Lesson

5-1 Multiplication of Algebraic Fractions

Vocabulary

algebraic fraction

BIG IDEA Algebraic fractions are multiplied in the same way you multiply numeric fractions.

In algebra, a division is represented by a fraction. An **algebraic fraction** is a fraction with a variable in the numerator, in the denominator, or in both. Here are some algebraic fractions.

$\frac{7t}{2}$ $\frac{-a}{6.4bc}$ $\frac{3m+4}{4m+3}$ $\frac{\frac{2}{3}+\frac{4}{5}}{x^2}$ $\frac{x-y}{\sqrt{x^2+y^2}}$

Mental Math

Evaluate.

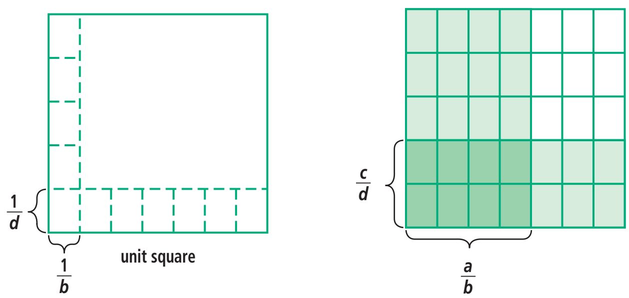
a. 0.5 • 4

b. 5 • 0.4

c. 0.5 • 0.4

Multiplying Algebraic Fractions

Algebraic fractions are multiplied just as you multiply numeric fractions. Below is a way to picture the product of the fractions $\frac{a}{b}$ and $\frac{c}{d}$. First draw a unit square as shown below. Split one side into b parts and the other side into d parts, and draw lines creating bd small rectangles. Then find $\frac{a}{b}$ of one side and $\frac{c}{d}$ of an adjacent side.



There are ac shaded rectangles out of bd small rectangles in the unit area. So the area is $\frac{ac}{bd}$. This describes the common rule for multiplying fractions, which applies to all algebraic fractions.

Multiplying Fractions Property

For all real numbers a, b, c, and d, with $b \neq 0$ and $d \neq 0$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

Example 1

Multiply $\frac{L}{4} \cdot \frac{W}{3}$.

Solution Use the Multiplying Fractions Property.

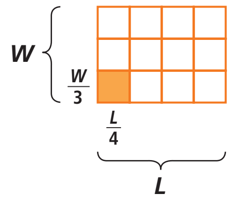
$$\frac{L}{4} \cdot \frac{W}{3} = \frac{LW}{12}.$$

Check 1 Substitute values for L and W, say $L = 20$ and $W = 8$.

Does $\frac{20}{4} \cdot \frac{8}{3} = \frac{20 \cdot 8}{12}$?

Yes, the left side is $5 \cdot \frac{8}{3}$, or $\frac{40}{3}$. The right side is $\frac{160}{12}$, or $\frac{40}{3}$.

Check 2 Use an area model. Draw a rectangle with length L and width W. Divide the length into fourths and the width into thirds. Shade a smaller rectangle with dimensions $\frac{L}{4}$ and $\frac{W}{3}$. Since the L by W rectangle with area LW is divided into twelfths, the shaded rectangle has area $\frac{1}{12} \cdot LW$ or $\frac{LW}{12}$.



The fraction $\frac{LW}{12}$ is another way of writing $\frac{1}{12}LW$. Example 2 below shows how the Multiplying Fractions Property can be used to explain why many different expressions with algebraic fractions are equivalent.

Example 2

Show that each of the following expressions equals $\frac{5x}{3}$.

a. $\frac{5}{3}x$ b. $\frac{1}{3} \cdot 5x$ c. $5 \cdot \frac{x}{3}$

Solutions Notice how each part below uses the property that $x = \frac{x}{1}$ and the Multiplying Fractions Property.

a. $\frac{5}{3}x = \frac{5}{3} \cdot \frac{x}{1} = \frac{5x}{3}$

b. $\frac{1}{3} \cdot 5x = \frac{1}{3} \cdot \frac{5x}{1} = \frac{5x}{3}$

c. $5 \cdot \frac{x}{3} = \frac{5}{1} \cdot \frac{x}{3} = \frac{5x}{3}$

This shows that $\frac{5x}{3}$, $\frac{1}{3} \cdot 5x$, and $5 \cdot \frac{x}{3}$ are all equal to each other.

Equal Fractions

As you know, every numerical fraction is equal to many other fractions. For example, $\frac{3}{5} = \frac{30}{50}$ and $\frac{30}{50} = \frac{9}{15}$. These equalities are examples of the *Equal Fractions Property.*

Equal Fractions Property

For all real numbers a, b, and k, if $b \neq 0$ and $k \neq 0$, then $\frac{a}{b} = \frac{ak}{bk}$.

The Equal Fractions Property holds for all fractions $\frac{a}{b}$ and values of k as long as the denominator is not zero. In $\frac{3}{5} = \frac{30}{50}$, $a = 3$, $b = 5$, and $k = 10$. That is, $\frac{3}{5} = \frac{3 \cdot 10}{5 \cdot 10} = \frac{30}{50}$. In $\frac{30}{50} = \frac{9}{15}$, $a = 9$, $b = 15$, and $k = \frac{10}{3}$. The Equal Fractions Property is true because of the Multiplying Fractions Property and the Multiplicative Identity Property. Here is how.

$\frac{a}{b} = \frac{a}{b} \cdot 1$ Multiplicative Identity Property

$= \frac{a}{b} \cdot \frac{k}{k}$ $\frac{k}{k} = 1, k \neq 0$

$= \frac{ak}{bk}$ Multiplying Fractions Property

Algebraic fractions, like numeric fractions, can sometimes be written in simpler form. To use the Equal Fractions Property to simplify algebraic fractions, find common factors in the numerator and denominator of the fraction.

Example 3

Simplify $\frac{112ab}{7a}$.

Solution 1 $7a$ is a common factor of the numerator and denominator.

$\frac{112ab}{7a} = \frac{7a \cdot 16b}{7a \cdot 1}$ Multiplying Fractions Property

$= \frac{16b}{1}$ Equal Fractions Property

$= 16b$ $x = \frac{x}{1}$ for all x.

Solution 2 People often skip steps. They sometimes show division of the common factors with slashes.

$$\frac{\overset{16}{\cancel{112}}\overset{1}{\cancel{a}}b}{\underset{1}{\cancel{7}}\underset{1}{\cancel{a}}} = \frac{16b}{1} = 16b$$

STOP QY1

QY1

Show that $\frac{25m}{30n}$ and $\frac{5m^2}{6mn}$ equal the same algebraic expression.

Activity

Use a CAS to simplify the following algebraic fractions. Check your work with a CAS or by substitution.

1. $\frac{5ab}{10b}$
2. $\frac{27x^2}{9x^2}$
3. $\frac{2y}{2yz}$
4. $\frac{48cd}{6ac}$
5. $\frac{12m^2}{18m}$
6. $\frac{8\pi x}{6x^2}$

Example 4

Assume $x \neq 0$ and $y \neq 0$. Multiply $\frac{4x}{27y}$ by $\frac{3y}{2x^2}$ and simplify the product.

Solution 1 Here we show all the major steps.

$$\frac{4x}{27y} \cdot \frac{3y}{2x^2} = \frac{4x \cdot 3y}{27y \cdot 2x^2} \quad \text{Multiplying Fractions Property}$$

$$= \frac{12xy}{54x^2y} \quad \text{Multiply.}$$

$$= \frac{6 \cdot 2 \cdot x \cdot y}{6 \cdot 9 \cdot x \cdot x \cdot y} \quad \text{Factor each expression.}$$

$$= \frac{6}{6} \cdot \frac{2}{9} \cdot \frac{x}{x} \cdot \frac{1}{x} \cdot \frac{y}{y} \quad \text{Multiplying Fractions Property}$$

$$= \frac{2}{9} \cdot \frac{1}{x} \quad \frac{k}{k} = 1 \text{ if } k \neq 0\text{; Identity Property}$$

$$= \frac{2}{9x} \quad \text{Multiplying Fractions Property}$$

Solution 2 Look for common factors in the numerator and denominator.

$$\frac{4x}{27y} \cdot \frac{3y}{2x^2} = \frac{\overset{2}{\cancel{4}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{y}}}{\underset{9}{\cancel{27}} \cdot \underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{x}} \cdot x} = \frac{2}{9x}$$

STOP QY2

READING MATH

The Equal Fractions Property is a property related to multiplication. It does not work when the same terms are *added* to the numerator and denominator.

QY2

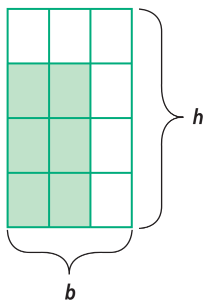
Multiply $\frac{-5a^2}{12b} \cdot \frac{2b^2}{6a}$ and simplify the product.

Questions

COVERING THE IDEAS

1. State the Multiplying Fractions Property.

2. The rectangle at the right has base b and height h.



   a. If all the small rectangles have the same dimensions, what is the area of the shaded region?

   b. What product of algebraic fractions is represented by the area of the shaded region?

In 3 and 4, multiply the fractions.

3. $\frac{a}{7} \cdot \frac{b}{2}$

4. $\frac{x}{30} \cdot \frac{3y}{z^2}$

5. Determine whether $\frac{1}{5}n = \frac{n}{5}$ is *always, sometimes but not always*, or *never* true.

6. Explain why $\frac{3}{8}x$ is equal to $\frac{3x}{8}$.

7. **Multiple Choice** Which expression does *not* equal the others?

   A $\frac{5n}{8}$ B $\frac{5}{8}n$ C $5n \cdot \frac{1}{8}$ D $\frac{5}{n} \cdot 8$

In 8–10, use the Equal Fractions Property to simplify the fraction.

8. $\frac{1{,}875}{225}$ 9. $\frac{-4n}{24n^2}$ 10. $\frac{10mn}{15np}$

In 11–14, multiply and simplify the result.

11. $\frac{1.2m}{n} \cdot \frac{1.2n}{m}$ 12. $\frac{7v}{x^2} \cdot \frac{x^2}{7v}$

13. $\frac{4abc}{27c} \cdot \frac{3}{2a^2b^3}$ 14. $\frac{1}{4} \cdot 2n \cdot \frac{3n}{6}$

15. a. One rectangle is half as wide and one-fourth as long as another rectangle. How do their areas compare?
    b. Draw a figure to illustrate your answer.

16. a. Show that $\frac{30 + x}{10 + x}$ and $\frac{30}{10}$ are *not* equivalent by letting $x = 7$.
    b. Show that $\frac{30 + x}{10 + x}$ and $\frac{3 + x}{1 + x}$ are *not* equivalent by letting $x = -4$.
    c. Why can't the Equal Fractions Property be applied in Parts a and b?

17. The Brock and Pease families have rectangular vegetable gardens. The length of the Brocks' garden is $\frac{2}{3}$ the length and $\frac{3}{4}$ the width of the Peases' garden.
    a. How do the areas of the gardens compare?
    b. Check your answer by using a specific length and width for the Peases' garden.

APPLYING THE MATHEMATICS

18. ***Skill Sequence*** Compute in your head.
    a. $\frac{5}{3} \cdot 3b$ b. $\frac{9}{x} \cdot xc$ c. $\frac{a}{b} \cdot bd$ d. $n^2 \cdot \frac{a}{n^2}$

19. Combine and simplify $\frac{4n - 5}{n} + \frac{5}{n}$.

In 20–22, multiply and simplify where possible.

20. $\frac{a}{b} \cdot \frac{b}{c} \cdot \frac{c}{a}$ 21. $\frac{-30r^3}{7s} \cdot \frac{-28s}{120r^4}$ 22. $\frac{-2y}{3} \cdot \frac{5y}{6} \cdot z$

23. Find two *algebraic* fractions whose product is $\frac{36a^2}{5x}$.

24. **Multiple Choice** Find the fraction that is *not* equal to the other three.
    A $\frac{110}{130}$ B $\frac{121}{143}$ C $\frac{121}{169}$ D $\frac{550}{650}$

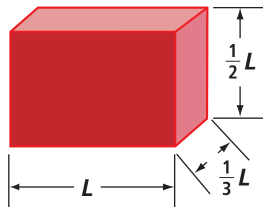
Recent surveys show the average size of a garden is between 500 and 1,000 square feet.

Source: www.oldhouseweb.com

25. **a.** Find the volume of the brick at the right.

b. Check your answer by letting $L = 12$.

c. Think of a cube with sides of length L. How many of these bricks would fit into the cube? How can you tell?



REVIEW

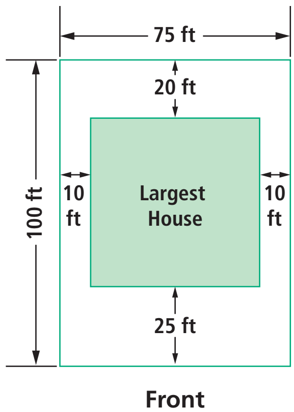
26. If 5% of a number is 12, what is 12% of that number? **(Lesson 4-1)**

27. Solve $150x + 200x = 14{,}000$. **(Lessons 3-4, 2-2)**

28. ***Skill Sequence*** State the reciprocal of each number. **(Lesson 2-8)**

a. 5 **b.** $\frac{1}{100}$ **c.** $\frac{-2}{3}$

29. A single-story house is to be built on a lot 75 feet wide by 100 feet deep. The shorter side of the lot faces the street. The house must be set back from the street at least 25 feet. It must be 20 feet from the back lot line, and 10 feet from each side lot line. What is the maximum area the house can have? **(Lesson 2-1)**



30. Consider $\frac{3}{4} \div \frac{9}{32}$.

a. Rewrite the problem as the multiplication of two fractions.

b. What is the answer in lowest terms?

c. What is the answer as a decimal? **(Previous Course)**

EXPLORATION

31. **a.** Calculate the products at the right.

b. Write a sentence or two describing the patterns you observed in Part a.

c. Predict the following products.

$$\frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdot \ldots \cdot \frac{2{,}010}{2{,}009}$$

$$\frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdot \ldots \cdot \frac{n+1}{n}$$

$$\frac{2}{1} \cdot \frac{3}{2}$$

$$\frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3}$$

$$\frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4}$$

QY ANSWERS

1. $\frac{25m}{30n} = \frac{5 \cdot 5m}{5 \cdot 6n} = \frac{5m}{6n}$ and $\frac{5m^2}{6mn} = \frac{m \cdot 5m}{m \cdot 6n} = \frac{5m}{6n}$.

2. $\frac{-5ab}{36}$