Chapter

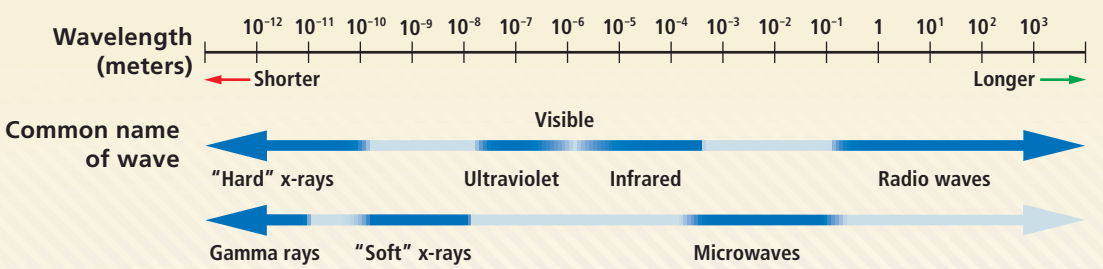
8 Powers and Roots

Contents

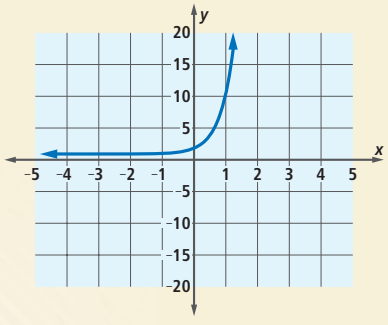
Visible light, infrared and ultraviolet radiation, x-rays, microwaves, and radio waves are all parts of the electromagnetic spectrum. Waves differ only in their wavelengths. Radio waves can be as long as 1,000 meters or longer; x-rays can be as short as one billionth of a meter or less.

To describe these numbers, we use powers of 10. On the scale shown below, each tick mark is 10 *times* the length of the tick mark to its left. You have seen these powers of 10 used in scientific notation.

The Electromagnetic Spectrum



Here is a graph of $y = 10^x$ on the window $-5 \le x \le 5$, $-10 \le y \le 20$. You can see that as x increases by 1, y is multiplied by 10. The coordinates of some points on this graph are easily found by hand. When $x = 0$, $y = 10^0 = 1$. When $x = 1$, $y = 10^1 = 10$. When $x = 2$, $y = 10^2 = 100$. When $x = 3$, $y = 10^3 = 1{,}000$, too large to be on the graph. But the graph contains values of the function when x is negative. What is the meaning of 10^{-1}, 10^{-2}, 10^{-3}, and so on?



Also, the graph computes values for y when x is not an integer. Although we do not discuss all the powers of x in this chapter, the meanings of $\frac{1}{2}$ and $\frac{1}{3}$ are discussed and found to be related to square roots and cube roots. Additionally, there is the question of how all these powers and roots are related. For example, how is x^7 related to x^{-7}? Are the 7th powers of different numbers related in any way? The answers to these questions provide additional understanding of some of the applications of powers that you saw in the preceding chapter. They also shed light on some important formulas for lengths, area, and volume found in geometry.