Lesson

5-8 Probability Without Counting

BIG IDEA The probability that a point lands in a particular region can be calculated by taking the ratios of measures of regions.

Mental Math

Compare using >, =, or <.

a. $-50 + 74$ and $-74 + 50$

b. -5 and $(-5)^2$

c. $y + 7$ and $y + 6$

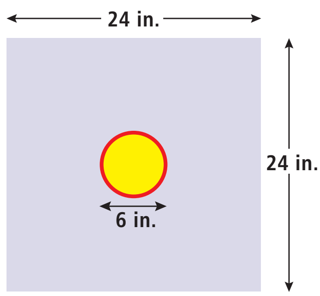
d. $|x|$ and -4

When a situation has equally likely outcomes, the probability of an event is the ratio of the number of outcomes in the event to the total number of outcomes. But sometimes the number of outcomes is infinite and not countable. In such cases, probabilities may still be found by division.

Probabilities from Areas

Example 1

Suppose a dart is thrown at a 24-inch square board containing a target circle of radius 3 inches, as shown at the right. Assuming that the dart hits the board and that it is equally likely to land on any point on the board, what is the probability that the dart lands in the circle?



Solution Recall that the area of a circle with radius r is πr^2. Compare the area of the circle to the area of the square.

$$\text{Probability the dart lands in the circle} = \frac{\text{area of circle}}{\text{area of square}}$$
$$= \frac{\pi \cdot 3^2}{24 \cdot 24}$$
$$= \frac{9\pi}{576}$$
$$\approx 0.049, \text{ or about } 5\%$$

So, the probability of the dart landing in the circle is about 5%.

Example 1 illustrates the Probability Formula for Geometric Regions.

Probability Formula for Geometric Regions

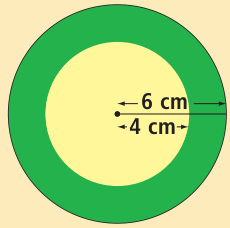
Suppose points are selected at random in a region and part of that region's points represent an event E of interest. The probability P of the event is given by

$$\frac{\text{measurement of region in the event}}{\text{measure of entire region}}.$$

STOP QY1

QY1

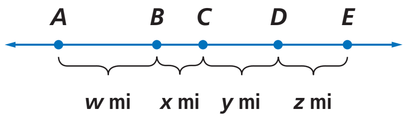
A target consists of two concentric circles as shown below. The smaller circle (called the "bull's eye"), has a radius of 4 cm and the larger circle has a radius of 6 cm. If a point is selected at random from inside the target, what is the probability it *misses* the bull's eye?



Probabilities from Lengths

Example 2

Points *A*, *B*, *C*, *D*, and *E* below represent exits on an interstate highway.



If accidents occur at random along the highway between exits *A* and *E*, what is the probability that when an accident occurs, it happens between exit *C* and exit *D*?

Solution First find the length of the entire segment.

Length of $\overline{AE} = w + x + y + z$

Probability the accident is in $\overline{CD} = \frac{y}{w + x + y + z}$

Traffic safety engineers might compare the probabilities in Example 2 with the actual relative frequency of accidents. If the relative frequency along one stretch of the highway is greater than predicted, then that part of the highway might be a candidate for repair or new safety features.

STOP QY2

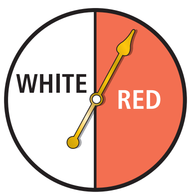
QY2

What is the probability that an accident occurring between exits *A* and *E* happens between exits *C* and *E* from Example 2?

Probabilities can also be determined by finding ratios of angle measures.

Activity

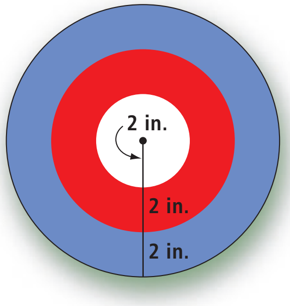
A basic spinner used in many games is shown here. Suppose the spinner is equally likely to point in any direction. There is a 50% probability the spinner lands in the red region. Draw a different spinner that still has a 50% probability of landing in a red region.



Sometimes the calculation of the measures needed to compute a probability requires you to do some addition or subtraction first.

Example 3

A target consisting of three evenly spaced concentric circles is shown below. If a point is selected at random from inside the circular target, what is the probability that it lies in the red region?



Solution Probability of a point in the red region $= \dfrac{\text{area of red region}}{\text{area of largest circle}}$

Area of the red region = the difference in the areas of the circles with radii 4 inches and 2 inches

$$\text{Area of red region} = \pi(4)^2 - \pi(2)^2$$
$$= 16\pi - 4\pi$$
$$= 12\pi \text{ in}^2$$

The radius of the largest circle is 6 inches.

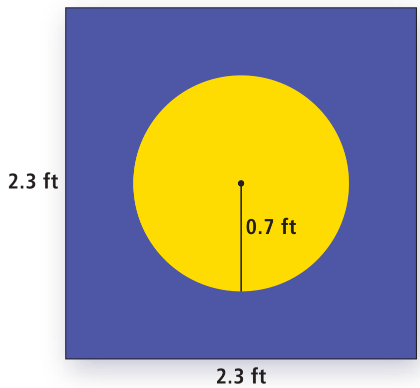
Area of largest circle $= \pi(6)^2 = 36\pi \text{ in}^2$

Thus the probability of choosing a point in the red region is $\frac{12\pi}{36\pi} = \frac{1}{3}$.

Questions

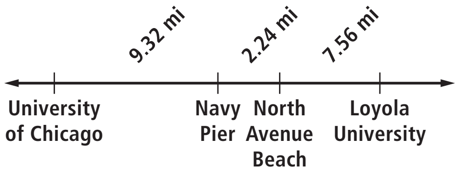
COVERING THE IDEAS

1. Consider the square archery target board at the right.
   a. What is the area of the bull's eye?
   b. What is the area of the entire target board?
   c. To the nearest percent, what is the probability that an arrow shot at random that hits the board will land in the bull's eye?
   d. What is the probability that the arrow hitting the board will land on the target outside the bull's eye?



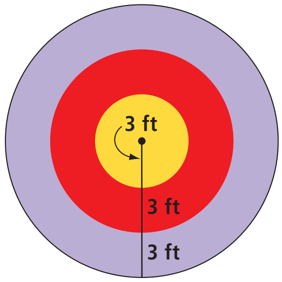
2. Draw three different spinners that have a $\frac{2}{3}$ probability of landing in a blue region.
3. An electric clock with a continuously-moving second hand is stopped by a power failure. What is the probability that the second hand stopped between the following two numbers?
   a. 12 and 2 b. 5 and 6 c. 7 and 11

In 4 and 5, use the following scenario and diagram. A student from the University of Chicago wanted to ride her bike north to Loyola University. Along the bike trek, she planned on making stops at Navy Pier and North Avenue Beach. If the student has a flat tire on the trip, what is the probability it occurs between each pair of locations?



4. Navy Pier and North Avenue Beach
5. University of Chicago and North Avenue Beach

In 6–8, refer to the target at the right. Suppose a point on the target is chosen at random.



6. What is the probability that it lies inside the bull's eye?
7. What is the probability that it lies in the outermost ring?
8. What is the probability that it lies in the middle ring?

APPLYING THE MATHEMATICS

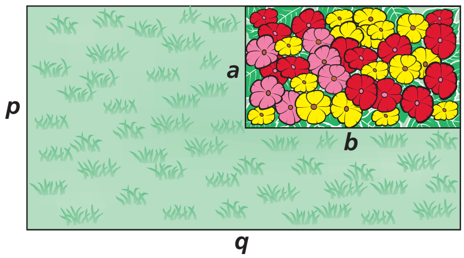
9. The land area of Earth is about 57,510,000 square miles and the water surface area is about 139,440,000 square miles. Give the probability that a meteor hitting the surface of the earth will

   a. fall on land. **b.** fall on water.

Ocean waters cover nearly 71% of Earth's surface, whereas fresh waters in lakes and rivers cover less than 1%.

Source: NASA

10. In a rectangular yard of dimensions q by p, there is a rectangular garden of dimensions b by a. If a newspaper is thrown randomly into the yard, what is the probability that it lands on a point in the garden?

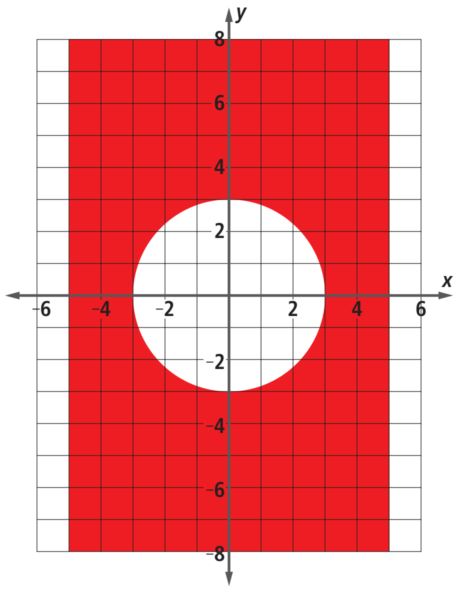


11. The table below displays the membership in the Drama Club. Design a spinner that can be used to select a representative group from the club.

Grade	Members
9	5
10	15
11	17
12	23

12. One student, seeing that the answer to Example 3 is $\frac{1}{3}$, said that if there were five concentric circles instead of 3, then the middle ring would have $\frac{1}{5}$ the area of the largest circle. Is the student correct? Why or why not?

13. What is the probability that a point selected from the region within the red rectangle at the right is also inside the circle?



REVIEW

14. A die is tossed once. **(Lessons 5-7, 5-6)**
   a. If the die is assumed to be fair, which event is more likely, "the number showing is less than 3" or "the number showing is odd"?
   b. If a 4 showed on the die, what was the relative frequency that a 2 showed?

15. A 14-foot-long metal rod is cut so that the two pieces formed have lengths in a ratio of $\frac{7}{3}$. How long is each piece? **(Lesson 5-5)**

16. Write an equation of a line that passes through the points (1, –8) and (1, 1). **(Lesson 4-2)**

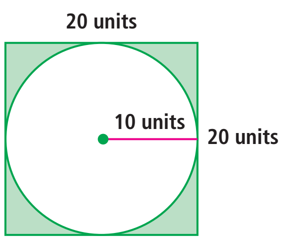
17. a. If $42n = 0$, then $\frac{2n}{5} = \underline{\ \ ?\ \ }$.
   b. What property was used to answer Part a? **(Lesson 2-8)**

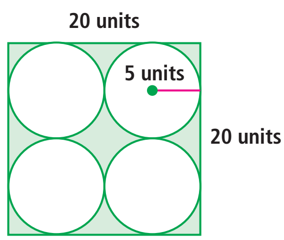
In 18–20, rewrite the fraction in lowest terms. (Previous Course)

18. $\frac{20}{25}$　　19. $\frac{42}{54}$　　20. $\frac{112}{28}$

EXPLORATION

21. a. A circle with radius of 10 units is drawn inside a square with sides of 20 units, as shown below. What part of the area inside the square is outside the circle?





   b. Four circles with radii of 5 units are drawn inside a square with sides of 20 units, as shown at the right. What part of the area inside of the square lies outside the four circles?
   c. Generalize Parts a and b. Explain why you believe your generalization to be true.

QY ANSWERS

1. $0.\overline{55}$ or about 56%
2. $\frac{y + z}{w + x + y + z}$