Lesson 8-3

Quotients of Powers

BIG IDEA Because of the relationship between multiplication and division, quotients of powers can be themselves written as powers.

As you know, $\frac{24}{3} = 8$ because $8 \cdot 3 = 24$. Similarly, $\frac{24}{3} = \frac{8 \cdot \cancel{3}^{1}}{1\cancel{3}} = 8$. Both of these methods can be helpful in understanding quotients of powers.

For example, suppose $\frac{x^{10}}{x^2} = x^?$. By rewriting this statement to read $x^? \cdot x^2 = x^{10}$, we can apply the Product of Powers Property, $x^{?+2} = x^{10}$. You can see that the unknown exponent is 8 because $8 + 2 = 10$. So $\frac{x^{10}}{x^2} = x^8$. Another way of finding the unknown exponent in $\frac{x^{10}}{x^2} = x^?$ is to write both the numerator and denominator in expanded form and simplify the fraction.

$$\frac{x^{10}}{x^2} = \frac{{}^{1}\cancel{x} \cdot {}^{1}\cancel{x} \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{{}_{1}\cancel{x} \cdot \cancel{x}_{1}}$$
$$= x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$$
$$= x^8$$

Activity

1. Simplify each expression.
   a. $\frac{a^7}{a^4}$
   b. $\frac{m^{14}}{m^5}$
   c. $\frac{y^{12}}{y}$
   d. $\frac{n^{13}}{n^{13}}$
2. When dividing powers of the same base, how is the exponent of the answer related to the exponents of the original division?

The general pattern established in the Activity is the *Quotient of Powers Property*.

Quotient of Powers Property

For all m and n, and all nonzero b, $\frac{b^m}{b^n} = b^{m-n}$.

STOP QY

Mental Math

True or false?

a. $x^{100} + x^{101} = x^{201}$

b. $x^{100} \cdot x^{101} = x^{201}$

c. $x^{100} + x^{101} = 2x^{101}$

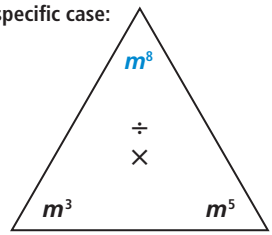
d. $x^{100} + x^{100} = 2x^{100}$

QY

Simplify $\frac{z^{50}}{z^{10}}$.

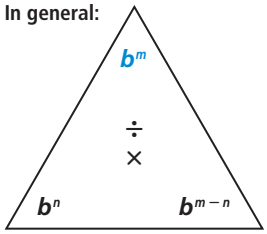
Fact triangles are another way of representing the Quotient of Powers Property.

A specific case:



$$\frac{m^8}{m^3} = m^5$$
$$\frac{m^8}{m^5} = m^3$$
$$m^3 \cdot m^5 = m^8$$
$$m^5 \cdot m^3 = m^8$$

In general:



$$\frac{b^m}{b^n} = b^{m-n}$$
$$\frac{b^m}{b^{m-n}} = b^n$$
$$b^n \cdot b^{m-n} = b^{n+m-n} = b^m$$
$$b^{m-n} \cdot b^n = b^{m-n+n} = b^m$$

The Zero Power

In Question 1d of the Activity, you should have seen that $\frac{n^{13}}{n^{13}} = 1$. More generally, consider the fraction $\frac{b^m}{b^m}$. By the Quotient of Powers Property, $\frac{b^m}{b^m} = b^{m-m} = b^0$. But you also know that any nonzero number divided by itself is 1. So $1 = \frac{b^m}{b^m} = b^0$. This is another way of showing why $b^0 = 1$.

An Application of the Quotient of Powers Property

The Quotient of Powers Property is useful in dividing numbers written in scientific notation.

Example 1

The Gross Domestic Product (GDP) of a country is the total value of all the goods and services produced in the country. When the GDP is divided by the population of the country, the result is the GDP per person, often called the *GDP per capita.* In 2006, Denmark had a population of about 5.5 million and a GDP of $243.4 billion. What is Denmark's GDP per capita?

Solution Since GDP per capita is a rate unit, the answer is found by division.

$$\frac{\$243{,}400{,}000{,}000}{5.5 \text{ million people}} = \frac{2.434 \times 10^{11}}{5.5 \times 10^6}$$ Write in scientific notation.

$$= \frac{2.434}{5.5} \cdot \frac{10^{11}}{10^6}$$ Multiplying Fractions Property

$$\approx 0.44 \cdot 10^5$$ Quotient of Powers Property

$$\approx \$44{,}000/\text{person}$$ Write in base 10.

Approximately 5.5 million people live in Denmark, making it one of the most densely populated nations in Northern Europe.

Source: Danish Tourist Board

Check Change the numbers to decimal notation and simplify the fraction.

$$\frac{243{,}400{,}000{,}000}{5{,}500{,}000} = \frac{2{,}434{,}000}{55} \approx 44{,}000$$

Dividing Powers with Different Bases

To use the Quotient of Powers Property, the bases must be the same. For example, $\frac{a^5}{b^2}$ cannot be simplified further. To divide two algebraic expressions that involve different bases, group powers of the same base together and use the Quotient of Powers Property to simplify each fraction.

Example 2

Simplify $\frac{30a^3n^6}{5a^2n}$.

Solution 1

$$\frac{30a^3n^6}{5a^2n} = \frac{30}{5} \cdot \frac{a^3}{a^2} \cdot \frac{n^6}{n}$$ Multiplying Fractions Property

$$= \frac{30}{5} \cdot a^{3-2} \cdot n^{6-1}$$ Quotient of Powers Property

$$= 6 \cdot a^1 \cdot n^5 = 6an^5$$ Arithmetic

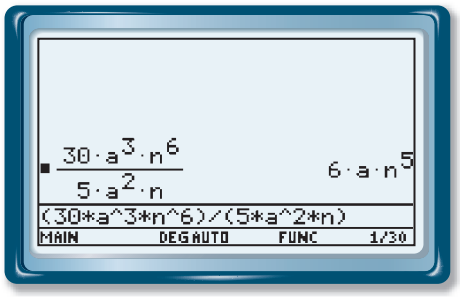
Solution 2

$$\frac{30a^3n^6}{5a^2n} = \frac{30 \cdot a \cdot a \cdot a \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n}{5 \cdot a \cdot a \cdot n}$$ Repeated Multiplication Property of Powers

$$= \frac{6 \cdot a \cdot n \cdot n \cdot n \cdot n \cdot n}{1}$$ Equal Fractions Property

$$= 6an^5$$ Arithmetic

Check Use a CAS to check your answer, as shown below.



Questions

COVERING THE IDEAS

In 1–3, write the quotient as a single power.

1. $\frac{2^7}{2^4}$
2. $\frac{8^5}{8^m}$
3. $\frac{3^m}{3^n}$

In 4–9, use the Quotient of Powers Property to simplify the fraction.

4. $\frac{x^{12}}{x^2}$
5. $\frac{a^{20}}{a^{20}}$
6. $\frac{6.5 \times 10^{21}}{3.1 \times 10^{19}}$
7. $\frac{12a^2b^{12}}{2ab^7}$
8. $\frac{2a^5b^9}{8a^3b}$
9. $\frac{24a^{10}b^5}{6a^4b^5}$

10. In 2006, the African country of Burundi had a population of about 8.1 million and a GDP of about $5.7 billion. What is Burundi's GDP per capita?
11. Why can't $\frac{a^3}{b^4}$ be simplified?
12. If $\frac{b^n}{b^m} = 1$, how must m and n be related?

APPLYING THE MATHEMATICS

In 13–15, write the quotient as a single power.

13. $\frac{4^3}{2^6}$
14. $\frac{16 \cdot 2^m}{2^6}$
15. $\frac{4^3}{8 \cdot 2^6}$

In 16–19, rewrite the expression so that it has no fraction.

16. $\frac{(7m)^5}{(7m)^3}$
17. $\frac{(7+3m)^7}{(7+3m)^6}$
18. $\frac{x^{5a-10}}{x^{3-3a}}$
19. $\frac{2a^6 + 6a^6}{2a^5}$

20. In Norway in 2002, $4.7 \cdot 10^7$ kilograms of ground coffee were consumed, and the total population was approximately 4,525,000. To brew a typical cup of coffee, you need 10.6 grams of ground coffee. Determine the coffee consumption (in terms of cups per person) in Norway in 2002.
21. In 2005, the world's population was approximately $6.446 \cdot 10^9$ people. In the same year, global oil output was approximately $8.0 \cdot 10^7$ million barrels per day. A barrel is equivalent to 35 gallons of oil.
    a. How many barrels of oil was this per person per day?
    b. At this rate, how many gallons of oil were consumed per person during 2005?
22. Write an algebraic fraction that can be simplified to $12a^2b$ using the Quotient of Powers Property.

The number of cups of coffee consumed per capita per year in the Nordic countries of Norway, Sweden, Denmark, and Finland is among the highest in the world.

Source: nationmaster.com

REVIEW

In 23–26, simplify the expression. (Lesson 8-2)

23. $3x \cdot x^2$
24. $n \cdot n^2 \cdot n^3$
25. $2h^3 \cdot 6h^4 + 3h \cdot 4h^6$
26. $a^x \cdot a^y \cdot a^z$

27. Suppose each question on a 5-question, multiple-choice quiz has four choices. **(Lesson 8-1)**
    a. Give the probability of guessing all the correct answers as the reciprocal of a power.
    b. Give the probability of guessing all wrong answers.

28. Distances after various times when traveling at 65 miles per hour are shown on the spreadsheet below. **(Lesson 6-1)**

◇	A	B
1	Time (hours)	Distance (miles)
2	1.0	?
3	1.5	97.5
4	2	130
5	2.5	162.5
6	3	?
7	3.5	?

    a. Complete the spreadsheet.
    b. Name two ways to get the value in cell B7.

29. Calculate the total cost in your head. **(Lesson 2-2)**
    a. 8 cans of tuna fish at $2.99 per can
    b. 5 tickets to a movie at $10.50 per ticket

EXPLORATION

30. The average 14 year old has a volume of about 3 cubic feet.
    a. Consider all the students in your school. Would their total volume be more or less than the volume of one classroom? Assume the classroom is 10 feet high, 30 feet long, and 30 feet wide.
    b. Assume the population of the world to be 6.4 billion people and the average volume of a person to be 4 cubic feet. Is the volume of all the people more or less than 1 cubic mile? How much more or less? (There are $5{,}280^3$ cubic feet in a cubic mile.)

QY ANSWER

z^{40}