

## Lesson

## 5-5

## Ratios

## Vocabulary

ratio  
tax rate  
discount rate

► **BIG IDEA** The quotient of two measures with the same type of units is a ratio.

## What Is a ratio?

A *ratio* describes how many times larger one number is compared to another. For example, dogs come in a huge variety of shapes and sizes. Consider the three types pictured below, which are very different from each other. To compare the three dog breeds, you could use corresponding lengths on their bodies to form ratios.



Dachshund



Labrador Retriever



Greyhound

## Mental Math

Give the coordinates of a point on

- the  $x$ -axis.
- the  $y$ -axis.
- both the  $x$ - and  $y$ -axes.

## Activity

Measure the pictures to answer each question.

- Which dog has the greatest  $\frac{\text{head length}}{\text{front leg length}}$  ratio?
- Which dog has the least  $\frac{\text{head length}}{\text{body length}}$  ratio?
- Which dog has the least  $\frac{\text{front leg length}}{\text{body length}}$  ratio?
- Which dog has the  $\frac{\text{front leg length}}{\text{body length}}$  ratio that is closest to 1?

The direction of the comparison is important. The dachshund's  $\frac{\text{head length}}{\text{front leg length}}$  ratio is much greater than 1 because its head is much longer than its front legs. But the reciprocal ratio  $\frac{\text{front leg length}}{\text{head length}}$  is much less than 1.

A **ratio** is a comparison of two quantities with the same type of units. For example,  $a$  to  $b$  is written  $\frac{a}{b}$ . Similarly, the ratio  $\frac{b}{a}$  compares  $b$  to  $a$ . Notice the difference between a rate and a ratio. In the rate  $a$  per  $b$ , the units for  $a$  and  $b$  are different. In a ratio, the units are the same.  $\frac{40 \text{ km}}{2 \text{ hr}} = 20 \frac{\text{km}}{\text{hr}}$  is a rate.  $\frac{40 \text{ km}}{8 \text{ km}} = 5$  is a ratio.

### Example 1

It takes Max  $\frac{1}{4}$  of an hour to ride his bike to school, and it takes Riley 21 minutes to walk to school. Write a ratio comparing Max's time to Riley's time.

**Solution** The units of measure for 21 minutes and for  $\frac{1}{4}$  of an hour are not the same, so we need to change hours to minutes. Since Max's time is to be compared to Riley's, his time is in the numerator.

$$\frac{\text{Max's Time}}{\text{Riley's Time}} = \frac{\frac{1}{4} \text{ hr}}{21 \text{ min}} = \frac{\frac{1}{4} \text{ hr} \cdot \frac{60 \text{ min}}{\text{hr}}}{21 \text{ min}} = \frac{15 \text{ min}}{21 \text{ min}} = \frac{5}{7}$$

This means that it takes Max  $\frac{5}{7}$  of the time to ride his bike as it takes Riley to walk to school.



About 20 million bicycles were sold in the United States in 2005.

Source: National Bicycle Dealers Association

## Ratios and Percents

Ratios can be expressed as fractions, decimals, or percents. In Example 1,  $\frac{5}{7} \approx 0.\overline{714285}$ , so you could say that it takes Max about 71% or  $\frac{71}{100}$  of the time it takes Riley to go to school.

While in this book we distinguish rates from ratios, in the real world some ratios are called rates. In money matters, the **tax rate** is the ratio of the tax amount to the selling price. The **discount rate** is the ratio of the discount to the original price.

### Example 2

A TV that normally sells for \$400 is on sale for \$340. The tax on the reduced price is \$23.80, so the total cost with tax is \$363.80.

- What is the discount rate?
- What is the tax rate?
- Including tax, how much would a customer save by buying the TV on sale?

#### Solutions

$$\text{a. discount rate} = \frac{\text{amount of discount}}{\text{original price}} = \frac{\$400 - \$340}{\$400} = \frac{\$60}{\$400} = 0.15 = 15\%$$

(continued on next page)

- b. tax rate =  $\frac{\text{amount of tax}}{\text{discounted price}} = \frac{\$23.80}{\$340} = 0.07 = 7\%$
- c. Paying full price, the 7% tax is paid on \$400. Recall that price + 7% of price =  $1.07 \cdot \text{price}$ . So at full price, the customer would pay  $1.07(400) = \$428$ . Buying the TV on sale saves the customer  $\$428 - \$363.80 = \$64.20$ .

## Using Ratios to Set Up Equations

When two numbers are in the ratio  $a$  to  $b$ , then they are also in the ratio  $ka$  to  $kb$  because  $\frac{a}{b} = \frac{ka}{kb}$  for any nonzero value of  $k$ . For example, if the ratio of boys to girls in a band is  $\frac{7}{6}$ , the ratio is also  $\frac{14}{12}$ , or  $\frac{21}{18}$ .

If you know the original ratio, then you can multiply the numerator and denominator by the same constant and never change the value of the ratio.

Directions for mixing foods or chemicals are frequently given as ratios. You can use equations to determine actual quantities from the ratios.

### Example 3

One Internet site warns painters: “Always • always • ALWAYS! make a note of the paint ratios when you mix paints.” In painting a pink wall, Bethany mixed 1 part red paint with 3 parts white paint. How much of each color will be needed for a wall that will use about 5 gallons of paint in all?

**Solution** The red paint and white paint are in the ratio of 1 to 3. So their ratio is  $\frac{1}{3}$ . Now we use the fact that  $\frac{1}{3} = \frac{1k}{3k}$  for any nonzero  $k$ . So let  $k$  be the number of gallons of red paint and  $3k$  be the number of gallons of white paint. The total paint needed is 5 gallons, so you can set up the following equation and solve for  $k$ .

$$1k + 3k = 5$$

$$4k = 5$$

$$k = \frac{5}{4} = 1.25$$

$$\text{So } 3k = 3 \cdot 1.25 = 3.75$$

She will need 1.25 gallons of red paint and 3.75 gallons of white paint.

**Check**  $\frac{1.25}{3.75} = \frac{1}{3}$  and  $1.25 \text{ gallons} + 3.75 \text{ gallons} = 5 \text{ gallons}$ . It checks.



One gallon of paint generally covers 400 square feet of wall space.

Source: Paint and Decorating Retailers Association

### QY

To paint a house light green, you want a ratio of 1 part green to 4 parts white. Find out how much of each color you must buy if you need 12 gallons of paint.

## Changing Ratios

It is natural to ask how a ratio will change if the numerator or denominator (or both) change. For example, right now Myron is 6 years old and his little sister Ella is 2. The ratio of Myron's age to Ella's is  $\frac{6 \text{ years}}{2 \text{ years}} = \frac{3}{1}$ . Now, Myron is 3 times as old as his sister. But when they get 2 years older, the ratio will have changed. In two years,  $\frac{6 + 2 \text{ years}}{2 + 2 \text{ years}} = \frac{8}{4} = \frac{2}{1}$ , so Myron will be twice as old as Ella.

### Example 4

Suppose a team has won 15 of its first 38 games. How many games must it win in a row to bring its winning percentage to at least 0.500?

**Solution** If the team now wins  $r$  games in a row, then the team will have played  $38 + r$  games and have won  $15 + r$  of them. So we want to know when  $\frac{\text{number of wins}}{\text{number of games played}} = \frac{15 + r}{38 + r} \geq 0.500$ .

To clear this inequality of fractions, multiply both sides by  $38 + r$ . This is a positive number so the sense of the inequality remains the same.

$$\begin{array}{ll} 15 + r \geq 0.500(38 + r) & \text{Multiply each side by } (38 + r). \\ 15 + r \geq 19 + 0.5r & \text{Distributive Property} \\ r \geq 4 + 0.5r & \text{Add } -15 \text{ to both sides.} \\ 0.5r \geq 4 & \text{Add } -0.5r \text{ to both sides.} \\ r \geq 8 & \text{Divide each side by } 0.5. \end{array}$$

The team must win 8 games in a row to bring its winning percentage up to at least .500.

## Questions

### COVERING THE IDEAS

- Grass snakes and rattlesnakes are found in the Great Plains. Grass snakes grow up to 20 inches in length, and rattlesnakes grow to 45 inches. Express the ratio of rattlesnake length to grass snake length
  - as a fraction.
  - as a percent.
- An item is on sale for \$16. It originally cost \$28.
  - What is the discount rate?
  - If the tax on the sale price is \$1.04, what is the tax rate?



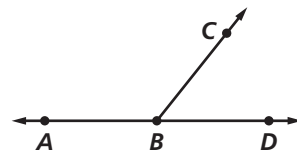
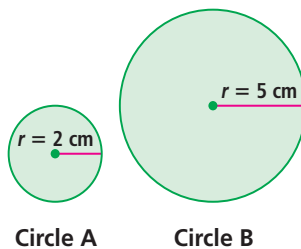
Rattlesnakes are usually between 7 and 15 inches long at birth

Source: San Diego Zoo

3. A recipe for fruit juice punch calls for 1 part tropical fruit-punch and 2 parts ginger ale. If a person wants to make 5 gallons (20 quarts) of fruit-juice punch, how many quarts of each ingredient are needed?
4. A paint mixture calls for 7 parts of linseed oil, 5 parts of solvent, and 1 part of pigment. How much of each ingredient is needed to make 60 gallons of paint?
5. A team has won 11 of 17 games.
  - a. If the team wins its next 2 games, what will its winning percentage be? Round your answer to three decimal places.
  - b. If the team loses its next  $n$  games, what will its winning percentage be?
  - c. How many games could the team lose and still have a winning percentage above .600?

### APPLYING THE MATHEMATICS

6. At a local concert,  $\frac{1}{6}$  of the attendees are teenagers and  $\frac{5}{8}$  of the people are between the ages of 20 and 40. The rest of the people are older. Write the ratio (in lowest terms) of the number of people over 40 to the number of people between 20 and 40.
7. The ratio of adults to children at a concert is expected to be 2 to 5. If 200 people attend, how many are expected to be children?
8. The circles at the right have radii 2 centimeters (Circle A) and 5 centimeters (Circle B).
  - a. Find the circumference and area of each circle using  $C = \pi d$  and  $A = \pi r^2$ .
  - b. Find the ratio of the diameter of Circle A to the diameter of Circle B in lowest terms.
  - c. Give the ratio of the area of Circle A to the area of Circle B in lowest terms.
9.
  - a. In 4 minutes, Liana can type 140 words. If she has an essay that is 700 words long, about how long will it take Liana to type it?
  - b. In 5 minutes, Elan can type 160 words. About how many words can he type in 12 minutes?
10. Recall that two angles are called *supplementary angles* if the sum of their measures is  $180^\circ$ . In the diagram at the right,  $\angle ABC$  and  $\angle CBD$  are supplementary angles whose measures have a ratio of 7 to 3. Find the measure of each angle.





## REVIEW

11. An airplane is scheduled to fly from LaGuardia Airport in New York to Los Angeles International Airport in 6 hours and 18 minutes. If the distance between the two airports is 2,779 miles, what is the average speed of the airplane? (Lesson 5-3)
12. For what values of  $c$  is  $\frac{16-4c}{2c+9}$  undefined? (Lesson 5-3)
13. Tell whether each fraction equals  $\frac{99}{100}$ . (Lessons 5-3, 5-2)
- a.  $\frac{999}{1,000}$       b.  $-\frac{-99}{-100}$       c.  $\frac{1}{1-\frac{1}{100}}$       d.  $\frac{\frac{99}{7}}{\frac{7}{100}}$
14. **Skill Sequence** Simplify each expression. (Lessons 5-2, 5-1, 2-1)
- a.  $\frac{x}{4} + \frac{x}{5}$       b.  $\frac{x}{4} - \frac{x}{5}$       c.  $\frac{x}{4} \cdot \frac{x}{5}$       d.  $\frac{x}{4} \div \frac{x}{5}$
15. The volume of a box is to be less than 20 cubic meters. If the base has dimensions 185 cm by 250 cm, what inequality describes possible heights of the box? (Lesson 3-7)
16. What fraction of a complete turn is a rotation of  $75^\circ$ ? (Previous Course)

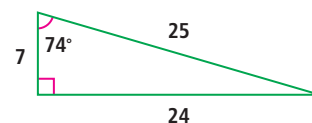


Los Angeles International Airport is serviced by more than 85 major airlines and is the third largest airport in the world in terms of passenger traffic.

Source: The New York Times

## EXPLORATION

17. The right triangle shown here has side lengths of 7, 24, and 25. One of its angles has a measure of about  $74^\circ$ . The lengths of the three sides can form six different ratios.
- Write the values of all six of these ratios.
  - These ratios have special names. One of these is called the *sine*. For this triangle it is written  $\sin 74^\circ$ . Put your calculator in degree mode and compute  $\sin 74^\circ$  using the key sequence  $74$   $\boxed{\text{SIN}}$  or  $\boxed{\text{SIN}}$  $74$  (depending on your calculator), and determine which of the six ratios in Part a is the sine.
  - Another special ratio is called the *cosine*. Compute  $\cos 74^\circ$  on your calculator and determine which of the six ratios in Part a is the cosine.
  - A third special ratio is called the *tangent*. Compute  $\tan 74^\circ$  on your calculator and determine which of the six ratios in Part a is the tangent.
  - Which of the three ratios in Parts b, c, and d is a ratio of the other two?



## QY ANSWER

2.4 gal of green paint,  
9.6 gal of white paint