Lesson

4-5 Solving $ax + b < cx + d$

BIG IDEA An inequality of the form $ax + b < cx + d$ can be solved by applying the same operations to both sides as in solving the equation $ax + b = cx + d$.

Just as an equation can have the variable on both sides of the equal sign, an inequality can have the variable on both sides of the inequality sign. The methods from the last lesson can also be used to solve linear inequalities of the form $ax + b < cx + d$.

Consider the following situation. Years ago, the Moseley family planted a 12-foot beech tree and a 4-foot maple tree. Beech trees grow at a rate of about $\frac{1}{2}$ foot per year. Maple trees grow about 1 foot per year. So t years after planting, the height of the beech tree is $12 + 0.5t$ feet and the height of the maple tree is $4 + t$ feet.

Mental Math

a. How long is a 9-inning baseball game that averages 20 minutes per inning?

b. How long is a 4-quarter basketball game that averages 40 minutes per quarter?

c. How long is a 3-period hockey game that averages 50 minutes per period?

Example 1

Mrs. Moseley looked at an old photograph of the two trees. In it, the beech tree was taller than the maple tree. She wondered when the photo was taken.

Solution 1 Use an algebraic solution. Write an inequality relating the heights. The beech tree was taller than the maple tree, so the heights satisfy the inequality $12 + 0.5t > 4 + t$.

Solve this inequality as you would solve an equation.

$12 + 0.5t > 4 + t$	Write the equation.
$12 + 0.5t + (-0.5t) > 4 + t + (-0.5t)$	Add $-0.5t$ to each side.
$12 > 4 + 0.5t$	Add like terms.
$12 + (-4) > 4 + (-4) + 0.5t$	Add -4 to each side.
$8 > 0.5t$	Simplify.
$\frac{8}{0.5} > \frac{0.5t}{05}$	Divide each side by 0.5.
$16 > t$	Simplify.

So $t < 16$. The photo was taken less than 16 years after the Moseleys planted the trees.

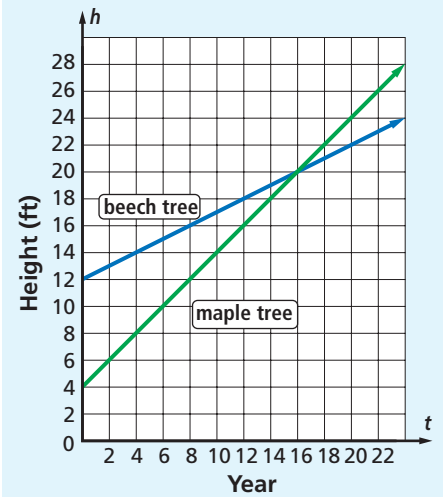
Solution 2 Use a table as shown on the next page. You can see the beech tree was taller when $t < 16$. This verifies that the photo was taken less than 16 years after the Moseleys planted the trees.

National Arbor Day was founded by J. Sterling Morton in Nebraska in 1872.

Source: National Arbor Day Foundation

Solution 3 Use a graph. Write an equation describing each tree height. For the beech tree, $h = 12 + 0.5t$. For the maple tree, $h = 4 + t$. Graph each equation and find the point of intersection.

t	Height h (ft)	
Number of Years	Beech Tree	Maple Tree
0	12	4
4	14	8
8	16	12
12	18	16
16	20	20
20	22	24
24	24	28



The lines intersect at the point (16, 20). So sixteen years after they were planted, the trees were both 20 feet tall. We are looking for the times when the beech tree was taller than the maple tree. So look for values of t where the beech's line is *above* that for the maple. These times lie to the left of the intersection point (16, 20). This is where $t < 16$. The photo was taken less than 16 years after the trees were planted.

The Addition and Multiplication Properties of Inequality can also be used to solve any inequality of the form $ax + b < cx + d$. In Guided Example 2, two algebraic solutions are given. Solution 2 involves dividing the inequality by a negative number. Recall that multiplying or dividing an inequality by a negative number reverses the inequality sign.

GUIDED

Example 2

Solve $7 - 11x \geq 4x + 12$.

Solution 1

$7 - 11x \geq 4x + 12$	Write the inequality.
$7 - 11x + \_?\_ \geq 4x + 12 + \_?\_$	Add $\_?\_$ to each side.
$7 \geq 15x + 12$	Add like terms.
$7 - \_?\_ \geq 15x + 12 - \_?\_$	Subtract $\_?\_$ from each side.
$-5 \geq 15x$	Simplify.
$\_?\_ \geq \_?\_$	$\_?\_$ each side by 15.
$\_?\_ \geq x$	Simplify.

(continued on next page)

Solution 2

$7 - 11x \geq 4x + 12$	Write the inequality.
$7 - 11x - \underline{?} \geq 4x + 12 - \underline{?}$	Subtract $\underline{?}$ from each side.
$7 - 15x \geq 12$	Add like terms.
$7 - 15x - \underline{?} \geq 12 - \underline{?}$	Subtract $\underline{?}$ from each side.
$-15x \geq 5$	Simplify.
$\frac{-15x}{-15} \ \underline{?} \ \frac{5}{-15}$	Divide each side by −15. Be sure to $\underline{?}$ the inequality sign.
$x \ \underline{?} \ -\frac{1}{3}$	Simplify.

Check Recall that checking an inequality requires two steps.

Step 1 Try the boundary value of x. Check that $x = \underline{?}$ makes both sides of the original sentence equal.

$7 - 11(\underline{?}) \geq 4(\underline{?}) + 12$

Step 2 Try a number that satisfies $x \leq \underline{?}$. Test to see if this number makes the original inequality true.

$7 - 11(\underline{?}) \geq 4(\underline{?}) + 12$

Example 3

Five times a number is less than three times the same number. Find the number.

Solution It may seem that there is no such number. But let's work it out and see. Let n be such a number. Then n must be a solution to $5n < 3n$. Solve this as you would any other linear inequality.

$5n < 3n$	Write the inequality.
$5n - 3n < 3n - 3n$	Add $-3n$ to each side.
$2n < 0$	Combine like terms.
$\frac{2n}{2} < \frac{0}{2}$	Divide each side by 2.
$n < 0$	Simplify.

So n must be less than zero. Any negative number will work.

Check

Step 1 Check the boundary value. If $n = 0$, $5 \cdot 0 = 3 \cdot 0$, and $0 = 0$. It checks.

Step 2 Pick $n < 0$ to check that the inequality is true. We let $n = -4$.

Is $5(-4) < 3(-4)$?

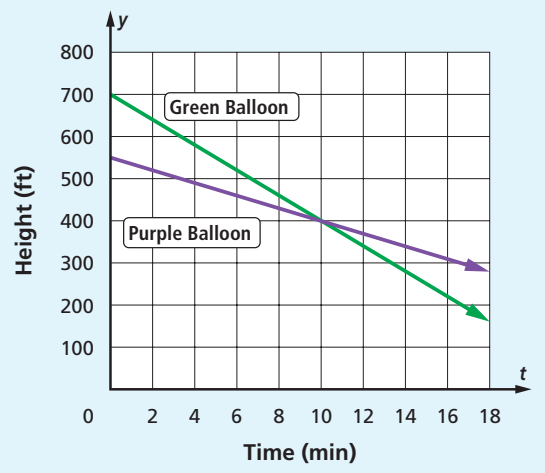
$-20 < -12$? Yes, it checks.

5 times n will be less than 3 times n exactly when n is a negative number.

Questions

COVERING THE IDEAS

1. Two hot air balloons are descending from their cruising altitude at a constant rate to the ground. Both balloons begin descending at the same time, but from different elevations. The green balloon starts at 700 feet and the purple balloon starts at 550 feet as shown in the graph below.



   a. Give three values of t for which the green balloon is higher.
   b. Write an inequality that describes when the green balloon is higher.
   c. Write an inequality that describes when the purple balloon is higher.

2. In a typical year, a willow tree grows 3.5 feet, while a Chitalpa tree grows 2 feet. Suppose a 6-foot willow tree and a 13-foot Chitalpa tree were planted at the same time.
   a. Which tree is taller 2 years after they were planted?
   b. Which tree is taller 10 years after they were planted?
   c. Write an inequality to describe when the willow tree is taller.
   d. Solve the inequality you wrote in Part c.

The Chitalpa tree is a hybrid created in Uzbekistan by Nikolai Rusanov in 1964 and was first introduced into the United States in 1977.

Source: Mid-Columbia Community Forestry Council

APPLYING THE MATHEMATICS

3. a. Solve $5m + 4 > 8m + 14$ by first adding $-5m$ to each side.
   b. Solve $5m + 4 > 8m + 14$ by first adding $-8m$ to each side.
   c. Should you get the same answers for Parts a and b? Why or why not?
   d. Describe how the steps in the solutions to Parts a and b are different.

In 4–7, a. solve the inequality, and
b. check the inequality.

4. $7a + 4 \geq 3a + 28$
5. $-53 + 15x \leq -8 + 20x$
6. $2y + 13 < -5y - 8$
7. $14 - 4n > 29 + 6n$
8. Three times a number is less than two times the same number. Find such a number.

In 9 and 10, solve the inequality.

9. $9 - 4(x + 2) < 10x$
10. $-5y \geq -y + 8 + 7y$
11. In solving the inequality $12x - 16 > 20x + 15$, you must decide which term to eliminate first, $12x$ or $20x$. Explain why this decision matters more for solving an inequality than an equation.
12. Suppose two skyscrapers are being built. The Edwards Building was already 155 feet high when work began on the King Tower. The Edwards Building is going up at an average of 24 feet per day, while the King Tower construction is progressing at an average of 29 feet per day. Let t represent the number of days since construction started on the King Tower.
    a. How tall will each building be when $t = 10$?
    b. Write an inequality for the following question: *When will the King Tower be taller than the Edwards Building?*
    c. Solve your inequality in Part b.
    d. When each building is completed, the graph of its height stops increasing and becomes horizontal. Suppose that the King Tower will be 1,230 feet tall and the Edwards Building will be taller than the King Tower. Draw a graph to show when the Edwards Tower will again be taller than the King Tower.

A typical tower crane can lift up to 18 metric tons of material at one time.

Source: howstuffworks.com

13. Angelina Wright has a Web site. She allows two music companies to advertise on it, Elevator Tunes and Sleepy Songs. Elevator Tunes pays her \$5.00 plus 6¢ each time their ad gets a hit. Sleepy Songs pays her \$3.50 plus 10¢ each time their ad gets a hit. (A hit is when someone clicks on the ad from Angelina's Web site.) For how many hits are the Elevator Tunes' ads more profitable than the Sleepy Songs' ads? Justify your answer.

REVIEW

In 14–17, solve the equation. (Lesson 4-4)

14. $2t + 38 = 5(t + 1)$

15. $\frac{7}{3} - \frac{1}{4}z = 7 + \frac{2}{5}z$

16. $3(x - 6) + 12(2x + 5) = 10(2x + 7)$

17. $2.83 - 0.4r = 9.02 - 4.2r$

18. According to the Census Bureau, in 2005, Delaware had a population of approximately 840,000, which was increasing at a rate of about 12,000 people a year. Montana had a population of approximately 935,000, increasing at a rate of about 6,700 per year. If these rates continue in the future, in how many years after 2005 will the populations be equal? **(Lesson 4-4)**

19. **a.** On a coordinate grid, graph $x = -5$ and $y = 9$. **(Lesson 4-2)**
    b. Give the coordinates of the point the two lines have in common.

20. Suppose a sweater on sale costs \$35 and a pair of jeans on sale costs \$27. If they originally cost a total of \$86, what is the percent of discount, rounded to the nearest percent? **(Lesson 4-1)**

21. **Multiple Choice** Suppose $k > m$. What is true about $k - m$? **(Lesson 3-6)**
    A $k - m$ is always negative.
    B $k - m$ is always positive.
    C $k - m$ can be either positive or negative.

EXPLORATION

22. The square of a number is less than the product of one less than the number and two greater than the number.
    a. Find one such number that makes the statement true.
    b. Find one such number that makes the statement false.
    c. Find all such numbers that make the statement true.