Lesson 5-2

Division of Algebraic Fractions

▶ **BIG IDEA** Algebraic fractions are divided in the same way as you divide numeric fractions.

Remember that two numbers are *reciprocals* if the product of the numbers is 1. The reciprocal of the number $\frac{c}{d}$ is $\frac{d}{c}$, with $c \neq 0$ and $d \neq 0$, because $\frac{c}{d} \cdot \frac{d}{c} = \frac{c \cdot d}{d \cdot c} = 1$.

When 32 ounces of orange juice are shared equally among 5 people, each person gets $\frac{1}{5}$ of a quart. This is an example of the Algebraic Definition of Division: dividing by a number is the same as multiplying by its reciprocal.

Mental Math

Use the triangle below.

a. What is x?

b. What is y?

Dividing Algebraic Fractions

Consider the division of fractions $\frac{a}{b} \div \frac{c}{d}$. Since dividing by a number is the same as multiplying by its reciprocal, dividing by $\frac{c}{d}$ gives the same result as multiplying by $\frac{d}{c}$.

Dividing Fractions Property

For all real numbers a, b, c, and d, with b, c, and $d \neq 0$,
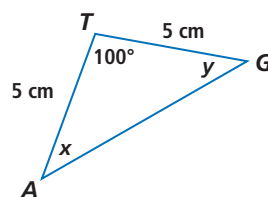$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}.$$

For example, $\frac{5}{3} \div \frac{7}{11} = \frac{5}{3} \cdot \frac{11}{7} = \frac{55}{21}$. The Dividing Fractions Property is sometimes called the "invert and multiply" rule.

Example 1

Simplify $\frac{x}{4} \div \frac{3}{5}$.

Solution Dividing by $\frac{3}{5}$ is the same as multiplying by $\frac{5}{3}$.

$$\frac{x}{4} \div \frac{3}{5} = \frac{x}{4} \cdot \frac{5}{3} = \frac{5x}{12}$$

Check Substitute some value for x. Use this number to evaluate the original expression and your answer. Suppose $x = 2$. Does $\frac{2}{4} \div \frac{3}{5} = \frac{5 \cdot 2}{12}$? To determine this, change each fraction to a decimal. Does $0.5 \div 0.6 = \frac{10}{12}$? Yes, each side equals $0.8\overline{3}$.

Simplifying Complex Fractions

Recall that a horizontal fraction bar indicates division. The division $\frac{a}{b} \div \frac{c}{d}$ can be written as $\dfrac{\frac{a}{b}}{\frac{c}{d}}$. Fractions of the form $\dfrac{\frac{a}{b}}{\frac{c}{d}}$ are called *complex fractions.* A **complex fraction** consists of three fractions; One is the numerator and the second is the denominator of a third "bigger" fraction.

fraction in numerator $\qquad \dfrac{\frac{a}{b}}{\frac{c}{d}}$ "big fraction"
fraction in denominator

Since a fraction is a division, one way to simplify a complex fraction is as follows: $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d}$.

GUIDED

Example 2

Simplify $\dfrac{\frac{6x}{5}}{\frac{9x}{10}}$.

Solution Rewrite the fraction as a division.

$$\dfrac{\frac{6x}{5}}{\frac{9x}{10}} = \underline{\;\;?\;\;} \div \underline{\;\;?\;\;}$$

$$\frac{6x}{5} \div \underline{\;\;?\;\;} = \frac{6x}{5} \cdot \underline{\;\;?\;\;} \quad \text{Dividing Fractions Property}$$

$$= \frac{\underline{\;\;?\;\;} x}{\underline{\;\;?\;\;} x} \quad \text{Multiply the fractions.}$$

$$= \frac{4}{3} \quad \text{Simplify.}$$

Check Let $x = 20$. Then $\frac{6x}{5} = \underline{\;\;?\;\;}$ and $\frac{9x}{10} = \underline{\;\;?\;\;}$.

Does $\frac{6x}{5} \div \frac{9x}{10} = \frac{4}{3}$? $\underline{\;\;?\;\;}$

When entering complex fractions into a calculator, be sure to group the numerator fraction in parentheses and the denominator fraction in parentheses. Otherwise, the calculator will follow the order of operations and the result will be incorrect.

For $\dfrac{\frac{2}{3}}{\frac{5}{12}}$, enter $(2/3)/(5/12)$. If you enter $2/3/5/12$, you will obtain $\dfrac{1}{90}$, which is incorrect. The correct quotient is $\dfrac{8}{5}$ or 1.6.

🛑 **QY**

> ▶ **QY**
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> Write $\dfrac{\frac{3}{8}}{\frac{3}{11}}$ as a decimal.

Questions

COVERING THE IDEAS

1. In this lesson, it is noted that $\dfrac{5}{3} \div \dfrac{7}{11} = \dfrac{55}{21}$. Check this result by approximating all three fractions by decimals.

2. State the Algebraic Definition of Division.

In 3 and 4, fill in the blanks.

3. a. $\dfrac{m}{n} = m \div \underline{\quad?\quad}$ b. $\dfrac{m}{n} = m \cdot \underline{\quad?\quad}$

4. a. $\dfrac{\frac{p}{q}}{\frac{r}{s}} = \dfrac{p}{q} \div \underline{\quad?\quad}$ b. $\dfrac{p}{q} \div \dfrac{r}{s} = \dfrac{p}{q} \cdot \underline{\quad?\quad}$

In 5–11, simplify the expression.

5. $\dfrac{\frac{4}{5}}{\frac{5}{6}}$ 6. $\dfrac{\frac{4}{x}}{\frac{x}{y}}$ 7. $\dfrac{\frac{3a}{2}}{\frac{a}{2}}$ 8. $\dfrac{1}{2} \div x$

9. $\dfrac{m}{30} \div \dfrac{n}{84}$ 10. $\dfrac{8v}{5} \div \dfrac{2v}{25}$ 11. $\dfrac{\frac{3\pi}{5}}{6\pi}$

APPLYING THE MATHEMATICS

12. Cody and Troy solved $\dfrac{3}{8}x = 15$ using different methods. Explain why Cody and Troy got the same solution.

   Cody's Method

   $$\dfrac{\frac{3}{8}x}{\frac{3}{8}} = \dfrac{15}{\frac{3}{8}}$$
   $$x = 15 \div \dfrac{3}{8}$$
   $$x = 15 \cdot \dfrac{8}{3}$$
   $$x = 40$$

   Troy's Method

   $$\dfrac{8}{3} \cdot \dfrac{3}{8}x = 15 \cdot \dfrac{8}{3}$$
   $$x = 40$$

13. The area of a rectangle with side lengths of m and $\dfrac{4}{23}$ is 16. Find the value of m.

14. Half of a pizza was divided equally among 3 people. How much of the original pizza did each person receive?

15. Le Parfum Company produces perfume in 200-ounce batches and bottles it in quarter-ounce bottles.

    a. Write a division problem that will tell you how many bottles will be filled by one batch.

    b. Find the answer.

16. A dozen bagels are bought for a group of x people. On average, how many bagels are there per person?
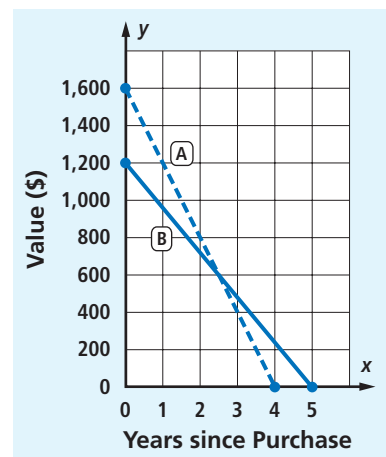
In 17–19, simplify the expression.

17. $b \div \frac{1}{b}$

18. $\frac{xy}{21} \div \frac{x}{47}$

19. $\frac{\frac{12m}{5}}{\frac{mn}{20}}$

Machines and attached tanks distill perfume in the Molinard Perfumerie in France.

20. a. Evaluate $x \div y$ and $y \div x$ for each of the following.

    i. $x = 12$ and $y = 2$ ii. $x = 20$ and $y = -5$ iii. $x = \frac{2}{3}$ and $y = \frac{4}{5}$

    b. Do your answers in Part a indicate that division is commutative? Explain your answer.

    c. Describe how $x \div y$ and $y \div x$ are related in general.
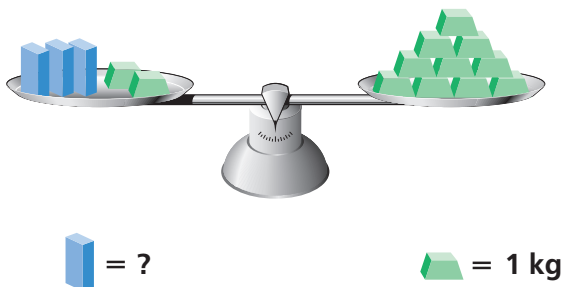
REVIEW

21. Multiply and simplify $\frac{5}{13}d \cdot \left(\frac{d}{5} \cdot \frac{5}{13}d \right)$. **(Lesson 5-1)**

22. The graph at the right compares the values of two computers A and B over time. **(Lesson 4-3)**

    a. Which computer is decreasing in value faster?

    b. About how much does the value of the computer you found in Part a change each year?

    c. After about how many years do the computers have the same value?

    d. Suppose you buy these two computers and you wish to sell one of them after 3 years. For which computer will you get more money? About how much more will you get for it?

23. a. Solve $V + 0.06V - 100 = 14{,}289.16$.

    b. **Fill in the Blanks** The equation in Part a could arise from this situation. After a discount of $100 and with a ___?___ tax, the car cost __?__. Find V, the cost of ___?___. **(Lessons 4-1, 3-5)**
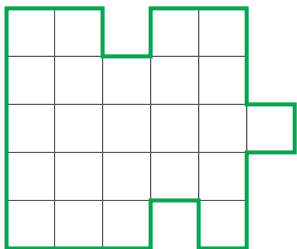
24. Let $y =$ the depth of a point in Lake Baikal in Siberia, the deepest lake in the world. **(Lesson 3-6)**

   a. Give a reasonable domain for y.

   b. It is known that the deepest point in the Lake Baikal is 1,940 meters below the surface. What inequality does y satisfy?

   c. Graph the solution set to Part b.

25. Use the picture of the balance below. The boxes are equal in weight and the other objects are one-kilogram weights. **(Lesson 3-3)**

📦 = ?   🔺 = 1 kg

   a. Write an equation describing the situation, with B representing the weight of one box.

   b. What is the weight of one box?

26. A circle has a radius of 1.2 meters. Find its area to the nearest tenth of a square meter. **(Previous Course)**

EXPLORATION

27. Congruent figures are figures with the same size and shape. Split this region into 6 congruent pieces.

Lake Baikal is situated nearly in the center of Asia in a huge stone bowl set 445 meters above sea level.

Source: www.irkutsk.org

QY ANSWER

1.375