programs have featured the battling 'bots. The competitions take place in an enclosed arena that is laid out in a grid pattern like the one below.



To get from point A to point B in the arena, robots can be maneuvered manually by their "driver," but because the shortest distance between A and B is the straight line segment, robot designers like to program direction and distance commands into their robots. The distance traveled is an application of the Pythagorean Theorem because side $\overline{AB}$ is the hypotenuse of a right triangle.

Distances along Vertical and Horizontal Lines

To find the distance between any two points in the coordinate plane, we begin by examining the situation where points are on the same vertical or horizontal line.

You can find the distance between two points on vertical or horizontal lines by thinking of them as being on a number line. The distance can be obtained by counting spaces or by subtracting appropriate coordinates. Consider the rectangle $DEFG$ graphed on the next page.

Mental Math

What is 20% of each quantity?

a. $40x$

b. $5y$

c. $40x + 5y$

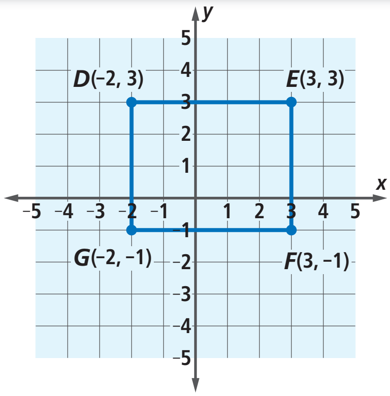
Japan's humanoid robot VisiON NEXTA kicks a ball during a penalty kick competition at the RoboCup 2005 in Osaka, Japan.

Horizontal Distance The distance DE can be found by counting spaces on the number line or it can be calculated by subtracting the x-coordinates and then taking the absolute value.

$$DE = |-2 - 3| = 5$$

Vertical Distance Similarly, the distance EF can be found by counting spaces or it can be calculated by subtracting the y-coordinates and taking the absolute value.

$$EF = |3 - (-1)| = 4$$



STOP QY1

▶ QY1

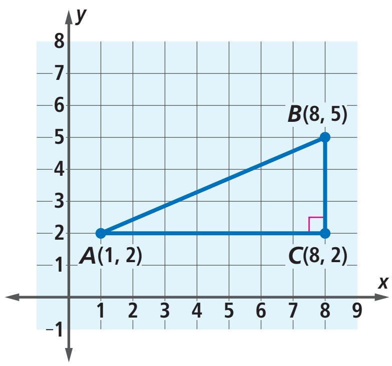
Find the length of the segment whose endpoints are (50, 21) and (50, 46).

The Distance between Any Two Points in a Plane

The Pythagorean Theorem enables you to find the distance between any two points in the plane.

Example 1

Find AB in $\triangle ABC$ at the right.



Solution $\overline{AB}$ is the hypotenuse of $\triangle ABC$ whose legs, $\overline{AC}$ and $\overline{BC}$, are horizontal and vertical, respectively. The length of the legs can be calculated by subtracting appropriate coordinates.

$AC = |8 - 1| = 7$ $\qquad$ $BC = |5 - 2| = 3$

Now apply the Pythagorean Theorem.

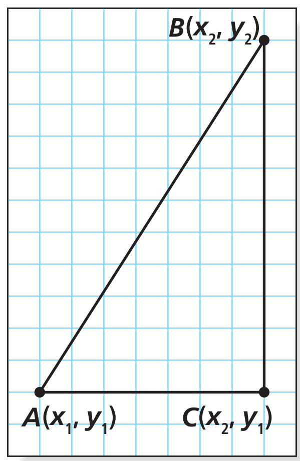
$(AB)^2 = (AC)^2 + (BC)^2$

$(AB)^2 = 7^2 + 3^2$

$(AB)^2 = 58$

$AB = \sqrt{58} \approx 7.62$

This method can be generalized to find the distance between any two points on a coordinate grid.



Let point $A = (x_1, y_1)$ and $B = (x_2, y_2)$, as shown at the right. Then a right triangle can be formed with a third vertex at $C = (x_2, y_1)$. Using these coordinates, $AC = |x_2 - x_1|$ and $BC = |y_2 - y_1|$. Now use the Pythagorean Theorem.

$$AB^2 = AC^2 + BC^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

Since a number and its absolute value have the same square, $|x_2 - x_1|^2 = (x_2 - x_1)^2$ and $|y_2 - y_1|^2 = (y_2 - y_1)^2$.

Thus $AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

Take the positive square root of each side. The result is a formula for the distance between two points in the coordinate plane.

Formula for the Distance between Two Points in a Coordinate Plane

The distance AB between the points $A = (x_1, y_1)$ and $B = (x_2, y_2)$ in a coordinate plane is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

GUIDED

Example 2

Find the distance between the points (23, 16) and (31, −11) to the nearest thousandth.

Solution Using the formula for the distance between two points in a plane, let $A = (x_1, y_1) = (\underline{\ ?\ }, \underline{\ ?\ })$ and $B = (x_2, y_2) = (\underline{\ ?\ }, \underline{\ ?\ })$.

$AB = \sqrt{(\underline{\ ?\ } - \underline{\ ?\ })^2 + (\underline{\ ?\ } - \underline{\ ?\ })^2}$

$= \sqrt{(\underline{\ ?\ })^2 + (\underline{\ ?\ })^2}$

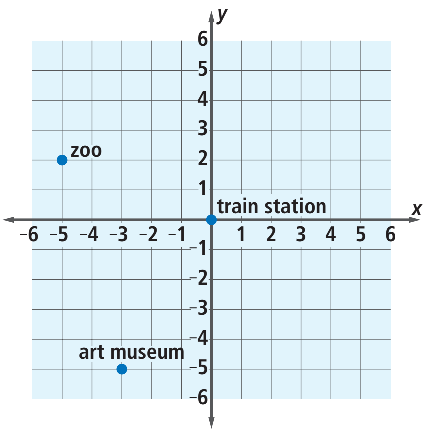
$= \sqrt{\underline{\ ?\ } + \underline{\ ?\ }} = \sqrt{\underline{\ ?\ }} \approx \underline{\ ?\ }$

Example 3

Use the map at the right. It shows streets and the locations of three buildings in a city. The streets are 1 block apart.

a. Give the coordinates of all three buildings.

b. Find the distance from the train station to the zoo.



Solutions

a. The coordinates are as follows.

train station (0, 0)

zoo (-5, 2)

art museum (-3, -5)

b. We need to find the distance from (0, 0) to (−5, 2).

distance $= \sqrt{(0 - -5)^2 + (0 - 2)^2}$

$= \sqrt{(5)^2 + (-2)^2}$

$= \sqrt{25 + 4}$

$= \sqrt{29} \approx 5.39$ blocks

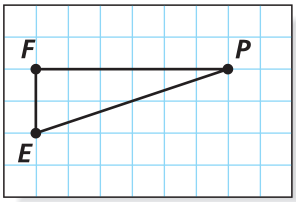
QY2

> **QY2**
> In Example 3, find the distance from the zoo to the art museum.

Questions

COVERING THE IDEAS

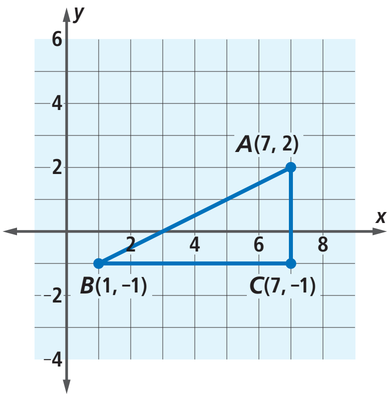
1. Refer to $\triangle EFP$ at the right.
   a. Determine FE.
   b. Determine FP.
   c. Use the Pythagorean Theorem to calculate EP.

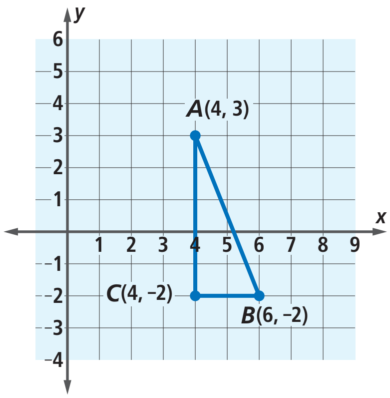


In 2–5, find *PT*.

2. $P = (3, 4)$ and $T = (3, 9)$
3. $P = (-3, 4)$ and $T = (4, 4)$
4. $P = (-\frac{2}{3}, \frac{1}{2})$ and $T = (\frac{5}{3}, \frac{1}{2})$
5. $P = (33, -4)$ and $T = (33, 18)$

In 6 and 7, find the length of $\overline{AB}$.

6. 

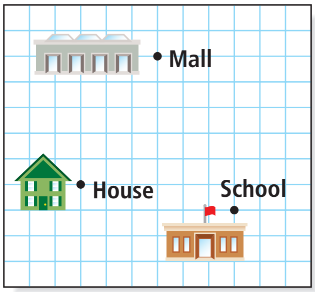
7. 

In 8–13, find the distance between the two points.

8. $E = (4, 9)$; $D = (8, 6)$
9. $F = (15, 2)$; $G = (20, -10)$
10. $H = (5, -1)$; $I = (11, 2.2)$
11. $J = (-6, -7)$; $K = (-2, 0)$
12. $L = (-1, 2)$; $M = (-3, 4)$
13. $N = (-0.43, -0.91)$; $P = (-0.36, -0.63)$

APPLYING THE MATHEMATICS

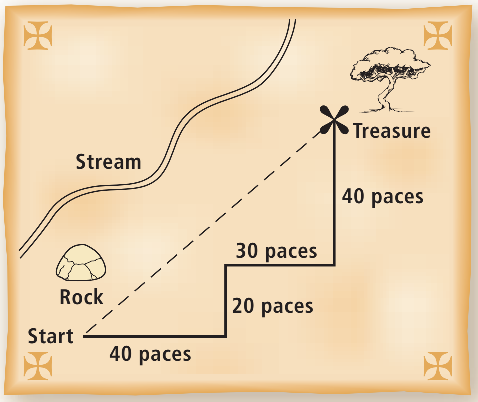
In 14 and 15, use the map at the right, which shows the locations of a house, school, and mall. Suppose each square of the grid is a half mile on a side.



14. **a.** What is the distance from the house to the school?
    b. Which is closer to the house, the mall or the school?

15. How far is it from school to the mall?

16. Pirate Slopebeard has the treasure map below. How far is the treasure from the start if you travel along a straight path "as the crow flies?"



17. Write an expression for the distance between the points (0, 0) and (b, d).

18. The vertices of a triangle are (3, 4), (6, 9), and (9, 4). Is the triangle equilateral? How do you know?

19. Write the distance formula using a power instead of a radical sign.

20. Does it matter which ordered pair is first when using the distance formula? Choose two ordered pairs. Do the calculation both ways to verify your answer.

REVIEW

21. Evaluate $3^{\frac{1}{2}} \cdot 4^{\frac{1}{2}} \cdot 12^{\frac{1}{2}}$ in your head. **(Lesson 8-7)**

22. Consider the function f with $f(x) = 1.5^x$. Find the following values to the nearest hundredth. **(Lessons 8-6, 7-6)**
    a. $f(0)$ **b.** $f\left(\frac{1}{2}\right)$ **c.** $f(1)$
    d. $f(2)$ **e.** $f(-1)$ **f.** $f(-2)$

23. If $d = \sqrt{6}$, find the value of $\frac{d^5}{(3d)^2} \cdot 10d$. **(Lessons 8-6, 8-5)**

24. Suppose Pythagoras Park is a rectangle 250 meters wide and 420 meters long. There are sidewalks around the edges of the park and a diagonal sidewalk connecting the southeast corner to the northwest corner. Esmeralda and Dory are standing at the southeast corner and want to get to the ice cream stand at the northwest corner. (**Lessons 8-6, 5-3**)

a. How many meters would Esmeralda and Dory have to walk if they traveled along the diagonal sidewalk? Round your answer to the nearest meter.

b. How many meters would Esmeralda and Dory have to walk if they traveled along the edge sidewalks?

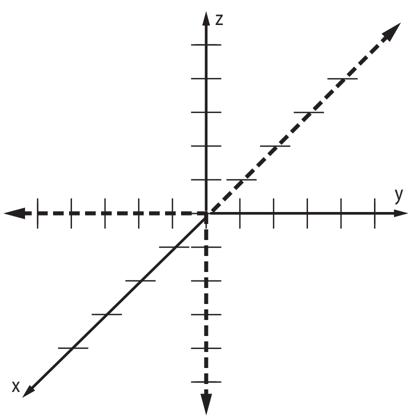
c. Esmeralda walks along the diagonal sidewalk at 60 meters per minute while Dory jogs along the edge sidewalks at 100 meters per minute. Who arrives at the ice cream stand first?

25. Suppose the graph of f is a line with slope $\frac{8}{5}$ and $f(7) = 1.2$. Write a formula for $f(x)$. (**Lessons 7-6, 6-2**)

26. Suppose $f(x) = 29x - 2$ and $g(x) = 2(34.5x + 17.75)$. For what value of x does $f(x) = g(x)$? (**Lessons 7-6, 4-4**)

EXPLORATION

27. You may have seen videos showing giant robotic arms maneuvering through space to perform a task. Did you ever wonder how the robot is controlled? One component is calculating how far to move the arm. To do this, designers extend the ideas in this lesson from 2 dimensions (x, y) to 3 dimensions (x, y, z) and calculate the distance between two *ordered triples* that describe locations in space. The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in space is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. Calculate the distance between (3, 5, 2) and (–4, 3, –2). Draw a picture of the two points and the distance on the graph below.



According to the U.N.'s 2004 World Robotics Survey, most industrial robots are used on assembly lines, chiefly in the auto industry.

QY ANSWERS

1. 25

2. $\sqrt{53} \approx 7.28$ blocks