

Lesson

4-6

Situations That Always
or Never Happen

► **BIG IDEA** Some linear sentences have no solution; others are true for all real numbers.

Comparing Situations

Which job would you take?

Job 1

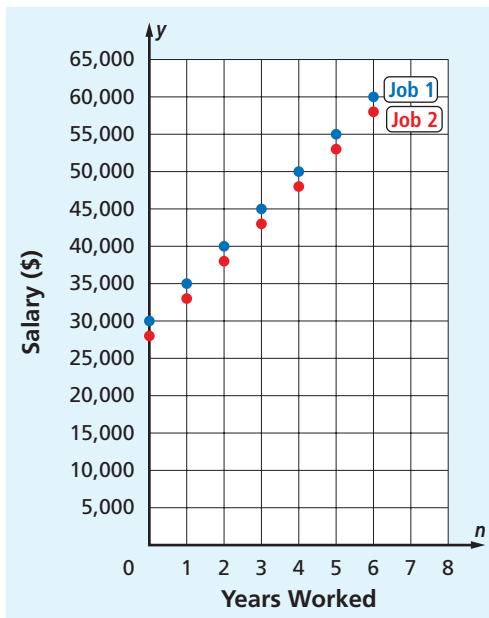
starting salary \$30,000
yearly raises of \$5,000

Job 2

starting salary \$28,000
yearly raises of \$5,000

Of course, the answer is obvious. Job 1 will always pay more than Job 2. Looking at the pay in a table of values or on a graph supports this conclusion.

Years Worked (n)	Job 1	Job 2
0	\$30,000	\$28,000
1	\$35,000	\$33,000
2	\$40,000	\$38,000
3	\$45,000	\$43,000
4	\$50,000	\$48,000
5	\$55,000	\$53,000
6	\$60,000	\$58,000



As you compare the salaries with the table, you can see that the money earned at Job 1 is always \$2,000 greater than the money earned at Job 2. When the points are graphed, notice that they lie on parallel lines. For any year you pick, the pay for Job 1 is greater than for Job 2. But what happens when this is solved algebraically?

Let n = number of years worked.

Mental Math

- How many cups are in 2 pints?
- How many cups are in 3 quarts?
- How many cups are in 4 gallons?

Salary in Job 1

$$30,000 + 5,000n$$

Salary in Job 2

$$28,000 + 5,000n$$

When is the pay in Job 1 better than the pay in Job 2? You must solve

$$30,000 + 5,000n > 28,000 + 5,000n.$$

Subtract $5,000n$ from each side.

$$30,000 + 5,000n - 5,000n > 28,000 + 5,000n - 5,000n$$

Now collect like terms.

$$30,000 > 28,000$$

The variable has disappeared! Since $30,000 > 28,000$ is always true, the disappearance of n signals that n can be any real number. Job 1 will always pay a better salary than Job 2, as expected. For any equation or inequality the following generalization is true.

Sentences That Are Always True

When solving a sentence, if you get a sentence that is *always* true, then the original sentence is always true.

When does Job 1 pay *less* than Job 2? Looking at the table or graph on page 216, this never appears to be true. To answer this algebraically, you could solve $30,000 + 5,000n < 28,000 + 5,000n$.

$$30,000 + 5,000n < 28,000 + 5,000n$$

$$30,000 + 5,000n - 5,000n < 28,000 + 5,000n - 5,000n$$

$$30,000 < 28,000$$

It is never true that 30,000 is less than 28,000. So Job 1 never pays less than Job 2, something which was obvious from the pay rates. The following generalization is also true.

Sentences That Are Never True

When solving a sentence, if you get a sentence that is never true, then the original sentence is never true.

True and False Sentences

Suppose the sentence you are solving has only one variable and the variable disappears. The chart below summarizes the possibilities.

Solving leads to a	Possible Examples	Solutions
<i>True</i> statement	$0 = 0$ or $92 \leq 92$	any real number
<i>False</i> statement	$0 = 3$ or $-9 > 14$	no real number

Example 1Solve $18m - 20 + 3m = 3(7m + 2)$.**Solution**

$$18m - 20 + 3m = 3(7m + 2) \quad \text{Write the equation.}$$

$$21m - 20 = 21m + 6 \quad \text{Simplify each side.}$$

$$21m - 21m - 20 = 21m - 21m + 6 \quad \text{Subtract } 21m \text{ from each side.}$$

$$-20 = 6 \quad \text{Simplify.}$$

Since -20 does not equal 6 , this statement is not true, so the original equation has no solution.

GUIDED**Example 2**Solve $42k < 80k + 6 - 38k$.**Solution**

$$42k < 80k + 6 - 38k \quad \text{Write the inequality.}$$

$$42k < 42k + 6 \quad \text{Combine like terms.}$$

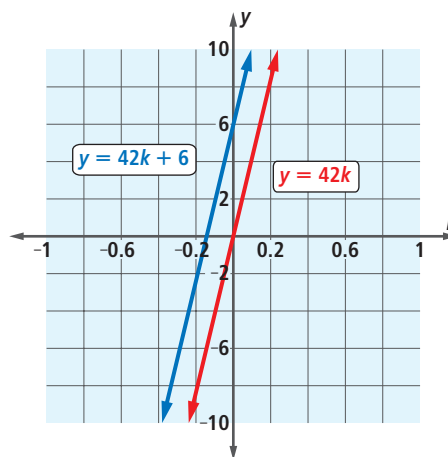
$$\underline{\quad ? \quad} < \underline{\quad ? \quad} \quad \text{Subtract } 42k \text{ from each side.}$$

$$0 < 6 \quad \underline{\quad ? \quad}$$

Since $0 < 6$ is always true, $\underline{\quad ? \quad}$ is true for any real number k .

Check Graph both sides of the inequality, as shown at the right.

The lines appear to be parallel and never intersect. The graph of $42k$ is always below the graph of $80k + 6 - 38k$. This supports the conclusion that $42k < 80k + 6 - 38k$ is true for any real number k .

**Questions****COVERING THE IDEAS**

- Hamburger Heaven pays a starting salary of \$6.90 an hour, and each year increases it by \$1 an hour. Video King starts at \$7.50 an hour, and also increases \$1 an hour per year.
 - Create a table for the data.
 - Write an inequality to represent this situation.
 - When does Hamburger Heaven pay more?
- Add $-12y$ to both sides of the inequality $12y + 11 > 12y - 1$.
 - What inequality results?
 - Describe the solutions to this inequality.



In 2006, the federal minimum wage for covered, nonexempt employees was \$5.15 per hour.

Source: U.S. Department of Labor

3. Add $2g$ to both sides of the equation $40 - 2g = 11 + 29 - 2g$.
 - a. What equation results?
 - b. Describe the solution(s) to this equation.
 - c. Check your solution(s) using a graph.
4. Check Guided Example 2 by substituting any real number for k .

In 5–8, solve the sentence.

5. $-6 + 4n > 4(n + 11)$
6. $65w = 11 + 65w$
7. $12x + 33 = 33 + 12x$
8. $-3(-9d + -2) > 30d - 3d$
9. The population of Yorkville is about 40,000 and growing at about 800 people a year. Newburgh has a population of about 200,000 and is growing at about 800 people a year.
 - a. What will be the population of Yorkville in y years?
 - b. What will be the population of Newburgh in y years?
 - c. When will their populations be the same?

APPLYING THE MATHEMATICS

10. Apartment A rents for \$810 per month including utilities. Apartment B rents for \$700 per month but the renter must pay \$110 per month for utilities and a one-time \$50 fee for a credit check.
 - a. What sentence could you solve to find out when apartment A is cheaper?
 - b. Solve this inequality.
 - c. If you wanted to rent one of these apartments for two years, which one would be cheaper?

In 11 and 12, create an example of an equation different than those in this lesson with the given solution.

11. There is no real solution.
12. The equation is true for all real numbers.

REVIEW

In 13 and 14, solve the inequality. (Lessons 4-5, 3-7)

13. $-3(5p + 2) > 24$
14. $2n \geq -7 - (n + 4)$
15. Sending a package by Wedropem shipping service costs \$3.50 plus \$0.25 per ounce. Brokefast Company charges \$4.75 plus \$0.10 per ounce. If your package weighs 16 ounces, which service is cheaper? Explain your reasoning. (Lesson 4-5)



Nationally, the median rent for a one-bedroom apartment in 2003 was \$550.

Source: National Multi-Housing Council

16. Mama's Pizza charges \$12 for a large cheese pizza and \$0.90 for each additional topping. Pizza Palace charges \$14 for a large cheese pizza but only \$0.40 for each additional topping. (Lessons 4-4, 4-3)
- Write an equation for each pizza shop describing the cost c of a large pizza in terms of the number of toppings t .
 - For how many toppings are the pizza prices at the two shops equal?
17. In the 2000 United States presidential election, 105,396,641 people voted, according to the *World Almanac and Book of Facts*. In the 2004 presidential election, 122,293,332 people voted. By what percent did the number of voters in the election increase from 2000 to 2004? (Lesson 4-1)
18. Four instances of a general pattern are given below. (Lesson 1-2)
- $$-(43 - 20) = 20 - 43$$
- $$-(5.21 - 8.49) = 8.49 - 5.21$$
- $$-\left(\frac{6}{5} - \frac{3}{7}\right) = \frac{3}{7} - \frac{6}{5}$$
- $$-(w - 16) = 16 - w$$
- Write the pattern using the variables x and y .
 - Is the pattern true for all real number values of x and y ? Justify your answer.

EXPLORATION

19. When solving the equation $ax + b = cx + d$ for x , there may be no solution, exactly one solution, or infinitely many solutions. What must be true about a , b , c , and d to guarantee each of the following?
- There is exactly one solution.
 - There are no solutions.
 - There are infinitely many solutions.