Lesson

4-3

Using Tables and Graphs to Solve

▶ **BIG IDEA** Tables and graphs are often useful for comparing values from two or more situations.

GUIDED

Example

You are the manager of an office. Your company needs to lease a copy machine. You must choose between two office supply firms. Acme Copiers offers a copier for $250 per month with an additional charge of $0.01 per copy.

Best Printers offers the same machine for $70 per month with a per-copy charge of $0.03.

Describe the *break-even point,* the situation for which the costs are the same.

Solution 1 Use a table. The cost of the machine depends upon the number of copies made. To compare, you must look at prices for many situations.

1. Let x = the number of copies made per month.

   Price for x copies from Acme Copiers = $250 + 0.01x$

   Price for x copies from Best Printers = $70 + 0.03x$

   Use your calculator to make a table showing the costs for 0; 2,000; 4,000; 6,000; ...; 20,000 copies per month. Use the table to draw conclusions about which company's pricing plan best suits your needs.

Number of Copies (x)	Acme Copiers Price $250 + 0.01x$	Best Printers Price $70 + 0.03x$
0	?	?
2,000	?	?
4,000	?	?
6,000	?	?
8,000	?	?
10,000	?	?
12,000	?	?
14,000	?	?
16,000	?	?
18,000	?	?
20,000	?	?

Mental Math

a. Half a serving of soup is 150 mL. How many mL are 2 servings?

b. How many mL are 3 servings?

c. How many mL are 4 servings?

2. When $x \geq$ __?__, Acme's price is lower. When $x \leq$ __?__, Best's price is lower.

3. From this table you cannot tell the exact break-even point. However, it seems to occur between __?__ copies and __?__ copies.

4. Another table with x between the values in Step 3 above may yield a solution. Complete this table.

Number of Copies (x)	Acme Copiers Price $250 + 0.01x$	Best Printers Price $70 + 0.03x$
8,000	?	?
8,500	?	?
9,000	?	?
9,500	?	?
10,000	?	?

5. The break-even point occurs when $x =$ __?__ and the price is __?__.

6. It is important to describe the two variables for the break-even point.

   a. The x value: How many copies are made when Best and Acme charge the same amount? __?__

   b. The y value: What is the amount charged by each copier when the prices are the same amount? __?__

A graph can help you interpret the information in a table and can help answer many questions.

Solution 2 Use a graph.

1. Let the prices be as in Step 1 of Solution 1.

   Graph $y_1 = 250 + 0.01x$ on the window $0 \leq x \leq 20{,}000, 0 \leq y \leq 500$.

   Graph $y_2 = 70 + 0.03x$ on the same window.

2. Use the INTERSECT command to determine the point at which the graphs intersect. (__?__ , __?__)

3. What do the coordinates of the point of intersection mean?

   The x-coordinate, __?__, means __?__.

   The y-coordinate, __?__, means __?__.

One of the challenges of finding solutions using tables or graphs is that the result is a coordinate point with both an x and a y value. You need to pay close attention to the question you are being asked to determine whether the x- or the y-coordinate is the final answer.

Graphs, tables, and algebraic sentences each have advantages. Graphs can display a great deal of information and are useful for comparing values, but may be time-consuming to make. Tables might also be time-consuming to make. Graphing calculators and spreadsheets can make both tables and graphs, but still may not always give exact solutions. Solving equations and inequalities using algebraic properties is often preferred because they are efficient tools, and the results are precise. In the next two lessons, you will use properties to solve equations with variables on each side of the equal sign.

Questions

COVERING THE IDEAS

1. The table below lists the charges for color copies of digital photos at two different camera shops.

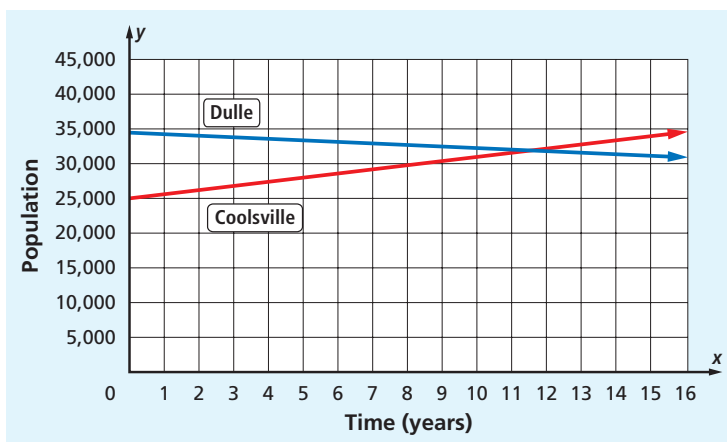| Number of Copies	Cost at Shop A	Cost at Shop B
1	$1.30	$1.20
2	$1.55	$1.50
3	$1.80	$1.80
4	$2.05	$2.10
5	$2.30	$2.40
6	$2.55	$2.70

   a. For what number of copies is Shop A's cost less?
   b. For what number of copies is Shop B's cost less?
   c. Graph the costs of the copies. Which graph is higher for 2 copies? Which graph is higher for 7 copies?
   d. If y is the cost of x copies, what are the (x, y) coordinates of the break-even point?

2. A bakery keeps a supply of flour and sugar. The graph at the right shows how many pounds of each are in the bakery storeroom over a period of days.
   a. Estimate when the bakery has the same amount of flour as sugar. How many pounds of each are in the storeroom?
   b. Give an example of a day for which there is more flour than sugar.
   c. Use an inequality to describe when there is more sugar than flour.
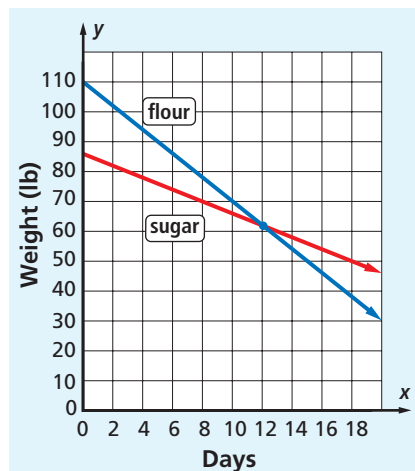
3. Rental-car companies sometimes charge a set fee plus an amount for each mile that the car is driven. Suppose Extra Value Cars charges $18.32 plus $0.32 per mile and Rhodes Rental charges $26.24 plus $0.20 per mile.

a. Use the graph to approximate the point of intersection of the two lines.

b. What does the x-coordinate of the point of intersection represent in the problem? What does the y-coordinate represent?

c. Using the graph, determine which company is less expensive if 80 miles are driven. Explain how you know this from the graph.
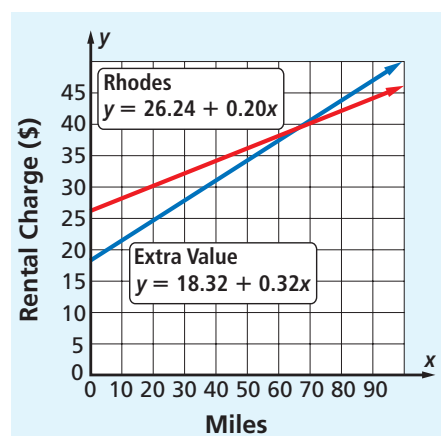
4. The population of Coolsville is currently 25,000 and is growing at a rate of 600 people per year. Across the river is the town Dulle, which currently has a population of 34,900 and is decreasing by 300 people per year. Use the graph and the table to answer the following questions.

Time in Years (x)	Coolsville Population (y_1)	Dulle Population (y_2)
0	25,000	34,900
1	25,600	34,600
2	26,200	34,300
3	26,800	34,000
4	27,400	33,700
5	28,000	33,400
6	28,600	33,100
7	29,200	32,800
8	29,800	32,500
9	30,400	32,200
10	31,000	31,900
11	31,600	31,600
12	32,200	31,300
13	32,800	31,000
14	33,400	30,700
15	34,000	30,400
16	34,600	30,100

a. Write an equation for y, the population of Coolsville after x years. Write a similar equation for the population of Dulle after x years.

b. Give the approximate coordinates of the intersection or break-even point. Explain what the two coordinates of this point represent in the problem.

c. **Fill in the Blanks** Until _?_ years, _?_ had the larger population. After _?_, _?_ had the larger population. At _?_ years, Coolsville and Dulle had the same population of _?_.

d. Write an inequality that represents the values of x for which the population of Coolsville is greater, and an inequality that represents the values of x for which the population of Dulle is greater.

APPLYING THE MATHEMATICS

5. Theo has $30 and is *saving* at a rate of $6 per week. Michelle has $150 and is *spending* at a rate of $5 per week.

   a. Write an expression for the amount Theo has after w weeks.

   b. Write an expression for the amount Michelle has after w weeks.

   c. Make a graph. Use it to determine when Theo and Michelle will have the same amount.

6. Alicia is offered two sales positions. With Company Q, she would earn $800 per month plus 5% commission on sales. (This means that 5% of the money her customers spend is added to her $800 salary.) With Company P, she would earn $600 per month plus a 6% commission on sales.

   a. If Alicia expects sales of about $20,000, which company would pay her more monthly?

   b. Complete the table at the right.

   c. Determine how much Alicia must sell to be paid more at Company P than at Company Q.

REVIEW

In 7 and 8, write the equation for the line pictured in the graph. (Lesson 4-2)

7.

8.

| Sales (S)	Earnings at Company Q	Earnings at Company P
$12,000	$1,400	$1,320
$14,000	$1,500	$1,440
$16,000	?	?
$18,000	?	?
$20,000	?	?
$22,000	?	?
$24,000	?	?
$26,000	?	?
$28,000	$2,200	$2,280
$30,000	$2,300	$2,400
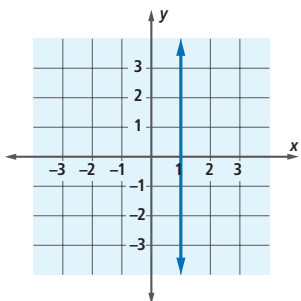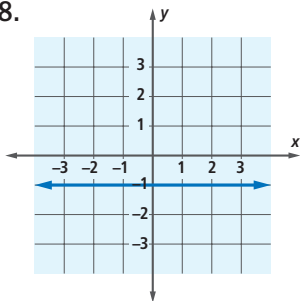
9. On a coordinate grid, graph the following three lines. (Lessons 4-2, 3-1)

   line ℓ: $y = 7$ line m: $x = 2$ line n: $y = 2x - 3$

   a. At what point do lines ℓ and m intersect?

   b. At what point do lines ℓ and n intersect?

   c. Find the area of the triangle formed by the three lines.

10. Ms. Chang invested $8,450 in stocks. After 1 year, the value of her stocks had fallen and her investment was now worth $7,625. By what percent did her investment fall? (Lesson 4-1)

In 11 and 12, compute in your head. (**Lesson 4-1**)

11. What is 25% of 60? **12.** 110 is what percent of 100?

13. Mary has a collection of foreign coins. $\frac{5}{12}$ of her coins are from China and $\frac{1}{4}$ are from Japan. If she has 248 Japanese and Chinese coins altogether, how many coins does Mary have in all? (**Lesson 3-8**)

14. *Skill Sequence* Solve each sentence. (**Lessons 3-7, 3-3**)
　　a. $0n = 8$　　**b.** $8n = 0$　　**c.** $8n > 0$　　**d.** $-8n > 0$

15. a. Determine whether the equations $(2x + 2)(-9) + 6x = 10$ and $2(-6x - 9) = 10$ are equivalent.
　　b. If the equations in Part a are equivalent, explain why they are equivalent. (**Lesson 3-3**)

16. Mika and June traveled from Denver, Colorado, to San Francisco, California, a distance of about 1,250 miles. Mika drove in a car, while June left later and flew by airplane. The graph shows the distance y each had traveled x hours after Mika began his trip. (**Previous Course**)
　　a. Who arrived in San Francisco first? How can you tell?
　　b. How long did June's trip take?

Here are various Japanese coins, ranging in value from 1 to 500 yen.
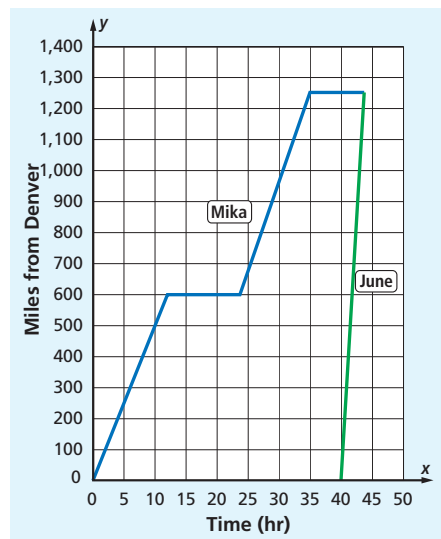
EXPLORATION

17. There are three kittens Tic, Tac, and Toe. They each have different colored fur: orange, gray, and yellow. They also live in different homes: a hotel, a condo, and a house. Using the following information, find the fur color of each kitten and where each lives.
　　(1) Tic does not live in the condo.
　　(2) The cat that lives in the house does not have orange fur.
　　(3) Tac lives in the hotel.
　　(4) Tic's favorite color is gray, but that is not her fur color.
　　(5) The cat that lives in the hotel has orange fur.

(*Hint:* It can help to record what you know in a table like the one shown below. You can eliminate an incorrect pairing by placing an X in the cell. Place an O in the cell for a correct pairing.)

	Orange	Gray	Yellow	Hotel	Condo	House
Tic	?	?	?	?	?	?
Tac	?	?	?	?	?	?
Toe	?	?	?	?	?	?