

## Lesson

## 4-7

## Equivalent Formulas

► **BIG IDEA** If a formula gives a first variable in terms of a second variable, it may be possible to transform the formula to give the second variable in terms of the first.

Two different temperature scales are in common use throughout the world. The scale used wherever people use the metric system is the **Celsius scale**, named after Anders Celsius (1701–1744), a Swedish astronomer and physicist. It is also sometimes called the **centigrade scale** because of the 100-degree interval between  $0^{\circ}\text{C}$  and  $100^{\circ}\text{C}$ , the freezing and boiling points of water. The other scale is the **Fahrenheit scale**, named after the German physicist Gabriel Fahrenheit (1686–1736). In the Fahrenheit scale, which is now used only in the United States and a few other countries, water's freezing and boiling points are  $32^{\circ}$  and  $212^{\circ}$ .

People outside the United States seldom use the Fahrenheit scale. An exception is when a visitor from the U.S. asks about the weather forecast. They have to change the Celsius temperature from their forecast into Fahrenheit using the formula  $F = 1.8C + 32$ . This formula uses the Celsius temperature  $C$  as the **input** and produces the Fahrenheit temperature  $F$  as the **output**. We say that it gives  $F$  in terms of  $C$ . However, visitors to the United States have to make the reverse conversion. They must convert Fahrenheit temperatures from the U.S. forecast into Celsius. If you are going to convert from Fahrenheit to Celsius often, it is convenient to have a formula that begins with  $F$  as input and gives  $C$  as output. To find this formula, we write 1.8 as the fraction  $\frac{9}{5}$ .

## GUIDED

**Example 1**

Solve  $F = \frac{9}{5}C + 32$  for  $C$ .

**Solution** Solve like any linear equation. Isolate  $C$  on the right side.

$$\begin{array}{ll}
 F = \frac{9}{5}C + 32 & \text{Write the formula.} \\
 \underline{\quad ? \quad} = \underline{\quad ? \quad} & \text{Add } -32 \text{ to each side.} \\
 \frac{5}{9}(\underline{\quad ? \quad}) = \frac{5}{9}(\underline{\quad ? \quad}) & \text{Multiply each side by } \frac{5}{9}. \\
 \underline{\quad ? \quad} = C & \text{Simplify.}
 \end{array}$$

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## Vocabulary

Celsius scale  
centigrade scale  
Fahrenheit scale  
input  
output  
equivalent formulas

## Mental Math

Evaluate  $x^2 - 2xy + y^2$  if

- $x = 5$  and  $y = 5$ .
- $x = -5$  and  $y = 5$ .
- $x = 5$  and  $y = -5$ .



The lowest recorded temperature in the U.S. occurred in Prospect Creek Camp, Alaska, on January 23, 1971. The temperature fell to  $-80^{\circ}\text{F}$ .

Source: *The World Almanac and Book of Facts*

**Check** In each formula, substitute 212 and 100 for the appropriate variables.

$$\text{Using } F = 1.8C + 32$$

$$\underline{\quad ? \quad} = 1.8 \cdot \underline{\quad ? \quad} + 32$$

$$\underline{\quad ? \quad} = \underline{\quad ? \quad} + 32$$

$$\underline{\quad ? \quad} = \underline{\quad ? \quad}$$

$$\text{Using } C = \frac{5}{9}(F - 32)$$

$$\underline{\quad ? \quad} = \frac{5}{9}(\underline{\quad ? \quad} - 32)$$

$$\underline{\quad ? \quad} = \frac{5}{9}(\underline{\quad ? \quad})$$

$$\underline{\quad ? \quad} = \underline{\quad ? \quad}$$

The numbers  $F = 212$  and  $C = 100$  satisfy both equations.

The formulas  $F = 1.8C + 32$  and  $C = \frac{5}{9}(F - 32)$  are called **equivalent formulas** because every pair of values of  $F$  and  $C$  that satisfies one equation also satisfies the other.

## GUIDED

### Example 2

A formula for the volume of a cone is  $V = \frac{1}{3}\pi r^2 h$ , where  $V$  is the volume,  $r$  is the radius of the base, and  $h$  is the height. Octavio solved this formula for  $h$ . His work is shown below. Explain what he did to get each equation.

$$V = \frac{1}{3}\pi r^2 h$$

a.  $3V = \pi r^2 h$        $\underline{\quad ? \quad}$

b.  $\frac{3V}{\pi} = r^2 h$        $\underline{\quad ? \quad}$

c.  $\frac{3V}{\pi r^2} = h$        $\underline{\quad ? \quad}$

## Activity

In 1–5, use a CAS to solve for the given variable. Tell what is done to both sides in each step.

1.  $-6x + 3y = 24$ , for  $y$

2.  $x - y = -14$ , for  $y$

3.  $S = \frac{C - 1}{3}$ , for  $C$  (formula for cap size  $S$  when  $C$  = circumference of the head in inches)

4.  $E = \pi ab$ , for  $b$  (formula for the area  $E$  of an ellipse when  $a$  is one-half the length of the major axis and  $b$  is one-half the length of the minor axis)

5.  $G = \frac{s + 2d + 3t + 4h}{a}$ , for  $h$  (formula for baseball slugging average  $G$  where  $s$  = number of singles,  $d$  = number of doubles,  $t$  = number of triples,  $h$  = number of home runs, and  $a$  = number of at bats)

Whether you are working with a CAS or by hand, different people may write formulas that look quite different. However, they might be equivalent. For example, here is what Alf and Beth wrote to solve  $p = 2\ell + 2w$  for  $w$ .

Alf's Solution		Beth's Solution	
$p = 2\ell + 2w$	Write the formula.	$p = 2\ell + 2w$	Write the formula.
$p - 2\ell = 2w$	Subtract $2\ell$ .	$p - 2\ell = 2w$	Subtract $2\ell$ .
$\frac{p - 2\ell}{2} = w$	Divide by 2.	$\frac{1}{2}(p - 2\ell) = w$	Multiply by $\frac{1}{2}$ .
$w = \frac{p - 2\ell}{2}$		$w = \frac{1}{2}(p - 2\ell)$	

**STOP** QY

**QY**

Explain how you know that  $w = \frac{p - 2\ell}{2}$  and  $w = \frac{1}{2}(p - 2\ell)$  are equivalent.

## Using a Graphing Calculator

One important use of equivalent formulas arises when using graphing calculators. Often formulas that are entered must give  $y$  in terms of  $x$ .



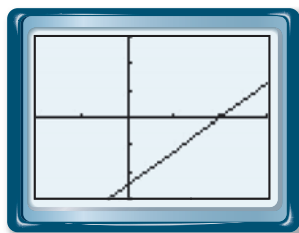
### Example 3

Use a graphing calculator to graph  $5x - 2y = 100$ .

**Solution** Solve for  $y$ .

$$\begin{aligned}
 5x - 2y &= 100 && \text{Write the equation.} \\
 -5x + 5x - 2y &= -5x + 100 && \text{Add } -5x \text{ to each side.} \\
 -2y &= -5x + 100 && \text{Combine like terms.} \\
 \frac{-2y}{-2} &= \frac{-5x + 100}{-2} && \text{Divide each side by } -2. \\
 y &= \frac{-5}{-2}x + \frac{100}{-2} && \text{Expand the fraction.} \\
 y &= 2.5x - 50 && \text{Simplify.}
 \end{aligned}$$

Now enter the equation  $Y1 = 2.5x - 50$  into the calculator. A window of  $-20 \leq x \leq 30$  and  $60 \leq y \leq 60$  is shown below.



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**Check 1** Use the TRACE feature to read the coordinates of some points on the line. Check that these satisfy the original equation. For example, our TRACE showed the point with  $x \approx 10.5$ ,  $y \approx -23.7$  on the graph.

Does  $5(10.5) - 2(-23.7) = 100$ ?

$$99.9 \approx 100 \text{ Yes. It checks.}$$

The point  $(10.5, -23.7)$  is very close to the graph of  $5x - 2y = 100$ .

**Check 2** Compute the coordinates of a point on the line. For example, when  $x = 0$ ,  $5(0) - 2y = 100$ .

$$5(0) - 2y = 100$$

$$-2y = 100$$

$$y = -50$$

The TRACE on our calculator shows that the point  $(0, -50)$  is on the line.

## Questions

### COVERING THE IDEAS

- There is one temperature at which Celsius and Fahrenheit thermometers give the same reading:  $-40^\circ$ . Verify that  $C = -40$ ,  $F = -40$  satisfies both  $F = 1.8C + 32$  and  $C = \frac{5}{9}(F - 32)$ .
- A person with a head circumference of 23.5 inches wears a size  $7\frac{1}{2}$  baseball cap. Verify that  $C = 23.5$  and  $S = 7\frac{1}{2}$  satisfy the formula  $S = \frac{C-1}{3}$ .
- Solve  $p = 2\ell + 2w$  for  $\ell$ .
  - Fill in the Blanks** In Part a you are asked to find a formula for  $\underline{\quad}$  in terms of  $\underline{\quad}$  and  $\underline{\quad}$ .
  - Check your solution to Part a by substituting values for  $\ell$ ,  $w$ , and  $p$ .

In 4 and 5, solve the equation for  $y$ .

- $8x + y = 20$
- $4x - 8y = -40$

In 6–9, solve the formula for the indicated variable.

- $r = \frac{d}{t}$  for  $d$
- $S = 180n - 360$  for  $n$
- $F = m \cdot a$  for  $a$
- $A = \frac{1}{2}(b_1 + b_2)h$  for  $h$

### APPLYING THE MATHEMATICS

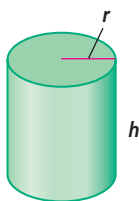
10. The formula  $S = 3F - 24$  gives the sizes of a person's shoe in terms of  $F$ , the length of a person's foot in inches.
- Solve this formula for  $F$ .
  - Estimate the length of a person's foot if the person wears a size 9 shoe.
11. Jocelyn and Alma were asked to solve the equation  $5x - 2y = 100$  for  $y$ . When solving the equation, they got the following answers.

$$\begin{array}{l} \text{Jocelyn} \\ y = \frac{100 - 5x}{-2} \end{array}$$

$$\begin{array}{l} \text{Alma} \\ y = 2.5x - 50 \end{array}$$

Is the work of either student correct? Explain how you know.

12. a. Solve the following equations for  $y$ .  $2x + 6y = 15$  and  $x = \frac{6y - 15}{-2}$   
 b. Graph each equation and  $y = 2.5 - \frac{x}{3}$  on a calculator.  
 c. Which of these equations appear to be equivalent? Provide evidence to support your answer.
13. A formula for the circumference  $C$  of a circle is  $C = \pi d$ , where  $d$  is the diameter.
- Solve this formula for  $\pi$ .
  - How could you use the formula to find a value of  $\pi$ ?
  - Use your answer to Part b to estimate  $\pi$  from the measurements of some circular object you have.
14. The formula  $S = 2\pi r^2 + 2\pi rh$  gives the total surface area of a cylindrical solid shown below with radius  $r$  and height  $h$ .



- Solve this formula for  $h$ .
  - Find the height to the nearest hundredth if the radius is 10 centimeters and the surface area is 2,000 square centimeters.
15. Solve for  $y$  and use a graphing calculator to graph the equations  $5x + 2y = 8$  and  $5x + 2y = 12$ . What is true about these graphs?

### REVIEW

In 16 and 17, solve the sentence. (Lessons 4-6, 4-4)

16.  $6(y - 4) = 2(y - 4) - 8(2 - y)$

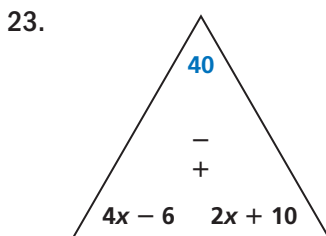
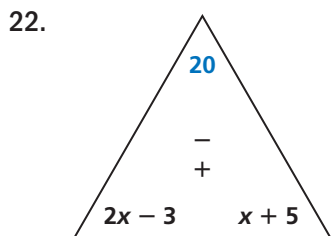
17.  $52v < 22v - 7 + 30v$
18. Five more than twice a number is three more than four times the number. What is the number? (Lesson 4-4)
19. a. Write an equation for the horizontal line through  $(5, -3)$ . (Lesson 4-2)  
 b. Write an equation for the vertical line through  $(5, -3)$ .
20. According to the *World Almanac and Book of Facts*, the Middle East is reported to have approximately 65% of the world's oil reserves. All together, the Middle East's crude oil reserves are estimated to total 686 billion barrels. How many barrels are estimated to be in the world's total crude oil reserves? (Lesson 4-1)
21. a. A pentagon has two sides of length  $2x + 22$  and three sides of length  $x - 1$ . Its perimeter is 55. Solve for  $x$ . (Lesson 3-5)  
 b. Suppose the pentagon had two sides of length  $x - 1$ , three sides of length  $2x + 22$ , and still had a perimeter of 55. Why is this impossible?



In 2004, the United States imported over 10,000,000 barrels of oil per day from foreign nations.

Source: *The World Almanac and Book of Facts*

In 22 and 23, write the related facts and determine the value of  $x$  for each fact triangle. (Lessons 3-5, 2-7)



### EXPLORATION

24. Ask a friend or relative for a formula used in his or her job. Explain what the variables represent and show an example of how it is used. Solve the formula for one of its other variables.

### QY ANSWER

The definition of division says that dividing by 2 is the same as multiplying by  $\frac{1}{2}$ .