Lesson 8-1

The Multiplication Counting Principle

Vocabulary

scientific notation

BIG IDEA Expressions involving powers result from certain counting problems and are used in scientific notation.

Mental Math

Simplify $\frac{n \cdot 3n \cdot 6n \cdot 9n}{n \cdot 2n \cdot 4n \cdot 6n}$.

The Jaipur Friendship and Knitting Society decided that it would call itself by the 3-letter acronym JFK. (An *acronym*, like USA or NCAA, is a "word" made up of the first letter of each word in a phrase.) They were disappointed to learn that JFK was already a popular acronym. It is the initials of President John Fitzgerald Kennedy and identifies one of the airports in New York City as well as some highways throughout the country. So the society decided to use JFKS.

Members of the society realized that there is only a certain number of 3-letter acronyms. So they wondered, how many 3-letter acronyms are there in the English language?

To answer this question, we apply a very useful problem-solving strategy. We consider a simpler problem we may be able to solve and then apply its solution to the problem we want to solve.

Knitting was first introduced to Europe in the 5th century CE.

Source: Fine Living TV Network

Example 1

How many 2-letter acronyms are there?

Solution Count the acronyms in an organized manner, alphabetically. AA, AB, AC, ..., AZ gives 26. BA, BB, BC, ... BZ gives another 26. There will be 26 groups of 26, so the total number is $26 \cdot 26 = 26^2 = 676$.

Notice that we gave the answer to Example 1 in three forms: as a *product* $26 \cdot 26$, as a *power* 26^2, in our customary *base-10 decimal* system as 676. Each of these forms is useful, so you need to be able to move back and forth from one way of writing a number to another.

Example 1 applies multiplication in a manner that is so important that it has a special name, the *Multiplication Counting Principle*.

Multiplication Counting Principle

If one choice can be made in m ways and then a second choice can be made in n ways, then there are mn ways of making the first choice followed by the second choice.

The Multiplication Counting Principle was applied in Example 1. There were 26 choices for the first letter. After that choice was made, there were 26 choices for the second letter. So $m = 26$ and $n = 26$, and the number of 2-letter acronyms is $26 \cdot 26$, or 676.

Example 2

How many 3-letter acronyms are there?

Solution 1 Apply the Multiplication Counting Principle to the result of Example 1, which found that there are 676 different 2-letter acronyms. For each one, there are 26 possible third letters. So the total number is $676 \cdot 26 = 17{,}576$.

Solution 2 Keep the result from Example 1 in factored form. There are $26 \cdot 26$ different 2-letter acronyms. Now, with 26 possible third letters, the total number is $26 \cdot 26 \cdot 26 = 26^3$. This is also equal to 17,576.

The idea behind the solutions to Example 2 is very powerful and can be continued. To get the number of 4-letter acronyms, you can work from the number of 3-letter acronyms. Each 3-letter acronym is the beginning of 26 4-letter acronyms, so the number of 4-letter acronyms is $26^3 \cdot 26$, or 26^4, and so on. This thinking is much like the Now/Next thinking you used in the spreadsheets of Lesson 7-8.

STOP **QY1**

Choosing From *n* Objects Repeatedly

A sequence of two objects is said to have "length" 2; a sequence of three objects has length 3, and so on. In the previous examples you counted ways to make acronyms of length 2 and 3. Order matters with acronyms. President John Fitzgerald Kennedy had initials JFK, but not KFJ. Order also matters with acronyms such as NASA (National Aeronautics and Space Administration) and SCUBA (self-contained underwater breathing apparatus).

The process used to find the number of different acronyms of length 2 or 3 can be generalized, leading to the following result.

Arrangements Theorem

If there are n ways to select each object in a sequence of length L, then n^L different sequences are possible.

STOP **QY2**

▶ **QY1**

The Russian alphabet has 33 letters. How many 4-letter Russian acronyms are possible?

▶ **QY2**

A test has 20 multiple-choice questions with 5 choices each. How many different sets of answers are possible?

In Example 3, not all of the objects have the same number of choices possible.

GUIDED

Example 3

Suppose a standardized test has 20 questions with 4 choices and 10 questions with 5 choices. What is the probability that a person could guess on every one of the 30 questions and answer them all correctly?

Solution Think of the test as having 2 parts. Count the number of ways each part can be created.

Part 1 This part has 20 questions with 4 choices. We want to know how many sequences m of length 20 there are with these 4 letters.

$$m = \underline{\quad ? \quad}$$

Part 2 This part has 10 questions with 5 choices. Let n be the number of sequences of length 10 with 5 letters.

$$n = \underline{\quad ? \quad}$$

Now apply the Multiplication Counting Principle and multiply the results from Part 1 and Part 2 to determine how many different sets of answers are possible.

$$mn = \underline{\quad ? \quad}$$

In base 10 this number is 10,737,418,240,000,000,000. If a person is guessing, then we assume that each one of these sets of answers is equally likely, and only one of them has all the correct answers. So the probability of having a perfect test is $\frac{1}{mn} = \underline{\quad ? \quad}$.

The national average mathematics score on The National Assessment of Educational Progress (NAEP) at grade 8 was 16 points higher in 2005 than in 1990.

Source: National Assessment of Educational Progress

Writing Large Numbers in Scientific Notation

Depending on your calculator and the mode it is in, if you enter mn to calculate the answer to Example 3, the result will be displayed either as the long base-10 number or in *scientific notation*. You should try this on your calculator. Presumably you have used scientific notation in other mathematics or science classes. Recall that in **scientific notation,** a number is represented as $x \cdot 10^n$, where n is an integer and $1 \leq x < 10$. In scientific notation, $10{,}737{,}418{,}240{,}000{,}000{,}000 \approx 1.0737 \cdot 10^{19}$.

A major advantage of scientific notation is that it quickly tells you the size of a number. The exponent is one less than the number of digits in the whole number. A whole number $x \cdot 10^n$ has $n + 1$ digits. Notice that the exponent is 19 in the scientific notation form of the number above, and the base-10 form has 20 digits.

Questions

COVERING THE IDEAS

1. a. Write all the 2-letter acronyms that can be made from the five vowels A, E, I, O, and U.
   b. **Fill in the Blanks** In Part a, you have found the number of sequences of length ___?___ of ___?___ objects.
2. The Greek alphabet is about 2,750 years old and is used by about 12 million people in Greece and other countries around the world. It contains 24 letters.
   a. How many 2-letter acronyms are there using Greek letters?
   b. How many 4-letter acronyms are there using Greek letters?
3. Write the number 5^6 in base-10 and in scientific notation.
4. a. Write an example of a 6-letter acronym made from the five letters A, B, C, D, and E.
   b. How many of these 6-letter acronyms are possible?
5. Suppose part of a spreadsheet has 6 columns and 15 rows.
   a. How many cells are in the spreadsheet?
   b. Explain how your answer applies the Multiplication Counting Principle by indicating how choices are involved in finding the number of cells.
6. a. Draw three horizontal lines and four vertical lines. In how many points do these lines intersect?
   b. If you drew 30 horizontal lines and 40 vertical lines, in how many points would they intersect?
   c. If you drew h horizontal lines and v vertical lines, in how many points would they intersect?
7. a. A quiz consists of 10 true-or-false questions. How many different sets of answers are possible? Write your answer in exponential form, in base-10, and in scientific notation.
   b. If you guess on all 10 questions, what is the probability of getting all the questions correct?
   c. Answer Parts a and b if there are Q true or false questions on the test.
8. A test has 5 true-or-false questions and 15 multiple-choice questions with 4 choices each.
   a. How many different sets of answers are possible? Write your answer in exponential form, in base-10, and in scientific notation.
   b. If you guessed on every question, what is the probability you would get all 20 questions correct?

9. Use the information on the electromagnetic spectrum on page 456. Write the number in base-10 notation.
   a. the longest wavelength marked on the spectrum
   b. the shortest wavelength marked on the spectrum

APPLYING THE MATHEMATICS

10. Radio station call letters in the United States must start with either W or K.
    a. How many choices are there for the first letter?
    b. How many choices are there for the second letter?
    c. How many different 4-letter station names are possible?

In June 2005, there were 2,019 commercial U.S. radio stations.

Source: Federal Communications Commission

11. The Cayuga Indians played a game called *Dish* using 6 peach pits. The pits were blackened on one side and plain on the other. When pits were tossed, they landed on the blackened and plain sides with about the same frequency. When the six pits were tossed, a player scored if either all blackened or all plain sides landed up. What is the probability that a player would score points on one toss of the pits?

12. How many 6-digit whole numbers are there? Answer the question in two ways.
    a. by subtracting the least 6-digit number from the greatest 6-digit number and working from that
    b. by thinking of the problem as a series of choices: 9 choices for the left digit (because it cannot be zero) and 10 choices for every other

13. Assume that everyone in the United States has a first name, a middle name, and a last name. Therefore, everyone has a 3-letter acronym of his or her initials. If your initials are typical, about how many of the 300 million people in the United States have your initials?

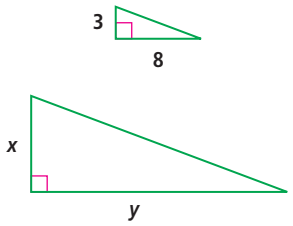
14. How many different sets of answers are possible for each of the following tests?
    a. a group of P true-or-false questions
    b. a group of Q questions that can be answered "sometimes," "always," or "never"
    c. a test made up of two parts: P true or false questions and Q *always*, *sometimes but not always*, or *never* questions

REVIEW

15. Do the ordered pairs (x, y) that satisfy $y < -4x + 8$ describe a function? Why or why not? **(Lesson 7-5)**

16. Consider these points (0, –2), (6, 4), and (–10, –12). **(Lessons 6-6, 3-4)**
    a. Write an equation for the line containing these three points.
    b. If the point $(x, 20)$ lies on this line, find the value of x.

17. Refer to the similar triangles at the right. If the ratio of similitude of the smaller triangle to the larger triangle is $\frac{1}{3}$, find the area of the larger triangle. **(Lesson 5-10)**



18. A biologist captured, tagged, and released 40 fish caught in a lake. Three weeks later, the biologist caught 28 fish. Of these, 8 had tags. Based on these findings, estimate the total number of fish in the lake. **(Lesson 5-9)**

19. Solve $\frac{1}{5}x - \frac{3}{10} = \frac{9}{10}$. **(Lesson 3-8)**

In 20 and 21, write the number in scientific notation. (Previous Course)

20. seven thousandths

21. 2.8 billion

EXPLORATION

22. Many of the cabinet-level departments in the United States government are identified by acronyms. Tell what department each of these acronyms stands for. As a hint, we have put the departments (not the acronyms) in alphabetical order.

    a. USDA b. DOC c. DOD d. ED
    e. DOE f. HHS g. DHS h. HUD
    i. DOJ j. DOL k. DOS l. DOI
    m. DOT n. VA

23. The Greek alphabet has 24 letters and the Russian alphabet has 33 letters.
    a. Are there more 4-letter Greek acronyms or 3-letter Russian acronyms?
    b. What is the least value of n for which there are *fewer* n-letter Greek acronyms compared to $(n - 1)$-letter Russian acronyms?

The Pentagon has three times the floor space of the Empire State Building in New York.

Source: Pentagon Tours

QY ANSWERS

1. $33^4 = 1{,}185{,}921$

2. $5^{20} =$
$95,367,431,640,625 \approx$
$9.5 \cdot 10^{13}$