

## Lesson

## 5-10

## Similar Figures

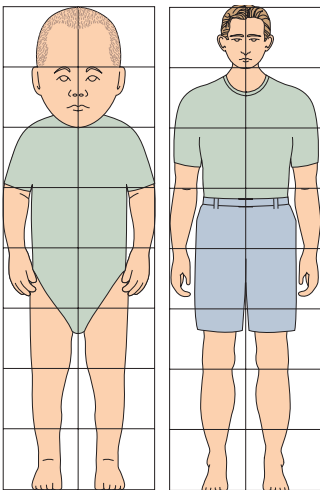
## Vocabulary

ratio of similitude

► **BIG IDEA** Ratios of lengths of similar geometric figures are equal, giving rise to many applications of proportions.

Model airplanes, architect's drawings, models of buildings, and photographs are all pictures of objects that have the same shape as the originals but not necessarily the same size. Mathematically, the original object and the model are *similar*. Blow-ups of photographs are also similar.

In some species of animals, babies are shaped much like their parents, like the elephants shown below. However, for humans this is not the case. Infants have very different shapes than adults. This drawing shows a baby and an adult, with each one's height divided into 8 equal parts. The divisions allow us to form ratios to compare the shapes of the infant and adult.



The largest land animal is the African bush elephant, standing 13 feet high and weighing 8 tons.

Source: *The World Almanac for Kids*

	Infant	Adult
Ratio of head length to height	$\frac{2 \text{ parts}}{8 \text{ parts}} = \frac{1}{4}$	$\frac{1 \text{ part}}{8 \text{ parts}} = \frac{1}{8}$
Ratio of trunk length to leg length	$\frac{3 \text{ parts}}{3 \text{ parts}} = \frac{1}{1}$	$\frac{3 \text{ parts}}{4 \text{ parts}} = \frac{3}{4}$

Notice that the ratios in an infant's body are quite different from an adult's body. An infant is not a scaled-down version of an adult.

## Mental Math

If you can bike to a friend's house in 15 minutes, averaging 10 miles an hour, how long will it take if you average 20 miles an hour?

## Activity

Pictured at the right is a coffeemaker that is 12.5 inches tall and 5.625 inches wide.

**Step 1** Measure  $AB$  and  $CD$  to find the height and width of the coffeemaker in the picture.

**Step 2** Calculate these ratios to the nearest tenth.

- $\frac{AB}{\text{actual height of coffeemaker}}$
- $\frac{CD}{\text{actual width of coffeemaker}}$

You should find that these ratios are about equal.

**Step 3** Measure  $EF$ , the height of the coffee pot.

**Step 4** Solve a proportion to find the height of the actual coffee pot.



You also should have found that the dimensions of the picture are  $\frac{1}{5}$  of the length of the corresponding dimensions of the coffeemaker. This illustrates a basic property of similar figures.

### Fundamental Property of Similar Figures

If two polygons are similar, then ratios of corresponding lengths are equal and corresponding angles have the same measure.

The ratio of the lengths of corresponding sides of two similar figures is called a **ratio of similitude**. In the activity the ratio of similitude is  $\frac{1}{5}$  because  $\frac{AB}{\text{actual height}} = \frac{CD}{\text{actual width}} = \frac{1}{5}$ .

### Finding Lengths in Similar Figures

When two figures are similar, a true proportion can be written using corresponding lengths. If three of the four lengths in the proportion are known, the fourth can be found by solving an equation.

**Example 1**

An adult African elephant can be 30 feet long and 11 feet high at the shoulder. Estimate the length of a baby elephant that is 3 feet high at the shoulder.

**Solution** Compare lengths on the adult with the corresponding lengths on the baby. Set up a proportion by forming two equal ratios. Let  $x$  be the length of the baby. Since the elephants are similar, the ratios are equal.

$$\frac{\text{height of adult}}{\text{height of baby}} = \frac{\text{length of adult}}{\text{length of baby}}$$

$$\frac{11}{3} = \frac{30}{x}$$

$$11x = 90$$

$$x = \frac{90}{11} \approx 8.2$$

We estimate that the baby elephant is slightly over 8 feet long.

**GUIDED****Example 2**

The two quadrilaterals at the right are similar, with corresponding sides parallel. Find  $x$ , the length of  $\overline{CD}$ .

**Solution** The side corresponding to the unknown length  $\overline{CD}$  is  $\underline{\quad?}$ .

There is a pair of corresponding sides whose lengths are both known. These are  $\underline{\quad?}$  and  $\underline{\quad?}$ . Because the figures are similar, the ratios of lengths of these corresponding sides are equal.

$$\frac{\overline{CD}}{?} = \frac{?}{?}$$

Write the proportion.

$$\frac{x}{?} = \frac{?}{?}$$

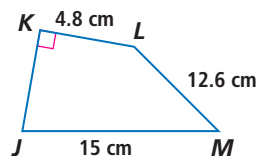
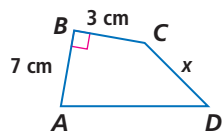
Substitute the known lengths.

$$\underline{\quad?} \cdot x = \underline{\quad?} \cdot \underline{\quad?}$$

Means-Extremes Property

$$x = \underline{\quad?} \text{ cm}$$

Divide by  $\underline{\quad?}$  and simplify.



## Using Similar Figures to Find Lengths without Measuring

Similar figures have many uses. For example, you can use similar triangles to find the height of an object you cannot measure easily. Suppose you want to find the height  $h$  of a flagpole. Here is how you can do it. Holding a yardstick parallel to the flagpole, measure the length of the yardstick's shadow. Then measure the length of the shadow of the flagpole. The picture on the next page illustrates one possible set of measurements.

**Example 3**

Use the measurements at the right to find the height  $h$  of the flagpole.

**Solution** Two similar right triangles are formed. Now, use ratios of corresponding sides to find  $h$ .

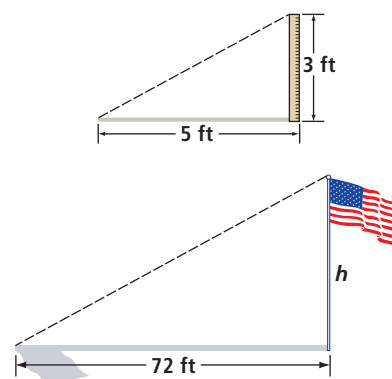
$$\frac{3}{h} = \frac{5}{72}$$

$$5h = 72 \cdot 3$$

$$5h = 216$$

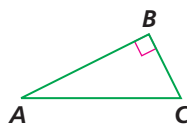
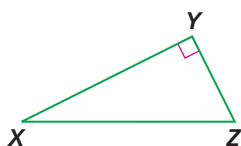
$$h = 43.2$$

The flagpole is about 43 feet tall.

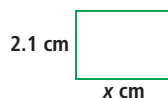
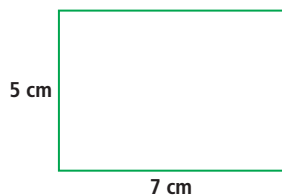
**Questions****COVERING THE IDEAS**

1. What is the fundamental property of similar figures?
2. An adult male African elephant is about 11 feet tall, with ears that measure 5 feet from top to bottom. If a baby elephant is 3 feet tall, find out how big its ears are.

In 3–5, refer to the two similar triangles below. Corresponding sides are parallel.



3. Which side of  $\triangle XYZ$  corresponds to the given side of  $\triangle ABC$ ?
  - a.  $\overline{AC}$
  - b.  $\overline{BC}$
  - c.  $\overline{AB}$
4. Find two ratios equal to  $\frac{XY}{AB}$ .
5. Suppose  $AB = 12$ ,  $BC = 5$ ,  $AC = 13$ , and  $XY = 18$ . Find
  - a.  $YZ$ .
  - b.  $XZ$ .
6. The quadrilaterals shown below are similar.



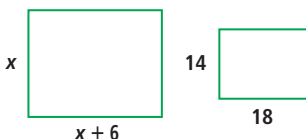
- a. Find  $x$ .
- b. Write two possible ratios of similitude.

7. A bookcase is pictured at the right. The actual bookcase is 36 in. wide.
- Measure the width of the bookcase in the picture.
  - What is the ratio of the similitude comparing the picture's width to the actual width?
  - Measure the height of the bookcase in the picture.
  - Use your answers to Parts b and c to determine the height of the actual bookcase.
8. Suppose a 3-foot yardstick casts a 4-foot shadow. A nearby building casts a shadow of 56 feet at the same time. What is the height of the building?



### APPLYING THE MATHEMATICS

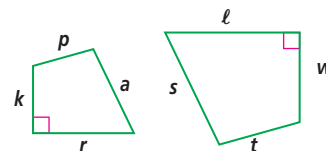
9. A person who is 160 cm tall is photographed. On the photo, the image of the person is 12 cm tall. What is the ratio of similitude?
10. The Crazy Horse Memorial in the Black Hills of South Dakota will be the world's largest mountain carving. From the chin to the top of the head is 87.5 feet. Use the picture, a ruler, and your knowledge of similar figures to approximate the length of the outstretched arm in the carving.
11. The two rectangles below are similar.



- Use a proportion to find the value of  $x$ .
  - Find the perimeter of the larger rectangle.
12. The quadrilaterals at the right are similar. Corresponding sides are parallel.
- Write a true proportion involving  $\frac{s}{a}$ .
  - Fill in the Blank** Complete  $\frac{k}{w} = \frac{?}{t}$  and solve for  $k$ .
13. At a certain time on a sunny day, Shadrack, who is 6 feet tall, casts a shadow that is 9 feet long. A nearby building that is  $t$  feet tall casts a shadow that is 24 feet long.
- Draw a diagram of this situation and label the lengths.
  - Write a proportion that describes the situation.
  - How tall is the building?



When completed, the Crazy Horse mountain carving (shown in the background) will be 641 feet long by 563 feet high. Crazy Horse's completed head is 87 feet 6 inches high. The horse's head, currently the focus of work on the mountain, is 219 feet, or 22 stories high.



14. For this question, you need to use a ruler and properties of similar figures. A scale drawing of a house, as seen from its front, is shown below. The actual width (across the front, not including the roof) of the house is 12 meters.



- Write a ratio comparing the width of the house in the drawing to the actual width of the house.
- Write a proportion you could use to find the actual distance from the ground to the peak of the roof.
- Solve the proportion in Part b.

### REVIEW

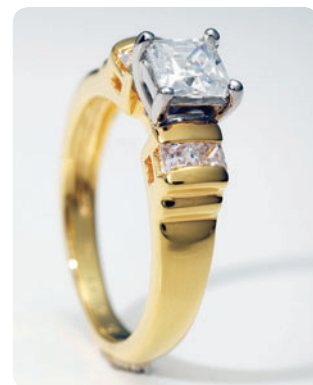
In 15 and 16, solve the proportion. (Lesson 5-9)

15.  $\frac{64}{3} = \frac{4x}{9}$

16.  $\frac{2}{1-a} = \frac{4}{a-3}$

In 17 and 18, use the fact that a *karat* is a measure of fineness used for gold and other precious materials. Pure gold is 24 karats. Gold of 18-karat fineness is 18 parts pure gold and 6 parts other metals, giving 24 parts in all. (Lessons 5-9, 4-1)

- A ring is 18-karat gold. What percent gold is this?
- A necklace weighing 6 ounces is 14-karat gold. How many ounces of pure gold are in the necklace?



Central banks of nations hold an estimated 32,000 tons of gold as official stock, and about 96,000 tons is privately held in bullion, coin, and jewelry.



19. In a 3-ounce serving of beef, there are about 26 grams of protein. About how many grams of protein are in an 8-ounce steak? (Lesson 5-9)
20. The scale of a map for Yellowstone National Park is 1.75 in. = 10 miles. If the distance between Old Faithful and Mammoth Hot Springs on the map is about 8.5 inches, what is the approximate distance between these two places in miles? (Lesson 5-9)
21. When rolling two 6-sided dice and recording their sum, there are two ways to get a 3—rolling a 1 on the first die and a 2 on the second die, or rolling a 2 on the first die and a 1 on the second die. (Lesson 5-6)
- How many ways are there to roll a 4?
  - Find the probability of rolling 4 if the dice are fair.
  - When rolling two 6-sided fair dice, a sum of seven is the most likely outcome. Explain why this is true.
22. a. Graph  $y = 6 + x$  and  $y = 2 + 3x$  on the same set of axes.  
b. According to the graph, for what value(s) of  $x$  is  $6 + x = 2 + 3x$ ? (Lesson 4-3)

In 23 and 24, consider the table at the right that shows the land area of three of the five largest countries in the world in area. (Lessons 4-1, 3-4)

Country	Square Miles
Russia	6,592,735
Canada	?
United States	3,717,792
China	?
Brazil	3,286,470

Source: infoplease.com

23. If the land area of Russia is 399,121 square miles less than the sum of the areas of China and Brazil, find the land area of China.
24. If the area of Canada is 3.6% larger than the area of the United States, estimate the area of Canada.

### EXPLORATION

25. Find the highest point of a tree, a building, or some other object, using the shadow method described in this lesson. Draw a diagram to illustrate your method.



An eruption of Old Faithful lasts anywhere from  $1\frac{1}{2}$  to 5 minutes, spraying 3,700 to 8,400 gallons of boiling water into the air.

Source: National Park Service