Chapter 5 Summary and Vocabulary

Fractions in algebra are generalizations of fractions in arithmetic. Every fraction can be treated as a division, and division by 0 is undefined (the denominator cannot be 0). For all real numbers a and b and $b \neq 0$, $\frac{a}{b} = a \div b = a \cdot \frac{1}{b}$.

Algebraic fractions are multiplied and divided just like fractions in arithmetic. For all real numbers $a, b, c,$ and $d,$ with $b, c,$ and $d \neq 0$,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \text{ and } \frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

Because fractions represent division, all the applications of division in arithmetic lead to applications of fractions in algebra. When quantities with different units are divided, the result is a rate. Rates are often signaled by the word "per" as in students per class, miles per hour, and people per square mile. Rates have rate units, and these units are multiplied and divided just as if they were fractions. Rate units are useful for converting from one unit to another.

When quantities with the same type of units are divided, the result is a ratio, a number without a unit. Ratios may be represented as fractions, percents, or decimals. The relative frequency of an event is the ratio of the number of times an event has occurred to the number of times it could have occurred. A probability is a number that is the expectation of what the relative frequency ratio would be in the long run. Both relative frequencies and probabilities are numbers from 0 to 1.

When two fractions, rates, or ratios are equal, the result is a proportion, an equation of the form $\frac{a}{b} = \frac{c}{d}$. So proportions are found wherever there are fractions. Proportions can be solved by applying the Multiplication Property of Equality or by using the Means-Extremes Property. In similar figures, the ratios of corresponding lengths are equal to a ratio of similitude k. Proportions are everywhere when there are similar figures.

Theorems and Properties

Multiplying Fractions Property p. 252
Equal Fractions Property p. 253
Dividing Fractions Property p. 258

Means-Extremes Property p. 302
Fundamental Property of Similar
  Figures p. 309

Vocabulary

5-1
algebraic fraction

5-2
complex fraction

5-3
rate
reciprocal rates

5-4
conversion rate

5-5
ratio
tax rate
discount rate

5-6
outcome
event
probability
P(x)
probability distribution
unbiased
fair
conditional probability
complement
odds of an event

5-7
relative frequency
pth percentile

5-9
proportion
extremes
means
population
sample
randomly
capture-recapture method

5-10
ratio of similitude

Chapter 5 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

In 1–3, simplify the expression.

1. $\dfrac{75c}{8p} \cdot \dfrac{2}{15c}$

2. $\dfrac{5}{a} \div \dfrac{9}{3a^2}$

3. $\dfrac{\frac{2x}{5}}{\frac{x}{5}}$

4. **Multiple Choice** Which of the following is *not* equal to $\dfrac{a}{b}$?

   A $\dfrac{1}{b} \cdot \dfrac{a}{1}$

   B $6 \div \dfrac{6a}{b}$

   C $\dfrac{a}{b} \cdot \dfrac{c}{c}$

   D $\dfrac{1}{2b} \div \dfrac{1}{2a}$

In 5–7, solve the proportion. Show your work.

5. $\dfrac{h}{17} = \dfrac{6}{101}$

6. $\dfrac{9}{5} = \dfrac{4u}{3}$

7. $\dfrac{x}{8} = \dfrac{2x-3}{24}$

8. Suppose $\dfrac{7}{x} = \dfrac{8}{15}$.

   a. What does the Means-Extremes Property tell you?

   b. Solve for x.

9. Mrs. Wright bought six boxes of pencils, with each box containing 10 pencils to be split among her four children. Writing your answer as a rate, find how many pencils each child received.

10. The Colorado Department of Public Health and Environment reported that for the years 2001–2005, 1,563 bats were examined for rabies. Of that number, 221 actually had rabies. Use this information to estimate the probability that if you see a bat in Colorado, it has rabies.

11. Teresa can run one mile in seven and a half minutes. Using the approximation 1.6 km ≈ 1 mi, how many seconds does it take her to run one kilometer?

12. The two rectangles below are similar with corresponding sides parallel. Solve for x.
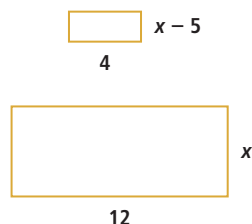
13. If the electricity goes out and a clock stops, what is the probability that the second hand stops between 3 and 4?

14. A DVD costs $18 after a discount of $4.50. What is the percent of discount?

15. In 2003, Markus Riese set a world record by bicycling backwards a distance of 50 kilometers in 1 hour and 47 minutes.

   a. To the nearest kilometer, what was his speed in kilometers per hour?

   b. Write the reciprocal rate to your answer from Part a.

In 16–18, consider the table below that displays the results of a survey of 50 people asked about their types of allergies.

Allergy	Number of People
Only Peanuts	7
Only Bees	16
Both Peanuts and Bees	2

16. What is the relative frequency of people who are allergic to peanuts?

17. What is the relative frequency of people who are allergic to neither peanuts nor bees?

18. What is the relative frequency of people who are allergic to either peanuts or bees, but not both?

19. In the expression $\frac{12k}{k-2}$, k cannot be what value? Explain.

20. A pet store has f goldfish, s snakes, c cats, and d dogs. What is the ratio of the number of cats to the total number of these animals?

21. If a bus travels 350 miles on 20 gallons of gas, about how far can it travel on 35 gallons of gas?

22. A circle with radius 21 inches is contained inside a rectangle that is 4 feet by 7 feet. If a point within the rectangle is chosen at random, what is the probability that it lies inside the circle?

21 in.

4 ft

7 ft

23. The Canadian National (CN) Tower in Toronto is one of the tallest towers in the world. Suppose the tower casts a shadow that is 1,210 feet long, and at the same time a 6-foot-tall man standing next to the tower casts a shadow that is 4 feet long. About how tall is the CN tower?

24. Suppose an 8th grade gym class recorded the amount of sit-ups each student could do in 1 minute. Janice was in the 75th percentile and only 6 students did more sit-ups than she did. How many people are in the gym class?
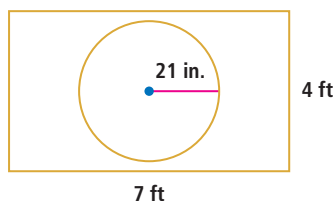
SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Multiply and simplify algebraic fractions. (Lesson 5-1)

In 1–4, multiply the fractions. Simplify if possible.

1. $\frac{7a}{2} \cdot \frac{4}{5}$

2. $\frac{12}{5x} \cdot \frac{2x}{3}$

3. $\frac{6}{5p} \cdot 10$

4. $\frac{2ax}{3} \cdot \frac{9x}{a}$

In 5 and 6, simplify the fraction.

5. $\frac{121bcd}{11cd}$

6. $\frac{-24x^3y}{32x^2y^2}$

OBJECTIVE B Divide algebraic fractions. (Lessons 5-2, 5-3)

In 7–10, find the quotient.

7. $\frac{c}{6} \div \frac{9}{2}$

8. $\frac{x}{y} \div \frac{x}{z}$

9. $\frac{\frac{4a}{15}}{-\frac{20a}{9}}$

10. $\frac{-625x}{\frac{50x}{8}}$

In 11 and 12, what value(s) can the variable *not* have?

11. $\frac{10 + x}{8 + x}$

12. $\frac{28v}{4v - 2.4}$

OBJECTIVE C Solve proportions. (Lesson 5-9)

In 13–16, solve the equation.

13. $-\frac{28k}{5} = \frac{14}{3}$

14. $\frac{6}{y - 4} = \frac{2}{5}$

15. $\frac{3(t - 5)}{4} = \frac{9t}{2}$

16. $\frac{a + 12}{a - 3} = 4$

17. **Fill in the Blank** 6 is to 54 as 54 is to ___?___.

PROPERTIES Principles behind the mathematics

OBJECTIVE D Use the language of proportions and the Means-Extremes Property. (Lesson 5-9)

18. Consider the proportion $\frac{7}{8} = \frac{28}{32}$.

   a. Which numbers are the means?

   b. Which numbers are the extremes?

19. If $\frac{6}{15} = \frac{x}{8}$, what does $\frac{8}{x}$ equal?

20. **Fill in the Blanks** If $\frac{a}{b} = \frac{x}{y}$, then by the Means-Extremes Property ___?___ = ___?___.

21. **Fill in the Blank** If $\frac{m}{n} = \frac{u}{v}$, then $\frac{v}{n} = $ ___?___.

USES Applications of mathematics in real-world situations

OBJECTIVE E Use rates in real situations. (Lesson 5-3)

22. Suppose a 225-mile train ride took 3 hours.

   a. What was the rate in miles per hour?

   b. What was the rate in hours per mile?

23. A 16-oz box of pasta costs $1.20 and a 32-oz box of pasta costs $1.80.

   a. Find the unit cost (cost per ounce) for the 16-oz box.

   b. Find the unit cost of the larger box of pasta.

   c. Which is the better buy?

24. It took Jamila 45 minutes to answer 32 questions on her Algebra test. On average, how much time did it take her to answer 1 question?

25. Which is faster, typing w words in $2m$ minutes or typing $4w$ words in $6m$ minutes? Explain your answer.

In 26 and 27, calculate a rate for the given situation.

26. In 22 almonds there are about 160 calories.

27. The red oak tree grew 12 feet in 8 years.

OBJECTIVE F Convert units and use reciprocal rates in real situations. (**Lesson 5-4**)

28. During a meteor shower, some meteors approach Earth's atmosphere at speeds of 95 kilometers per second. Using the fact that 1 mile ≈ 1.6 km, convert this rate into miles per hour.

29. The average human adult at rest takes 16 breaths per minute.

   a. At this rate, how many breaths would a human take in a week?

   b. If a cat takes 1,500 breaths per hour, does the cat or human breathe at a faster rate? Explain your answer.

30. Sliced turkey costs $6.50 per pound and there are 20 slices per pound. How many slices of turkey can you buy for $2.60?

31. It takes Clara 1 min to stuff $4n$ envelopes with letters. Melanie is half as fast.

   a. How many envelopes can Melanie stuff per minute?

   b. How many minutes does Melanie spend per envelope?

32. A more efficient halogen bulb can be used exactly six hours per day for a year before burning out. How many hours can the halogen bulb be used?

OBJECTIVE G Use ratios to compare two quantities. (**Lesson 5-5**)

33. **Multiple Choice** Which of the following is *not* equal to the ratio of 12 to 7?

   A $\frac{12x}{7x}$ **B** 60 : 35 **C** $\frac{24 \text{ ft}}{14 \text{ ft}}$ **D** 700 to 1,200

In 34 and 35, consider the table below that lists the types and number of televisions in stock at Eli's Electronic Store.

Type of TV	Number of TVs
High-Definition (HD)	12
Flat-Screen	36
Projection	8
Cathode-Ray Tube (CRT)	64

34. What is the ratio of HD televisions to all televisions?

35. What is the ratio of CRT televisions to projection televisions?

36. A pair of shoes that originally cost $53 is on sale for $42.40.

   a. What is the discount rate?

   b. Find the total amount saved, including tax, if the tax rate is 6.25%.

37. To make a certain shade of green paint, a painter mixes 5 parts blue paint with 3 parts yellow paint. If he needs 20 gallons of green paint, how many gallons of each paint color are needed in the mixture?

In 38 and 39, consider the following information. A baseball player's batting average can be viewed as the ratio of total number of hits divided by total number of at-bats. In 2005, Vladimir Guerrero got 20 hits in his first 57 at-bats for a batting average of .351.

38. If Guerrero got only 2 hits in his next 10 at-bats, what would have been his new batting average?

39. How many hits in a row would Guerrero have needed to raise his batting average to at least .400?

OBJECTIVE H Calculate relative frequencies and probabilities in situations with a finite number of equally likely outcomes.
(Lessons 5-6, 5-7)

40. A number is selected randomly from the integers {−1, 0, 1, ..., 8}. What is the probability that the number is greater than 1?

41. A fair die is thrown once. Find the probability of getting an even number greater than 2.

42. If the probability of winning a raffle is $\frac{1}{25,000}$, what is the probability of *not* winning?

43. A study shows that the relative frequency of people who eat cold cereal for breakfast in the United States is 31%. What is the relative frequency of people in the United States who do *not* eat cold cereal for breakfast?

44. Event X has a probability of 42%, event B has a probability of $\frac{5}{12}$, and event C has a probability of 0.45.
    a. Which event is most likely to happen?
    b. Which event is least likely to happen?

**OBJECTIVE I** Find probabilities involving geometric regions. (Lesson 5-8)

45. A 5-cm square inside a 6-cm square is shown below. If a point is selected at random from the figure, what is the probability that it lies in the shaded region?

5 cm

6 cm
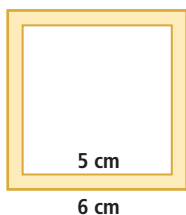
46. Tate drives to work every morning and follows the same route each day. The map below shows his path. One morning Tate runs out of gas while on the way to work. If each point on the map is equally likely, what is the probability that Tate ran out of gas on the highway?

18 mi of Highway

4 mi

Home

3 mi

Work
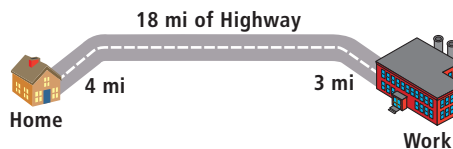
47. A target consists of a set of 4 concentric circles with radii of 4 inches, 8 inches, 12 inches, and 16 inches. The largest circle is inscribed in a square. A person with a bow and arrow randomly shoots at the target so that all points inside the square are equally likely to be hit. The arrow hits somewhere inside the square.

4 in.
8 in.
12 in.
16 in.
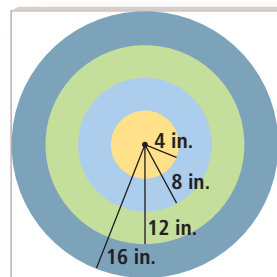
a. What is the probability that the arrow hits the bull's eye?
b. What is the probability that the arrow hits within one of the two middle rings (but not within the bull's eye)?

OBJECTIVE J Solve problems involving proportions in real situations.
(Lessons 5-9, 5-10)

48. If $\frac{1}{2}$ cup of brown sugar equals 24 teaspoons of brown sugar, how many teaspoons are there in $2\frac{1}{3}$ cups of brown sugar?

49. A school donating money to a charity decides that for every student donation of $5, the school will donate $12. If the total student donation amount is $490, how much money will the school donate?

50. On September 27, 2005, you could buy 10.89 pesos (the currency in Mexico) for one U.S. dollar. If a sombrero cost 290 pesos then, what was its cost in U.S. dollars, rounded to the nearest cent?

51. Suppose a ranger caught, tagged, and released 28 moose in a state park. Two months later, the ranger caught 20 moose, and 14 of these had tags. Based on these findings, estimate the total number of moose in the park.

OBJECTIVE K Interpret the meaning of percentile for benchmarks of 10th, 25th, 50th, 75th, and 90th percentiles. (Lesson 5-7)

In 52–55, a class of 20 students at O'Sullivan High School received the following SAT scores.

2330	2200	1900	1870	2050
1680	1790	1950	2110	2020
1880	1790	2230	2000	1970
2050	1680	1550	1780	1910

52. What is the median of the students' scores?

53. Nolan scored a 1790. At what percentile is he in his class?

54. Tia hopes that her score is at least at the 90th percentile of her classmates. What score must she have for this to be true?

55. How many scores are in each of the following percentiles?

    a. 10th   **b.** 25th   **c.** 75th

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE L Find lengths and ratios of similitude in similar figures. (Lesson 5-10)

56. One rectangular field has dimensions 900 m by 1,200 m; another rectangular field has dimensions 800 m by 1,100 m. Are the fields similar in shape? Explain your reasoning.

57. The quadrilaterals below are similar. Corresponding sides are parallel.
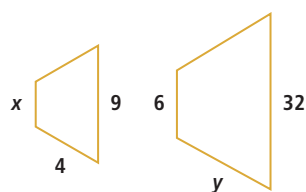
    a. Give the two possible ratios of similitude.

    b. Solve for y.   **c.** Solve for x.

58. A building casts a shadow that is 480 feet long. A yardstick casts a shadow n feet long at the same time. How tall is the building?

    a. Draw a sketch of the situation.

    b. Show how a proportion can be used to solve the problem.

59. Pentagons $PQRST$ and $VWXYZ$ are similar with ratio of similitude $\frac{4}{5}$. If $VWXYZ$ has area 60 square units, what is the area of $PQRST$?