

## Lesson

## 5-6

## Probability Distributions

► **BIG IDEA** The distribution of the probabilities of all possible outcomes from a situation can be displayed in tables and graphs.

In a situation such as asking a question or flipping a coin, the possibilities are called **outcomes**. No two outcomes can occur at the same time. A set of outcomes is an **event**. Recall from your earlier courses that the **probability** of an event is a number from 0 to 1 that measures how likely it is that the event will happen. The sum of the probabilities of all possible outcomes must add to 1.

If an outcome or event is identified as  $x$ , then the symbol **P(x)** stands for the probability of the outcome or event.

### How Are Probabilities Determined?

There are three common ways in which people determine probabilities.

1. Pick a probability close to the relative frequency with which the outcome or event has occurred in the past.
2. Deduce a probability from assumptions about the situation.
3. Give a best guess.

The gender of a newborn baby is a situation where there are two outcomes: B = a boy is born and G = a girl is born. Any one of the three common ways might be used to identify P(B), the probability that a boy is born.

1. According to the National Center for Health Statistics, there were 4,089,950 children born in the United States in 2003. The relative frequency of boys to births is found by dividing the number of boys born by the total number of births.

$$\text{relative frequency} = \frac{\text{number of baby boys born}}{\text{total number of births}} = \frac{2,093,535}{4,089,950} \approx 0.512 = 51.2\%$$

Using relative frequency, we might say that P(B) is about 51.2%.

2. However, some people would rather assume that the probabilities of a boy and a girl are equal. If the two probabilities are equal, then because the probabilities must add to 1,  $P(B) = P(G) = \frac{1}{2} = 50\%$ . More generally, if there are  $n$  outcomes in a situation and each has equal probability, then the probability of each is  $\frac{1}{n}$ .

### Vocabulary

outcome  
 event  
 probability  
 P(x)  
 probability distribution  
 unbiased  
 fair  
 conditional probability  
 complement  
 odds of an event

### Mental Math

Find the value of the variable.

- a. The temperature  $T$  is  $30^\circ$  cooler than  $82^\circ$ .
- b. A book's price  $p$  is 7 times as much as a \$2.25 pen.
- c. A plane departs 45 minutes later than its 6:55 A.M. departure time, at  $d$ .



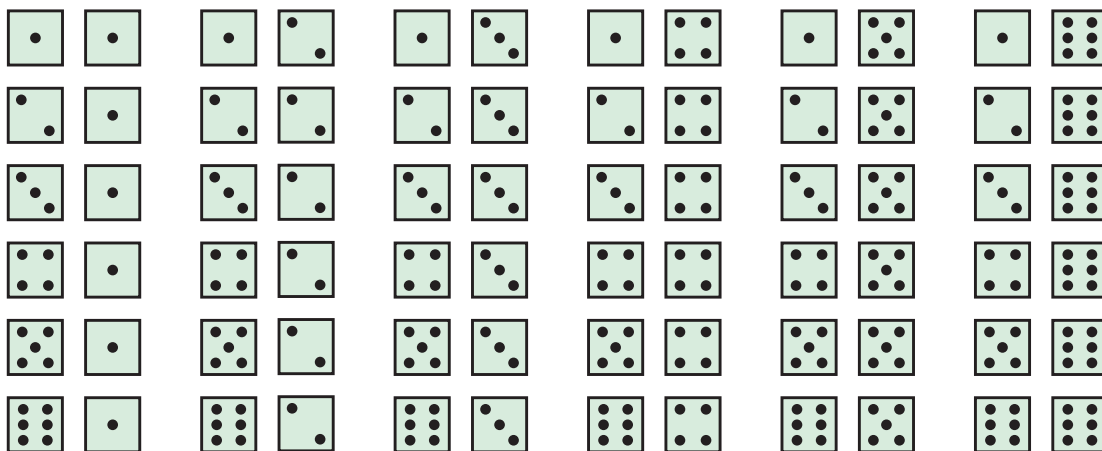
The birthrate gives the number of live births per 1,000 of population. The U.S. birthrate declined from 23.7 in 1960 to a record low of 13.9 in 2002.

Source: U.S. National Center for Health Statistics

3. In a family where more boys than girls have been born, some people might think that boys are much more likely than 50% or 51.2% to be born and guess that the probability that a boy will be born is much greater. But, usually we only guess when the event is a one-time event and there are no data from past experience. For example, if a new drug has been developed to cure a disease, at the start researchers may be able to only guess at the probability that the drug will actually work.

## Probability Distributions

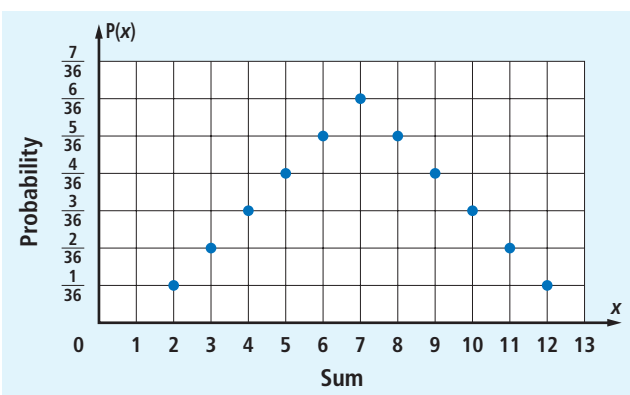
Some situations have many outcomes. A **probability distribution** is the set of ordered pairs of outcomes and their probabilities. For example, in many board games, two dice are thrown and the sum of the numbers that appear is used to make a move. Since the outcome of the game depends on landing or not landing on particular spaces, it is helpful to know the probability of obtaining each sum. The following diagram is helpful. It shows the 36 possible outcomes when two dice are thrown.



We call a situation **unbiased**, or **fair**, if each outcome has the same probability. If the dice are fair, then each of the 36 outcomes has a probability of  $\frac{1}{36}$ .

Let  $P(x)$  = the probability of getting a sum of  $x$ . In this case,  $x$  can only be a whole number from 2 to 12. There are 36 outcomes but only 11 possible sums. For example, the event “getting a sum of 7” has 6 possible outcomes: 1 and 6, 2 and 5, 3 and 4, 4 and 3, 5 and 2, and 6 and 1. So  $P(7) = 6 \cdot \frac{1}{36} = \frac{6}{36}$  or  $\frac{1}{6}$ . The probability distribution is shown on page 282 in the table and graph. A sum of 7 is the most likely outcome, so  $P(7)$  is plotted as the highest point on the graph.

$x$	$P(x)$
2	$\frac{1}{36} = 0.02\bar{7}$
3	$\frac{2}{36} = 0.0\bar{5}$
4	$\frac{3}{36} = 0.08\bar{3}$
5	$\frac{4}{36} = 0.1\bar{1}$
6	$\frac{5}{36} = 0.13\bar{8}$
7	$\frac{6}{36} = 0.1\bar{6}$
8	$\frac{5}{36} = 0.13\bar{8}$
9	$\frac{4}{36} = 0.1\bar{1}$
10	$\frac{3}{36} = 0.08\bar{3}$
11	$\frac{2}{36} = 0.0\bar{5}$
12	$\frac{1}{36} = 0.02\bar{7}$



Notice that the sum of all the probabilities adds to  $\frac{36}{36}$ , or 1. This is because these 11 events include each outcome exactly once.

### STOP QY1

You may have answered QY1 by adding  $P(9)$ ,  $P(10)$ ,  $P(11)$ , and  $P(12)$ . Another way to answer QY1 is to count the outcomes that give a sum of 9, 10, 11, and 12, and then divide that number by 36. This is because probabilities of events with equally likely outcomes satisfy the following property.

#### Probability of an Event with Equally Likely Outcomes

If a situation has a total of  $n$  equally likely outcomes and  $E$  is an event, then  $P(E) = \frac{\text{number of outcomes in } E}{n}$ .

## A Probability Distribution from Relative Frequencies

Your blood and the blood of all humans is one of eight types: O, A, B, or AB and either positive (+) or negative (-) for each type depending on the existence of a Rh antigen. Overall in the United States, the approximate percents of people with these types are shown in the table on page 283. Note: The percents do not add to 100% due to rounding.



Blood makes up about 7% of a person's body weight.

Source: www.bloodbook.com

We can take these relative frequencies as probabilities that a person at random in the United States has one of these types.

### Example 1

- What is the probability that a person in the United States has type-O blood?
- What is the probability that a person with type-O blood is O-?

#### Solutions

- A person has type-O blood if he or she is either O+ or O-.  

$$P(O) = P(O+) + P(O-)$$

$$= 38.4\% + 7.7\%$$

$$= 46.1\%$$
- Since 46.1% of the population have type-O blood, and 7.7% of the population is O-, the probability that a person with type-O blood is O- is  $\frac{P(O-)}{P(O)}$ , which is  $\frac{7.7\%}{46.1\%} \approx 0.167$ , or about 16.7%.

Type	Percent
O+	38.4
O-	7.7
A+	32.3
A-	6.5
B+	9.4
B-	1.7
AB+	3.2
AB-	0.7

Source: www.bloodbook.com

The probability calculated in Example 1b is called a **conditional probability**. It is the probability that a person has blood type O- given the condition that they are already known to be type O. You can write:  $P(O- \text{ given } O) = 16.7\%$ . In general, the conditional probability of event B given event A,  $P(B \text{ given } A) = \frac{P(A \text{ and } B)}{P(A)}$ . That is, it is the probability that both events occur divided by the probability of the first event.

## Complementary Events

If the probability of a tornado occurring on a weekend is  $\frac{2}{7}$ , then the probability of a tornado occurring on a weekday is  $\frac{5}{7}$  because there are five weekdays in a week. The events “occurring on a weekend” and “occurring on a weekday” are called *complements* of each other. Two events are **complements** if they have no elements in common and together they contain all possible outcomes.

The sum of the probability of an event and the probability of its complement is 1. The same goes for relative frequencies. Thus the probability or relative frequency of the complement of an event is found by subtracting the probability or relative frequency of the original event from 1.



The greatest incidence of tornadoes around the world is in North America.

Source: Oklahoma Climatological Survey

**STOP QY2**

The probabilities that weather forecasters use are found by using mathematical models. These models are created from a study of what happened in the past under conditions like those when the forecast is made.

**Odds**

Probabilities and complements of probabilities are used to compute *odds*. Odds are stated and used in different ways that are not always the same. One meaning is that the **odds of an event** occurring is the ratio of the probability that the event *will occur* to the probability that the event *will not occur*.

$$\text{odds of } E \text{ occurring} = \frac{P(E)}{P(\text{complement of } E)} = \frac{P(E)}{1 - P(E)}$$

For example, if you think that the odds of your being selected for a particular honor are 2 to 1, then you mean that you will be selected 2 out of 3 times, and that the probability of the event is  $\frac{2}{3}$ . This shows how to calculate a probability from odds. If the odds for the event are  $m$  to  $n$ , then the probability for the event is  $\frac{m}{m+n}$  and the probability the event will not occur is  $\frac{n}{m+n}$ .

**STOP QY3****Questions****COVERING THE IDEAS**

1. If  $E$  is an event, what does  $P(E)$  stand for?
2. Use the data for births in the United States found on page 280.
  - a. How many girls were born in the United States in 2003?
  - b. What was the relative frequency of female births in the United States in 2003?
  - c. Let  $G$  represent a girl being born. Using relative frequency, what is the value of  $P(G)$ ?
  - d. Let  $G$  represent a girl being born. If a baby has an equal probability of being a boy or a girl, what is  $P(G)$ ?
  - e. Suppose someone chooses one of the children in your family at random. What is the probability that the person chosen is a girl?

**► QY2**

A weather forecaster reports that the probability of rain tomorrow is 80%. What is the probability that it does not rain tomorrow?

**► QY3**

Cameron is managing a large project at work that is behind schedule. He estimates that there is a  $\frac{1}{4}$  chance it will be completed on time. What are the odds it will be done on time?



3. Suppose a multiple-choice question has 5 choices: A, B, C, D, and E. Jasmine guesses each answer randomly.
  - a. What is the probability that Jasmine will get a particular question correct?
  - b. What is the probability that Jasmine will miss the question?
4. Suppose you pick a number from 1 to 25 randomly out of a hat.
  - a. How many outcomes are possible?
  - b. What is the probability that you will pick the number 17?
  - c. Let  $E$  = you pick an even number. What is  $P(E)$ ?
  - d. Let  $D$  = you pick an odd number. What is  $P(D)$ ?
  - e. **Fill in the Blank**  $D$  and  $E$  are called \_\_\_\_?\_\_\_\_ events.
5. Examine the probability distribution in this lesson for the sum of the numbers on two fair dice when they are tossed.
  - a. What is  $P(2)$ ?
  - b. What is  $P(13)$ ?
  - c. What is  $P(\text{a number less than } 5)$ ?

In 6 and 7, use the information on blood types in the United States found on page 283.

6.
  - a. What blood type is the least common?
  - b. What is the most common blood type?
7.
  - a. What is the probability that a person has type-B blood?
  - b. What is the probability that a person with type-B blood is B+?
  - c. What is the probability that a person with type-B blood is B-?
  - d. **Fill in the Blank** The probability in Part c is called the \_\_\_\_?\_\_\_\_ probability that a person with type-B blood is B-.
8. When two equally matched teams play a best-of-5 series, the odds that one team will win in three games is 1 to 3. From this information, what is the probability that one team will win in three games?
9. Suppose the probability that an event will occur is  $\frac{5}{12}$ .
  - a. What are the odds in favor of the event occurring?
  - b. What are the odds against the event occurring?
10. **Fill in the Blank** Use *always*, *sometimes but not always*, or *never*. If  $p$  is the probability of an event and  $q$  is the probability of its complement, then the value of  $p + q$  \_\_\_\_?\_\_\_\_ equals 1.

In 11 and 12, find the complement of the event.

11. A heart is chosen from a standard deck of playing cards. A standard deck has 52 cards. Each card is one of four suits (clubs ♣, diamonds ♦, hearts ♥, and spades ♠) and has one of 13 values (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King).
12. You were born on a weekday.

### APPLYING THE MATHEMATICS

In 13–15, a card is picked randomly from a standard deck of playing cards.

13. Determine the probability of selecting each card.
  - a. the ace of spades
  - b. a 5
  - c. a 5 or a 9
14. Determine the probability of selecting each card.
  - a. a club
  - b. a club or a heart
  - c. a club or a 5 (This is a tricky one.)
15. If you know the card you have selected is a face card (Jack, Queen, or King), what is the probability that it is a King?
16. Detectives investigating a crime have narrowed the search for the criminal to five suspects. The table below lists each suspect's personal features.

Features	Suspect				
	1	2	3	4	5
Height	6'2"	5'8"	5'6"	6'3"	6'0"
Eye Color	Green	Blue	Green	Brown	Green
Gender	Male	Female	Female	Male	Female
Handedness	Right	Left	Right	Right	Right

- a. If chosen at random, what is the probability that Suspect 3 is the criminal?
- b. Suppose the detectives receive evidence that the criminal is female. If this new information is true, what is the probability that Suspect 3 is the criminal?
- c. Suppose the detectives also receive evidence that the female criminal is between 5'5" and 5'9" tall. Given the evidence from Part b and this new information, what is the probability that Suspect 3 is the criminal?

In 17 and 18, suppose that slips of paper containing the integers from 1 to 200 are put in a hat. A number  $x$  is drawn.

17. Determine each probability.
- $x = 135$
  - $x > 99$
  - $x < 1$
18. Determine  $P(x = 135)$  given each circumstance.
- The number  $x$  is odd.
  - The ones digit of  $x$  is 5.
  - The hundreds digit of  $x$  is 1.
  - The tens digit of  $x$  is 8.
19. A person buys a raffle ticket. The person says, "The probability of winning the raffle is  $\frac{1}{2}$  since either I will win or I won't." What is wrong with this argument?

### REVIEW

20. A television station has scheduled  $n$  hours of news,  $c$  hours of comedy,  $d$  hours of drama,  $s$  hours of sports, and  $x$  hours of other programs during the week. (Lesson 5-5)
- What is the ratio of hours of news to hours of drama?
  - What is the ratio of hours of sports to total number of hours of programs during the week?
21. The Jones family earned \$48,735 last year on their 95-acre farm.
- What is their income per acre?
  - Is the income per acre a ratio or a rate?  
(Lessons 5-5, 5-3)
22. The list price of a car is  $c$  dollars. Find the selling price according to the following conditions.
- You pay a 7% sales tax and there is no discount.
  - You get a 20% discount and there is no sales tax.
  - You pay a 7% sales tax and get a 20% discount.  
(Lessons 5-5, 4-1, Previous Course)

In 23 and 24, solve the equation. (Lessons 3-5, 3-4)

23.  $D - 8.5 - 0.25D = 7.5$       24.  $\frac{4}{3}w + 72 = 8$

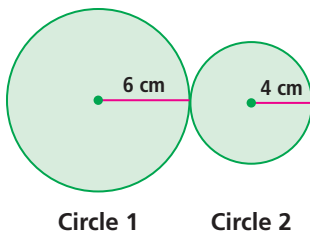


In 2004, 74 million acres of soybeans were harvested in the United States, a 31% increase since 1990.

Source: U.S. Department of Agriculture



25. Use the Distributive Property to compute \$0.50 times 299 in your head. (Lesson 2-1)
26. Two circles with radii 6 cm and 4 cm are shown below. Let  $A$  = area of Circle 1 and  $B$  = area of Circle 2. Calculate each expression. (Previous Course)



Circle 1

Circle 2

- a.  $A - B$                       b.  $\frac{B}{A}$                       c.  $\frac{A - B}{A}$

### EXPLORATION

27. a. Pick a letter of the alphabet. Estimate what percent of words used in the English language begin with that letter.
- b. Pick a reading selection that has more than 200 words. Determine the relative frequency that a word in your reading selection begins with your letter.
- c. Having done the experiment, decide whether or not you should change the probability you guessed in Part a. Explain your decision.

### QY ANSWERS

1.  $\frac{10}{36} = 0.2\bar{7}$
2. 20%
3. 1 to 3