Lesson 8-2 Products and Powers of Powers

▶ BIG IDEA Because of the relationship between repeated multiplication and powers, products and powers of powers can be themselves written as powers.

Multiplying Powers with the Same Base

When n is a positive integer, $x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}$. From this, a number of important properties can be developed. They all involve multiplication in some way because of the relationship between exponents and multiplication. Addition is different. In general, there is no way to simplify the sum of two powers. For example, $3^2 + 3^4 = 9 + 81 = 90$, and 90 is not an integer power of 3. But notice what happens when we multiply powers with the same base.

$$3^2 \cdot 3^4 = \underbrace{(3 \cdot 3)}_{2 \text{ factors}} \cdot \underbrace{(3 \cdot 3 \cdot 3 \cdot 3)}_{4 \text{ factors}} = \underbrace{(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}_{6 \text{ factors}} = 3^6$$

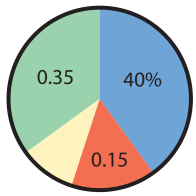
$$1.06^0 \cdot 1.06^3 = 1 \cdot \underbrace{(1.06 \cdot 1.06 \cdot 1.06)}_{3 \text{ factors}} = 1.06^3$$

$$(-6)^5 \cdot (-6)^5 = \underbrace{(-6 \cdot -6 \cdot -6 \cdot -6 \cdot -6)}_{5 \text{ factors}} \cdot \underbrace{(-6 \cdot -6 \cdot -6 \cdot -6 \cdot -6)}_{5 \text{ factors}} = (-6)^{10}$$

These three expressions involved multiplying powers of the same base, where the base was a specific number (3, 1.06, or –6). The same process is used to multiply powers of a variable.

Mental Math

Use the circle graph. Give each value in the indicated form.



a. blue sector, decimal

b. green sector, fraction

c. red sector, percent

d. yellow sector, decimal

e. yellow sector, fraction

Activity

Step 1 Evaluate each expression.

a. $z^7 \cdot z^4$ b. $y^6 \cdot y^8$ c. $y^2 \cdot y^4 \cdot y^3$ d. $x^3 \cdot x$

e. $x^4 \cdot x^2 \cdot x$ f. $t \cdot t \cdot t$ g. $t^3 \cdot t^4 \cdot t \cdot t$ h. $z^0 \cdot z^5 \cdot z^2 \cdot z^2$

Step 2 Check your answers using a CAS.

a. When multiplying powers with the same base, how is the exponent of the answer related to the exponents of the original factors?

b. Some of the variables do not have visible exponents, like $x^3 \cdot x$. Does the relationship you described in Part a apply in this case?

c. Refer to Part h. What does z^0 equal? How does this fit in with the answer to Part 2a?

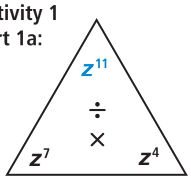
The general pattern established in the activity leads us to the *Product of Powers Property*.

Product of Powers Property

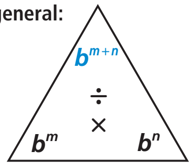
For all m and n, and all nonzero b, $b^m \cdot b^n = b^{m+n}$.

The Product of Powers Property can be illustrated with a multiplication fact triangle. Notice that the powers are multiplied, but the exponents are added.

Activity 1 Part 1a:



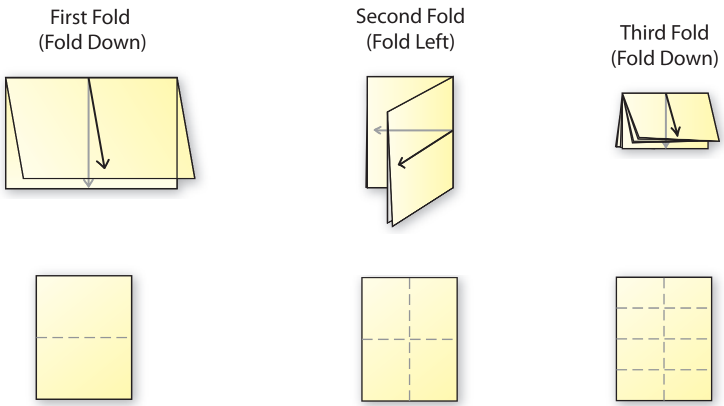
In general:



Here is a situation leading to multiplying powers with the same base.

Example 1

Suppose you fold an 8.5-inch by 11-inch piece of paper alternating the direction of the folds (fold down, fold to the left, fold down, fold to the left, and so on).



Imagine that you keep folding indefinitely. Write an expression for the number of regions created by first folding the paper n times, and then folding it *three times more.*

Solution You begin with 1 piece of paper, which is 1 region. Each time you fold the paper, you double the number of regions. After n folds, you have 2^n regions. Folding an additional 3 times doubles the number of regions 3 more times. The number of folds is $2^n \cdot 2 \cdot 2 \cdot 2$ or $2^n \cdot 2^3$. Applying the Product of Powers Property, $2^n \bullet 2^3 = 2^{n+3}$.

Multiplying Powers with Different Bases

The Product of Powers Property tells how to simplify the product of two powers with the same base. A product with different bases, such as $a^3 \cdot b^4$, usually *cannot* be simplified.

Example 2

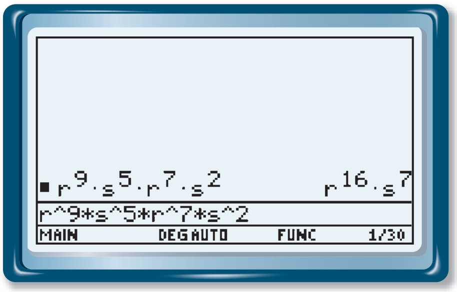
Simplify $r^9 \cdot s^5 \cdot r^7 \cdot s^2$.

Solution Use the properties of multiplication to group factors with the same base.

$r^9 \cdot s^5 \cdot r^7 \cdot s^2$	$= r^9 \cdot r^7 \cdot s^5 \cdot s^2$	Commutative Property of Multiplication
	$= r^{9+7} \cdot s^{5+2}$	Product of Powers Property
	$= r^{16} \cdot s^7$	Simplify.

$r^{16} \cdot s^7$ cannot be simplified further because the bases are different.

Check Perform the multiplication with a CAS.



A CAS indicates that $r^9 \cdot s^5 \cdot r^7 \cdot s^2 = r^{16} \cdot s^7$. It checks.

What Happens If We Take a Power of a Power?

When powers of powers are calculated, interesting patterns also emerge.

Example 3

Write $(5^2)^4$ as a single power.

Solution Think of 5^2 as a number that is raised to the 4^{th} power.

$(5^2)^4$	$= 5^2 \cdot 5^2 \cdot 5^2 \cdot 5^2$	Repeated Multiplication Model for Powering.
	$= 5^{2+2+2+2}$	Product of Powers Property
	$= 5^8$	Simplify.

The general pattern is called the *Power of a Power Property.*

Power of a Power Property

For all *m* and *n*, and all nonzero *b*, $(b^m)^n = b^{mn}$.

Some expressions involve both powers of powers and multiplication.

Example 4

Simplify $3m(m^4)^2$.

Solution 1 First rewrite $(m^4)^2$ as repeated multiplication.

$$3m(m^4)^2 = 3m^1 \cdot m^4 \cdot m^4 = 3m^9$$

Solution 2 First use the Power of a Power Property with $(m^4)^2$.

$$3m(m^4)^2 = 3m^1 \cdot m^8 = 3m^9$$

Questions

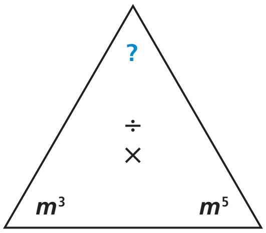
COVERING THE IDEAS

In 1 and 2, write the product as a single power.

1. $18^5 \cdot 18^4$
2. $(-7)^3 \cdot (-7)^2$
3. Write $w^4 \cdot w^3$ as a single power and check your answer by substituting 2 for w.

In 4–6, suppose you fold an 8.5-inch by 11-inch piece of paper as in Example 1. Calculate the number of regions created by folding the paper in the way described.

4. two times, then three more times
5. three times, then two more times, then two more times
6. m times, then n more times
7. Find the expression that completes the fact triangle at the right.



In 8–10, rewrite the expression as a single power.

8. $(2^3)^4$
9. $(m^5)^2$
10. $(y^2)^6$

In 11–16, simplify the power.

11. $3a^4 \cdot 5a^2$
12. $2(k^{10})^7$
13. $d(d^{13})$
14. $m^2 \cdot m^9 \cdot a^0 \cdot m^7 \cdot a^9$
15. $a^3(b^3a^5)$
16. $4k^2(k^3)^5$

APPLYING THE MATHEMATICS

17. A quiz has two parts. The first part has 5 multiple-choice questions. The second part has 3 multiple-choice questions. Each multiple-choice question has 4 choices. How many different sequences of answers are possible on the 8 questions?

In 18–20, solve the equation. Show all work.

18. $2^4 \cdot 2^n = 2^{12}$
19. $(5^6)^x = 5^6$
20. $(a^7 \cdot a^n)^2 = a^{24}$

21. Suppose a population P of bacteria triples each day.

a. Write an expression for the number of bacteria after 4 days.

b. How many days after the 4^{th} day will the bacteria population be $P \cdot 3^{20}$?

22. Does the Product of Powers Property work for fractions? Write each expression as a power and a simple fraction.

a. $\frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5}$ b. $\frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5}$ c. $\frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5}$

REVIEW

23. Abigail is going to buy a new car. She has to choose the body style (sedan, SUV, or convertible), transmission (automatic or standard), and color (white, black, red, blue, or green). (**Lesson 8-1**)

a. How many different ways can Abigail make her choices?

b. If another color choice of silver is given to her, how many more choices does she have?

In 2004, there were 4,236,736 passenger cars produced in the United States.

Source: Automotive News Data Center

24. If $f(x) = 3x + 2$ and $g(x) = 3x^2 - 2$, find each value. (**Lesson 7-6**)

a. $f(3)$ b. $g(-2)$ c. $f(5) - g(5)$ d. $g(-4) + f(-4)$

25. A band sold 1,252 tickets for a concert that were priced at \$35. The band decided to lower the ticket price to their next concert to \$30 in hopes of attracting a larger audience. After lowering the price, 1,510 tickets were sold. (**Lessons 6-6, 3-4**)

a. Write a linear equation that relates the price of the ticket x and the number of tickets sold y.

b. Use your answer to Part a to predict the number of tickets that will be sold if the price is lowered to \$20.

26. Consider the line $y = 4x - 5$. Find (**Lessons 6-4, 6-2**)

a. its slope. b. its y-intercept. c. its x-intercept.

27. Write 0.00324 in scientific notation. (**Previous Course**)

28. Write these numbers as decimals. (**Previous Course**)

a. $9.8 \cdot 10^0$ b. $9.8 \cdot 10^{-1}$

c. $9.8 \cdot 10^{-2}$ d. $9.8 \cdot 10^{-3}$

EXPLORATION

29. There are prefixes in the metric system for some of the powers of 10. For example, the prefix for 10^3 is kilo-, as in kilogram, kilometer, and kilobyte. Give the metric prefix for each power.

a. 10^6 b. 10^9 c. 10^{12} d. 10^{15} e. 10^{18}