Lesson 5-4

Multiplying and Dividing Rates

Vocabulary

conversion rate

BIG IDEA When rates are multiplied or divided, the unit of the answer follows rules of arithmetic on the units of the original rates.

Rates can be multiplied or divided by other quantities. The units are multiplied as if they were numeric fractions. Keeping the units in the calculations can help you understand the meaning.

If you bought 2.5 pounds of fish at \$8.49 a pound, it would cost $2.5 \text{ lb} \cdot 8.49 \frac{\text{dollars}}{\text{lb}} = 21.225$ dollars, or \$21.23.
Notice how the unit "lb" in "2.5 lb" cancels the unit "lb" in the denominator of $\frac{\text{dollars}}{\text{lb}}$, so that the result is in dollars.

Mental Math

Find a pair of numbers x and y that satisfy the equation.

a. $2x + 3y = 600$

b. $2x - 3y = 600$

c. $20x + 30y = 600$

Conversion Rates

A **conversion rate** is a rate determined from an equality between two quantities with different units. For example, since 1 hour = 60 minutes, dividing one unit by the other equals 1.

$$\frac{1 \text{ hour}}{60 \text{ min}} = \frac{60 \text{ min}}{1 \text{ hour}} = 1$$

Both $\frac{1 \text{ hour}}{60 \text{ min}}$ and $\frac{60 \text{ min}}{1 \text{ hour}}$ are conversion rates. Multiplying a quantity by a conversion rate does not change its value.

Example 1

On average, an adult human heart beats about 70 times per minute. At this rate, how many times does a heart beat in a 365-day year?

Solution We begin with the given information and multiply by conversion rates until one minute is converted into one year.

$$70 \frac{\text{beats}}{\text{minute}} = 70 \frac{\text{beats}}{\cancel{\text{minute}}} \cdot \frac{60 \cancel{\text{minutes}}}{1 \cancel{\text{hour}}} \cdot \frac{24 \cancel{\text{hours}}}{1 \cancel{\text{day}}} \cdot \frac{365 \cancel{\text{days}}}{1 \text{ year}}$$

$$= 70 \cdot 60 \cdot 24 \cdot 365 \frac{\text{beats}}{\text{year}}$$

$$= 36{,}792{,}000 \frac{\text{beats}}{\text{year}}$$

The Pike Place Market in Seattle, Washington, attracts 10 million visitors per year.

Source: Pike Place Market

Rates are used in many formulas. One of the more important formulas is $d = rt$, which gives the distance d traveled by an object moving at a constant rate r during a time t. (r is often called the speed.)

GUIDED

Example 2

Explain why a train traveling 68 $\frac{\text{mi}}{\text{hr}}$ is traveling about 100 $\frac{\text{ft}}{\text{sec}}$.

Solution The goal is to convert miles to ___?___ and an hour to ___?___.

Use conversion rates so that the units cancel in the numerator and denominator just like common factors.

$$\frac{68 \text{ mi}}{\text{hr}} = \frac{68 \text{ mi}}{\text{hr}} \cdot \frac{1 \text{ hr}}{\text{?}} \cdot \frac{\text{?}}{60 \text{ sec}} \cdot \frac{\text{?}}{1 \text{ mi}}$$

$$= \frac{68 \cdot \text{?}}{60 \cdot 60} \frac{\text{ft}}{\text{?}}$$

$$= \text{?}$$

So 68 $\frac{\text{mi}}{\text{hr}}$ is about 100 $\frac{\text{ft}}{\text{sec}}$.

STOP **QY**

▶ **QY**

The International Sports Medicine Institute has a formula for daily water intake. A nonactive person should consume $\frac{1}{2}$ ounce per pound of body weight. An athletic person should consume $\frac{2}{3}$ ounce per pound of body weight. (Note: 1 cup = 8 ounces.) If a person weighs p pounds, how many cups of water should the person drink each day if he or she is:

a. nonactive?

b. athletic?

Using Reciprocal Rates

Sometimes a rate does not help to simplify the computation but its reciprocal does. Guided Example 2 used $\frac{1 \text{ hr}}{60 \text{ min}}$, while Example 1 used $\frac{60 \text{ min}}{1 \text{ hr}}$.

Remember the algebraic definition of division: $\frac{a}{b} = a \cdot \frac{1}{b}$. So to divide by a rate, you can multiply by its reciprocal. For example, if a book has 672 pages and Marta reads an average of 35 pages per day, you can use division.

$$\frac{672 \text{ pages}}{\frac{35 \text{ pages}}{\text{day}}} = 672 \cancel{\text{ pages}} \cdot \frac{1}{35} \frac{\text{day}}{\cancel{\text{pages}}} = 19.2 \text{ days}$$

So it will take her a little more than 19 days to finish reading the book.

Not every reciprocal rate is meaningful in every situation. For example, in a school with an average of 23 $\frac{\text{students}}{\text{class}}$, the equivalent reciprocal rate of $\frac{1}{23} \frac{\text{class}}{\text{students}}$ does not have much meaning. Still, it might be useful in a computation.

There are approximately 8,000 to 10,000 professional online book dealers, selling books with an average price range of $10–$19 per book.

Source: www.bookologist.com

Questions

COVERING THE IDEAS

1. **a.** If a car travels 30 minutes at an average rate of 44 miles per hour, how far will it have gone?
   b. If a car travels m minutes at an average rate of 44 miles per hour, how far will it have gone?
2. Suppose ground beef is \$2.59 per pound.
   a. How much will it cost you to purchase P pounds?
   b. How many pounds can you purchase for x dollars?
3. One version of an English translation of the novel *War and Peace* by Leo Tolstoy has 1,370 pages. If you read 12 pages per day, show how to calculate how many months it will take you to read the entire novel. Use the conversion 1 month $\approx$ 30 days.

In 4 and 5, a rate is given.

a. Write a sentence using the rate to describe a situation.
b. Name the reciprocal rate.
c. Write a sentence using the reciprocal rate to describe a situation.

4. 12 feet per second
5. \$4 per kilogram

In 6 and 7, give the two conversion rates that result from the given equation.

6. 1,000 m = 1 km
7. 36 in. = 1 yd
8. A penny has a thickness of about 0.08 inch. How high, to the nearest 0.1 mile, would a stack of 1 million pennies be?
9. Convert 30 miles per hour into kilometers per hour using the conversion 1 mile $\approx$ 1.6 kilometers.

APPLYING THE MATHEMATICS

10. In 2005, Danica Patrick drove the fastest lap ever at the time during trials for the Indianapolis 500 auto race. In a South African newspaper, her speed for the lap was given as 360.955 km/hr.
    a. What is this speed to the nearest hundredth of a mile per hour? Use the conversion 1 mi = 1.609344 km.
    b. To the nearest hundredth of a minute, how long did it take her to drive one lap of the 2.5-mile track?

In 11 and 12, write a rate multiplication problem whose answer is the given quantity.

11. 200 inches
12. 4.2 $\frac{\text{min}}{\text{page}}$

Each penny costs 1.23 cents to make, but the U.S. Mint collects only one cent for it.

Source: U.S. Department of Treasury

13. Kenji can wash k dishes per minute. His sister Suna is twice as fast.
    a. How many dishes can Suna wash per minute?
    b. How many minutes does Suna spend per dish?

14. Suppose Chip is baking cookies for a school bake sale. His oven bakes 36 cookies every 10 minutes. He wants to bake d dozen cookies for the sale. How long will it take him to bake all of the cookies?

15. In 2006, Gordy Savela ran 300 miles across northern Minnesota in 12 days.
    a. What distance did Gordy average each day?
    b. If a marathon is 26.2 miles, how many marathons did Gordy run during his journey?

Bake sales have long been one of the most popular ways of raising funds for schools, social clubs, and other organizations.

16. People sometimes go for a walk after a big meal to burn off calories. To lose 1 pound, a person must burn 3,500 calories. Suppose a person burns about 300 calories per hour by walking.
    a. If a person walks for 2.5 hours, how many pounds will the person lose?
    b. If a person walks for 2.5 hours, how many ounces will the person lose?
    c. If a person walks for h hours, how many ounces will the person lose?
    d. If a person walks for m minutes, how many ounces will the person lose?

REVIEW

17. Over the past five days, Mykia has run 3.6 miles one day, 2.9 miles each of two days, 4.2 miles one day, and 1.9 miles one day. Calculate the average number of miles run per day. Write your answer as a rate. **(Lesson 5-3)**

18. For what value(s) of n is $\frac{19-2n}{19+2n}$ undefined? **(Lesson 5-3)**

19. A box is $\frac{1}{3}$ as long, $\frac{1}{3}$ as wide, and $\frac{1}{3}$ as high as a crate. How many of these boxes will fit in the crate? **(Lesson 5-1)**

In 20–22, use the formula $H = \frac{M+F+5}{2}$. It predicts the adult height H of a boy based on his mother's height M and his father's height F, where all measurements are given in inches. The formula $H = \frac{M+F-5}{2}$ applies to the adult height of girls. (Lesson 4-7)

20. Solve each formula for M.

21. Booker's father's height is 73 inches, and his mother's height is 68 inches. How tall does the formula predict Booker will be when he is an adult?

22. Predict your adult height using this formula.

Multiple Choice In 23–25, decide whether each equation has

A no solution.

B one solution.

C more than one solution. (Lesson 4-5)

23. $\frac{x}{5} + 5 = 5$

24. $\frac{y}{4} + 4 = \frac{y}{4}$

25. $\frac{z}{3} + 3 = 3\left(\frac{z}{3} + 3\right)$

26. Solve $8(-3d - 4) \leq 13(2 - d) - (7d - 2)$. (Lessons 4-4, 2-1)

EXPLORATION

27. **a.** Here is a problem found in many puzzle books. If a hen and a half can lay an egg and a half in a day and a half, how long will it take 24 hens to lay 24 eggs? Explain how you got your answer.

b. Generalize Part a. If a hen and a half can lay an egg and a half in a day and a half, how long will it take h hens to lay e eggs?

QY ANSWERS

a. $\frac{p}{16}$ c

b. $\frac{p}{12}$ c