

Lesson

5-7

Relative Frequency
and Percentiles

► **BIG IDEA** The distribution of the relative frequencies of the outcomes from a situation can be displayed in tables and graphs.

Suppose a particular event has occurred with a frequency of F times in a total of T opportunities for it to happen. Then the **relative frequency** of the event is $\frac{F}{T}$. Like other ratios, relative frequencies may be written as fractions, decimals, or percents.

The Differences between Probability and Relative Frequency

There are similarities and differences between probability and relative frequency. When a single die is tossed, we often think that it is equally likely that the die will land with each side up. If we think this way, then each side will land up $\frac{1}{6}$ of the time. So $P(1) = P(2) = \dots = P(6) = \frac{1}{6}$. But in an actual experiment, it is rare that outcomes occur exactly the same number of times even if their probabilities of occurring are the same. A person simulated the random tossing of a die 100 times and recorded the results in the table below.

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{6} = 0.1\bar{6}$	$\frac{1}{6} = 0.1\bar{6}$	$\frac{1}{6} = 0.1\bar{6}$	$\frac{1}{6} = 0.1\bar{6}$	$\frac{1}{6} = 0.1\bar{6}$	$\frac{1}{6} = 0.1\bar{6}$
Relative Frequency	$\frac{24}{100} = 0.24$	$\frac{16}{100} = 0.16$	$\frac{13}{100} = 0.13$	$\frac{14}{100} = 0.14$	$\frac{16}{100} = 0.16$	$\frac{17}{100} = 0.17$

Notice that the sum of the relative frequencies is 1, just as the sum of the probabilities of these events is 1. The probability of an event is what we would expect the relative frequency of the event to be close to “in the long run,” after a large number of experiments.

The table on page 290 summarizes some of the important similarities and differences between relative frequencies and probabilities.

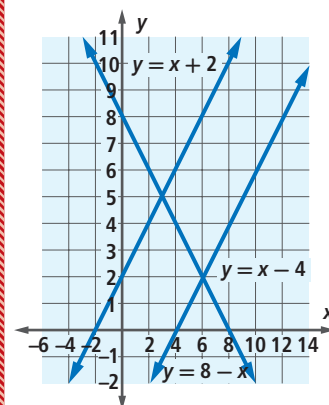
Vocabulary

relative frequency

p th percentile

Mental Math

Use the graph to answer the questions.



- Where do $y = 8 - x$ and $y = x + 2$ intersect?
- Where do $y = 8 - x$ and $y = x - 4$ intersect?
- Where do $y = x - 4$ and $y = x + 2$ intersect?

Relative Frequency	Probability
1. Calculated from data	1. Deduced from assumptions (like randomness) or assumed to be close to some relative frequency
2. The ratio of the number of times an event has occurred to the number of times it could occur	2. If outcomes are equally likely, the ratio of the number of outcomes in an event to the total number of possible outcomes
3. 0 means that an event did not occur. 1 means that the event occurred every time it could.	3. 0 means that an event is impossible. 1 means that an event is sure to happen.
4. The more often an event occurs relative to the number of times it could occur, the closer its relative frequency is to 1.	4. The more likely an event is, the closer its probability is to 1.
5. The sum of the relative frequencies of all outcomes in an experiment is 1.	5. The sum of the probabilities of all outcomes in an experiment is 1.
6. If the relative frequency of an event is r , then the relative frequency of its complement is $1 - r$.	6. If the probability of an event is p , then the probability of its complement is $1 - p$.

Relative Frequency Distributions

Just as there are probability distributions, there are distributions of relative frequency. Relative frequency distributions can be useful in answering questions about data when there are many possible answers. For example, how tall are professional basketball players?

GUIDED

Example 1

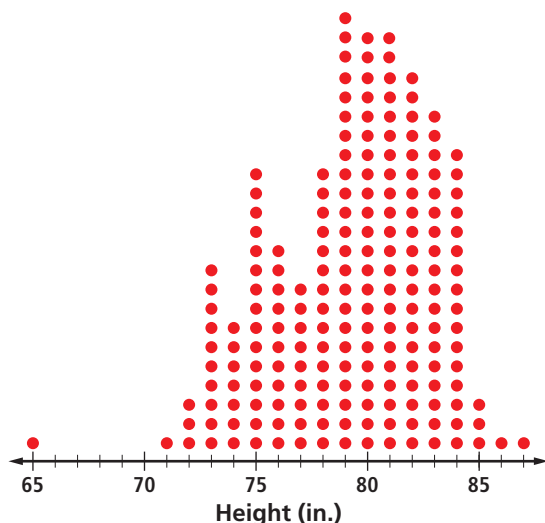
The dot plot on the next page shows the heights of 198 players in the National Basketball Association (NBA) during the 2005–06 season. Each player's height is represented by a dot. The information is summarized in the table at the right.

In the NBA, what is the relative frequency of a player

- being shorter than 6 feet tall?
- being at least 7 feet tall?
- being at most 6 feet 7 inches tall?
- having a height h given by $72 \leq h \leq 78$?

Height (in.)	Frequency
87	1
86	1
85	3
84	16
83	18
82	20
81	22
80	22
79	23
78	15
77	9
76	11
75	15
74	7
73	10
72	3
71	1
67	1

Source: NBA



Solutions

- a. Two players are under 6 feet. So the relative frequency is $\frac{2}{?} \approx 0.01 = 1\%$.
- b. Include all the players 7 ft or taller.
relative frequency = $\frac{?}{198} \approx 0.106 = 10.6\%$.
- c. Include players 6 ft 7 in. and shorter.
relative frequency = $\frac{23 + 15 + 9 + 11 + 15 + 7 + 10 + 3 + 1 + 1}{198}$
 $= \frac{?}{?} \approx \underline{\quad} \approx 48\%$
- d. These heights include $\underline{\quad}$ players. So the relative frequency is $\underline{\quad}\%$.

How tall are NBA players? Part c of the Example shows that in the NBA, almost half of the players (48%) are 6 ft 7 in. or shorter. Since 48% is very close to 50%, we know that 6 ft 7 in. is close to the median. Recall that the median is the middle number when a data set is arranged in order. For the NBA heights, the median is 6 ft 8 in. Another name for median is *50th percentile*. Fifty percent of the data are at or below the 50th percentile.

Percentiles

Sometimes it is interesting to describe the relative position of a person within a distribution. You have received standardized test reports in the past that report your score as a percentile. What does it mean if the report says you were at the 70th percentile? It means that 70 percent of the people that took the test had scores that were less than or equal to your score.

Percentile

The p th percentile of a data set is the smallest data value that is greater than or equal to p percent of the data values.

QY

Your school counselor tells you that your class rank or grade point average (GPA) is the 78th percentile. What does this mean?

STOP QY

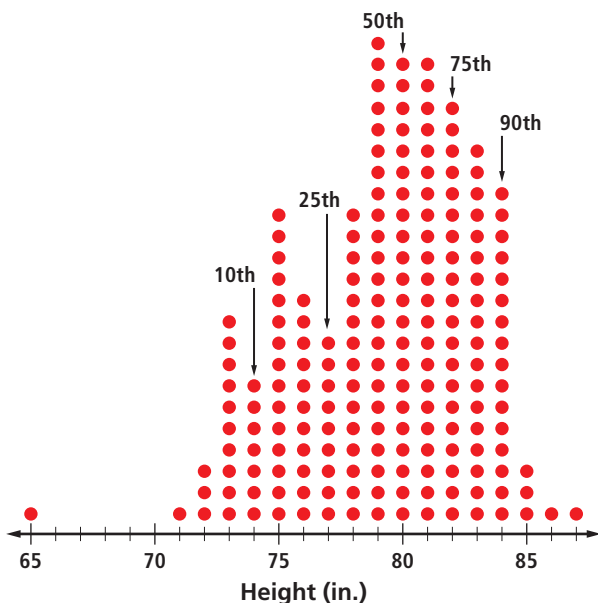
Percentiles cut the data set into 100 equal-size parts. This is similar to the way data are cut into four equal-size parts by the median and quartiles.

The 25th percentile is the same as the 1st quartile.

The 50th percentile is the same as the median.

The 75th percentile is the same as the 3rd quartile.

The dot plot of the NBA heights shown below has these three percentiles marked, along with the 10th percentile and the 90th percentile. A player who is 84 in. tall (7 ft) is at the 90th percentile. About 90% of the players are 7 ft or shorter and about 10% are taller.



Example 2

Find the 10th percentile in the NBA data and explain its meaning.

Solution The 10th percentile is the smallest data value that is greater than or equal to 10% of the data values. There are 198 values, and $10\% \text{ of } 198 = 19.8$, so look at the 20th height from the bottom of the list. This height is 74 in. So 74 in. is at the 10th percentile of NBA heights.

Questions

COVERING THE IDEAS

- What is the meaning of a relative frequency of 0?
- What is the meaning of a relative frequency of 1?
- What is the meaning when the probability of an event equals 0?
- What is the meaning when the probability of an event equals 1?
- A letter is picked randomly from the English alphabet.
 - What is the probability of picking a vowel (A, E, I, O, or U)?
 - Count the number of vowels and consonants in the following sentence.
The quick brown fox jumped over the lazy dog.
 - What is the relative frequency that a letter randomly picked in the sentence is a vowel?
 - Why aren't the answers to Parts a and c equal?
- A 2006 estimate was that 116 million households in the United States would have televisions in 2007. Of these homes, 82 million were thought to have TVs with digital capability.
 - What was the relative frequency of households with a TV having digital capability?
 - What was the relative frequency of households with a TV not having digital capability?
- What is meant by a test score that is at the 46th percentile?

In 8 and 9, refer to Example 2.

- What height is at the 50th percentile of NBA heights? Explain how you got your answer.
- What height is at the 80th percentile of NBA heights?

APPLYING THE MATHEMATICS

In 10 and 11, use the table which gives percentiles for weights of 3-year-old boys. Doctors use weight percentiles to see if children are growing properly.

- 75% of the boys are heavier than Fernando. How much does he weigh?
- Find possible values of a and b if 80% of the boys have weights in the interval $a \leq \text{weight} \leq b$.

Weight (lb)	Percentile
37	90th
34	75th
32	50th
29	25th
28	10th

12. In 1983 the American Veterinary Medical Association surveyed 20,000 households and found that 5,680 had a cat as a pet. In 2001, a similar survey of 54,000 households found that 31.6% had a cat. Was cat ownership becoming more popular?
13. **Multiple Choice** An event occurred c times out of t possible occurrences. The relative frequency of the event was 50%. Which equation is true?
 A $\frac{c}{t} = 0.5$ B $\frac{t}{c} = 0.5$ C $1 - t = 0.3$ D $1 - c = 0.3$
14. Of the people surveyed, $\frac{3}{5}$ thought the American League team would win the World Series. If $3n$ people were surveyed, how many thought the American League team would win?

In 15 and 16, use the following data, which show the level of education of U.S. adults through 2004. The numbers in the cells are in thousands.

Education Level	Age					Sum
	25–34	35–44	45–54	55–64	> 64	
Did not complete high school	5,072	5,232	4,251	3,856	9,339	27,750
Completed high school	11,244	13,739	12,910	9,436	12,482	59,811
1–3 years of college	7,583	7,420	7,210	4,824	4,771	31,808
4 or more years of college	15,304	17,183	16,699	10,258	8,067	67,511
Sum	39,203	43,574	41,070	28,374	34,659	186,880

Source: U.S. Census Bureau

15. What is the probability that an individual older than 64 completed at least one year of college?
16. What is the probability that a person between the ages of 25 and 44 did not graduate from high school?

REVIEW

17. The end-of-the-year raffle at Lincoln Elementary is the school's biggest fund-raiser. Last year, the school sold 578 tickets. The grand prize was a \$1,000 gift certificate to an electronics store. In addition, there were 3 prizes of computers, 5 bikes, and 10 sweatshirts with the school seal on them. (Lesson 5-6)
- Given that Rufus won a prize, what is the probability it was a computer?
 - If Randall bought one ticket, what is the probability that he won the grand prize?
 - How many times as likely is it to win a sweatshirt as a bike?

18. Joanie joined a bowling league to improve her game. Her scores over 10 weeks were: 52, 65, 59, 72, 70, 92, 85, 100, 95, and 93. (Lesson 5-6)
- What is the relative frequency of Joanie bowling over 70?
 - What is the relative frequency of Joanie bowling 70 or under?
19. In planning her trip from Toronto, Ontario, to Montreal, Quebec, Bianca looked at the legend of a map that says $\frac{3}{4}$ in. = 50 km. If the two cities are approximately 542 km apart, how far should the distance be between them on the map? (Lesson 5-4)
20. Simplify each expression. (Lessons 5-3, 5-2, 5-1)
- $\frac{63}{34} \div \frac{9}{2}$
 - $76 \text{ mi/hr} \cdot 2.15 \text{ hr}$
 - $\frac{7m^3(8n)}{4m^2n^2}$
21. Graph $y = |6 - 3x|$ and use your graph to solve $|6 - 3x| = 15$. (Lesson 4-9)

EXPLORATION

22.
 - Create an experiment in which randomness indicates certain probabilities that outcomes will occur.
 - Conduct the experiment a large number of times. Record the appropriate data.
 - Compare the relative frequencies you get with the probabilities predicted by randomness.
 - Do you think the outcomes occurred randomly? Why or why not?

QY ANSWER

78 percent of students in your class have GPAs less than or equal to your GPA.