Lesson

# 4-8 Compound Inequalities, *And* and *Or*

## Vocabulary

compound sentence

intersection

union

± notation

**BIG IDEA** Placing AND between two sentences means that you want the intersection of their solutions; placing OR between them means that you want the union of their solutions.

## The Language of Compound Sentences

A car's water temperature gauge is an indicator of its engine cooling system. This gauge is marked to highlight normal temperatures as shown. Temperatures below 40°C occur when the car is warming up. Temperatures above 120°C indicate that the engine is in jeopardy of breaking down. These temperatures can be graphed. Using $t$ to represent temperature, each graph can be described with a pair of inequalities.


**Normal Temperatures**

$t \geq 40°$ and $t \leq 120°$


The graph is an interval.

**Abnormal Temperatures**

$t < 40°$ or $t > 120°$


The graph is two rays without their endpoints.

A **compound sentence** is a single sentence consisting of two or more sentences linked by the words *and* or *or*. The above graphs are described by *compound inequalities*. The compound inequality at the left can be written as the *double inequality* $40° \leq t \leq 120°$.

## Intersection and Union of Sets

The graphs of the compound inequalities above come from the *intersection* and *union* of two sets. On the left, the interval showing normal engine temperatures is the set of points shared by the graphs of the two simple inequalities. It consists of the points where the two rays $t \geq 40°$ and $t \leq 120°$ overlap.

### Intersection of Sets

The **intersection** of sets A and B, written $A \cap B$, is the set of elements that are in both A and B.

### Mental Math

**a.** Katie's temperature is 4 degrees away from 98.6. What are her possible temperatures?

**b.** Jeremy was 7 points away from getting 90 points on a test. What are his possible scores?

The graph of abnormal engine temperatures also begins with two rays without their endpoints, which in this case show temperatures below 40° and those above 120°. However, we do not look for the overlap. Instead, we take the union of the two rays. The meaning of *union* in mathematics is similar to its meaning in other contexts. For example, the Preamble of the Constitution of the United States of America reads:

> *We the People of the United States, in Order to form a more perfect Union, establish Justice, insure domestic Tranquility, provide for the common defense, promote the general Welfare, and secure the Blessings of Liberty to ourselves and our Posterity, do ordain and establish this Constitution for the United States of America.*

Here the word union describes a new set (the United States) formed by joining together component sets (the thirteen colonies).

**Union of Sets**

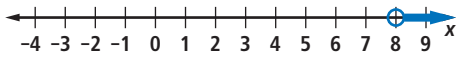
The **union** of sets A and B, written $A \cup B$, is the set of elements in either A or B or in both.

STOP **QY1**

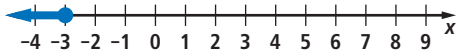
## Example 1

**Let A = the set of numbers for which $x > 8$.**
**Let B = the set of numbers for which $x \leq -3$.**
**Graph the set $A \cup B$. Describe the set with an *and* or an *or* statement.**

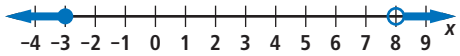
**Solution** Draw the graph of $x > 8$. The open circle at 8 indicates that 8 is not a solution.




Draw the graph of $x \leq -3$. The closed circle at −3 shows that −3 is a solution.

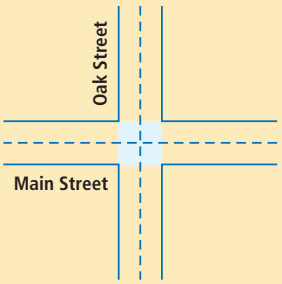



The union includes all points that satisfy either sentence or both sentences. The graph of $A \cup B$ is shown below. This is the set "$x > 8$ or $x \leq -3$."




**QY1**

Consider this situation: A police officer is directing traffic at the corner of Main and Oak Streets. Choose the correct word to complete each sentence.




The police officer was in the (union/intersection) of Main Street and Oak Street. This means that the officer was on Main Street (and/or) the officer was on Oak Street.
The officer was directing cars that were in the (union/intersection) of Main Street and Oak Street. This means that each car was on Main Street (and/or) on Oak Street.

## Example 2

**A family purchased some neon tetras to put in their new fish tank. They looked on the Internet to determine at what temperature to set the tank water. One site wrote to keep the water temperature from 72° to 78°F. A second site wrote 68° to 74° and a third site wrote 73° to 81°. To be safe, at what temperature should the family keep the tank?**

A neon tetra can live 10 years or more with the proper conditions.

Source: animal-world.com

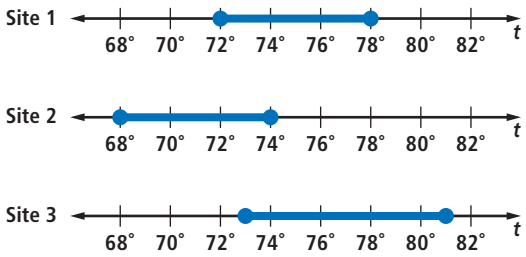
**Solution** Let $t$ represent the water temperature. Appropriate temperatures for tetras are those satisfying the following.

$$72° \leq t \leq 78° \text{ according to Site 1}$$

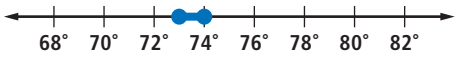
$$68° \leq t \leq 74° \text{ according to Site 2}$$

$$73° \leq t \leq 81° \text{ according to Site 3}$$

The family wondered if any temperatures could satisfy all three conditions. The graphs show the three intervals separately.

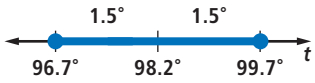



The best temperatures for the tetras are those that lie in all three intervals. This is the intersection in which the three graphs overlap. The tank should have a temperature satisfying $73° < t < 74°$.



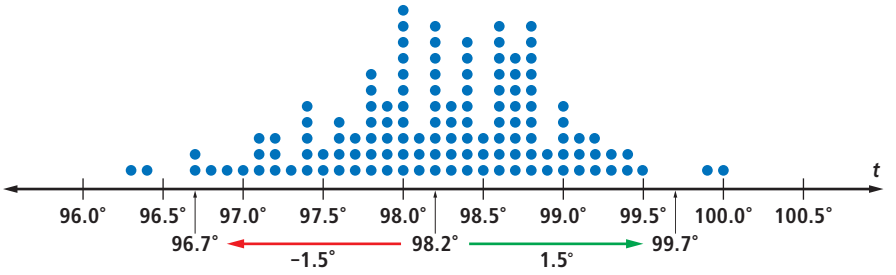

Site 1 ∩ Site 2 ∩ Site 3

## Describing the Intervals $a \leq x \leq b$




Most people would say that the average body temperature is 98.6°F. This figure was arrived at in the 19th century. Recent medical research has established that the mean temperature for healthy people is 98.2°F. However, there is some variability among healthy people. According to the new standard, the normal range varies above or below 98.2° by 1.5°. This means that the normal body temperatures $t$ of healthy people range from $98.2 + 1.5 = 99.7$ to $98.2 - 1.5 = 96.7$. So $96.7 \leq t \leq 99.7$.

You can combine $98.2 + 1.5$ and $98.2 - 1.5$ into one expression using **$\pm$ notation.** Then the interval of normal temperatures is written $98.2°\text{F} \pm 1.5°\text{F}$. The graph on the next page shows this interval and the temperatures of 129 men and women.




STOP QY2

▶ **QY2**

Write an expression of the form $a \pm d$ to represent the interval $40^\circ C \leq t \leq 120^\circ C$ for normal car engine temperatures from the beginning of this lesson.

## Solving Inequalities with *And* and *Or*

If the variable is not isolated in an inequality, its solutions are not evident. The first task then is to use the Addition and Multiplication Properties of Inequality to isolate the variable.

### Example 3

**Solve and graph $-8 \leq 5m + 4 < 19$.**

**Solution 1** $-8 \leq 5m + 4 < 19$ can be rewritten as $-8 \leq 5m + 4$ and $5m + 4 < 19$. Each inequality can be solved separately and then we take the intersection of their solutions.

$$\begin{array}{lcl} -8 \leq 5m + 4 & \text{and} & 5m + 4 < 19 \\ -8 + -4 \leq 5m + 4 + -4 & & 5m + 4 + -4 < 19 + -4 \\ -12 \leq 5m & & 5m < 15 \\ \frac{-12}{5} \leq \frac{5m}{m} & & \frac{5m}{m} < \frac{15}{5} \\ -2.4 \leq m & \text{and} & m < 3 \end{array}$$

Now combine these two statements to describe the interval.

$$-2.4 \leq m < 3$$


**Solution 2** Notice that after breaking the interval into two inequalities, the steps for solving each inequality are the same. In the future, the inequality does not need to be split apart at all. You perform the same operations to all three parts, as shown below.

$$\begin{array}{lll} -8 \leq 5m + 4 < 19 & & \text{Write the equation.} \\ -8 + -4 \leq 5m + 4 + -4 < 19 + -4 & & \text{Add } -4 \text{ to each part.} \\ -12 \leq 5m < 15 & & \text{Simplify.} \\ \frac{-12}{5} \leq \frac{5m}{m} < \frac{15}{5} & & \text{Divide by 5.} \\ -2.4 \leq m < 3 & & \text{Simplify.} \end{array}$$

GUIDED

## Example 4

Solve $13y + 86 > y + 2$ or $2y + 9 < 5y$.

**Solution** Solve each inequality separately.

| | | | |
|---|---|---|---|
| $13y + 86 > y + 2$ | | or | $2y + 9 < 5y$ |
| $12y + 86 >$ ? | | or | $-3y + 9 <$ ? |
| $12y >$ ? | | or | $-3y <$ ? |
| $y >$ ? | | or | $y$ ? ? |

The intervals overlap and their union is described by $y > -7$.

# Questions

## COVERING THE IDEAS

1. **Fill in the Blank** Fill in the blank with *and* or *or*.

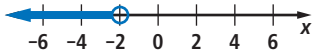
   $-8.2 < x \leq 4.75$ means $-8.2 < x$ ? $x \leq 4.75$.

**Matching In 2–5, match the inequality with its graph.**

2. $x < -2$ or $5 < x$
3. $-2 < x < 5$
4. $x < -2$ or $x < 5$
5. $x < -2$ and $x < 5$

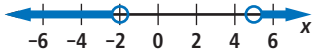
a.



b.



c.



d.



6. During the summer months, Mr. and Mrs. Boller have to agree on a temperature at which to set the air conditioner. Mr. Boller prefers the room temperature to be between 68° and 72°. Mrs. Boller likes temperature between 70° and 75°. Write an inequality to show the temperatures when Mr. and Mrs. Boller will both be comfortable.
7. During thyroid surgery, doctors make a 1.5 inch ± 0.5 inch incision. Write a double inequality showing the possible lengths of the incision. Then graph the sentence.
8. A movie theater gives a discount price to children under 3 years old and to senior citizens over 65 years old.
   a. Write a compound inequality to show the ages that receive the discounted price.
   b. Graph the ages that receive this discounted price.

9. Temperatures in space vary greatly. Astronauts on the International Space Station have to be able to endure outside temperatures that are $-18°C \pm 139°C$.
   a. Write an inequality to represent these temperatures in space.
   b. Graph the inequality.

The International Space Station (ISS) weighs 206,043.3 kilograms and has a habitable volume of 420 cubic meters.

Source: NASA

**In 10–13, solve the inequality and graph all solutions.**

10. $2x + 9 > 17$ or $8 - 5x \leq 13$
11. $\frac{1}{4}x - 7 < -2$ and $6x + 3.8 \geq 9.2$
12. $-4 \leq 2.5x - 9 < 15$
13. $20c + 5 \geq 30c - 15$ or $9 - c > 4 - 2c$

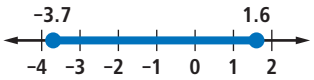
## APPLYING THE MATHEMATICS

14. Marcos and Lydia want to hire a band for their wedding. The band charges a flat fee of \$950 and an additional \$275 per hour. They are willing to spend at least \$1,200 on the band, but not more than \$2,300. For how many hours can they have the band play for their wedding?

**Fill in the Blanks** For 15 and 16, fill in the blanks to describe the interval shown on the graph with $\pm$ notation.

15. ? $\pm$ ?


16. ? $\pm$ ?




17. Make a table and graph of $y = -2x + 8$. Highlight the $x$-coordinates on the table and the part of the graph where $-2x + 8 > 12$ or $-2x + 8 < 1$.
18. Make a table and graph of $y = 3 + 5x$. Highlight the $x$-coordinates on the table and the part of the graph where $-2 \leq 3 + 5x \leq 23$.

## REVIEW

19. a. Solve the formula $d = rt$ for $r$, where $d$ is distance, $r$ is rate, and $t$ is time.
    b. What was the average speed of a truck that traveled 245 miles in 4 hours? **(Lesson 4-7)**

**In 20 and 21, solve the sentence.** (Lesson 4-6)

**20.** $7(k - 2) - 11 > (7k - 2) - 11$

**21.** $20p - 4(p + 2) = (6p - 8) + 10p$

**22.** Solve $-5 - 2y > 14 - 6y$ and graph the set of solutions. (Lesson 4-5)

**23.** Jennifer compared prices for the same pair of shoes at three different stores. The first store sold them at \$75 less a discount of 15% due to a storewide sale. Jennifer had a \$10 gift certificate for the second store, where the shoes cost \$72. She would have to pay 8% sales tax at these two stores. The shoes at the third store cost \$67 including tax. At which store can Jennifer buy the shoes for the cheapest price? (Lesson 4-1)

**24.** Evaluate $|-4xy + y - x|$ for each situation. (Lesson 1-6)

**a.** $x = 3, y = 4$

**b.** $x = -1, y = 2$

**c.** $x = -10, y = -5$

## EXPLORATION

**25.** In mathematics, the difference between the words *and* and *or* is determined by union and intersection, but the English usage of these words is not always as clear.

**a.** Consider this statement:
*In case of an emergency, women and children go first.*

Write how a mathematician would view the statement and then explain if there is a difference in what was probably meant by the statement.

**b.** Find at least two other situations in which the English usage of the words *and* and *or* varies from the mathematical usage.

**QY ANSWERS**

**1.** intersection; and; union; or

**2.** $80^\circ C \pm 40^\circ C$