## Lesson

4-2

Horizontal and Vertical Lines

▶ **BIG IDEA** Every horizontal line has an equation of the form $y = k$; every vertical line has the form $x = h$.

Equations for Horizontal Lines

In Lesson 2-3, you saw several number puzzles in which you began with a number and then performed a series of calculations to get a final answer. The puzzling part came from the surprising relationship between the starting number and the final result. Below you see ordered pairs for two of these puzzles in a table and on a graph. For each puzzle, the points lie on a line. In the "I Can Guess Your Age Puzzle," the result is always the same as the starting number. The line that is graphed has equation $y = x$.
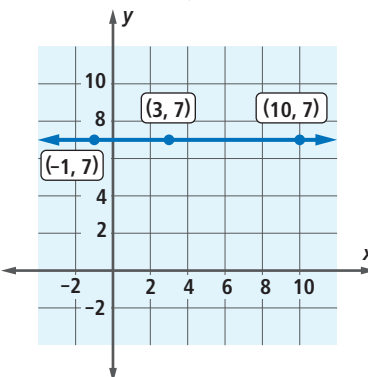
Mental Math

$\triangle ABC$ is equilateral.

a. What is x?

b. What is y?

I Can Guess Your Age Puzzle	
Starting Number (x)	Result (y)
3	3
10	10
–8	–8
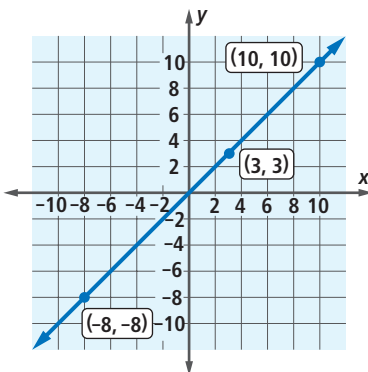
But the "Seven Is Heaven" graph is different.

Seven Is Heaven Puzzle	
Starting Number (x)	Result (y)
3	7
10	7
–1	7

The points lie on a *horizontal line*. For all of these points, the *y*-coordinate equals 7. In short, $y = 7$. Every *horizontal line* has an equation of this type.
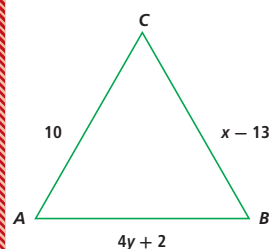
Equation of a Horizontal Line

Every **horizontal line** has an equation of the form $y = k$, where k is a real number.

Example 1

Find an equation describing all points on the line graphed at the right.

Solution The points are on a horizontal line that crosses the y-axis at 2. An equation for the line is $y = 2$.

Check Two of the points on the line have coordinates $(1, 2)$ and $(-3, 2)$. These numbers satisfy the equation $y = 2$.
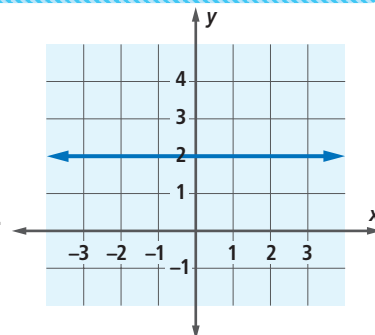
Equations for Vertical Lines

A vertical line is drawn at the right.

Notice that the x-coordinate of the ordered pairs is 4.5 regardless of the y-coordinate. Thus an equation for the line is $x = 4.5$. This means x is fixed at 4.5, but y can be any number.

Equation of a Vertical Line

Every **vertical line** has an equation of the form $x = h$, where h is a real number.

An equation with only one variable, such as $x = -4.5$ or $y = 7$, can be graphed on a number line (in which case its graph is a point), or on a coordinate plane (in which case its graph is a line). The directions or the context of the problem will usually tell you which type of graph to draw.
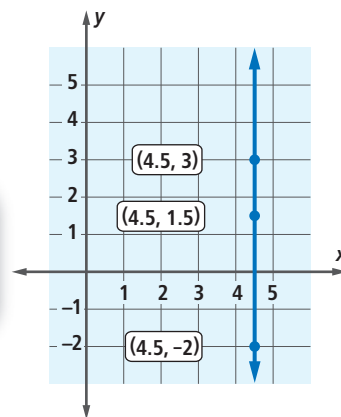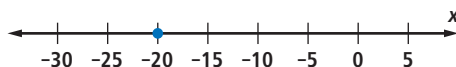
Example 2

a. **Graph $x = -20$ on a number line.**
b. **Graph $x = -20$ on a coordinate plane.**

Solutions

a. Draw a number line. Mark the point with coordinate equal to -20, as shown below. The graph of $x = -20$ on a number line is the single point with coordinate -20.

(continued on next page)

b. Draw a coordinate grid. Plot points whose x-coordinate is −20. Some points are (−20, 10), (−20, 4), and (−20, −8). Draw the line through these points.

The graph of $x = -20$ in a coordinate plane is the vertical line that crosses the x-axis at −20.
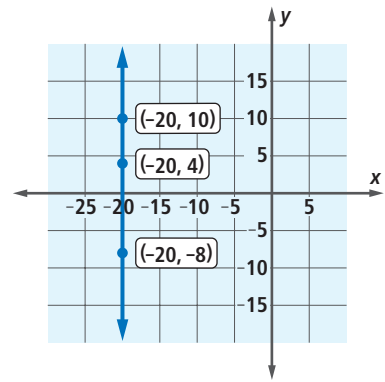
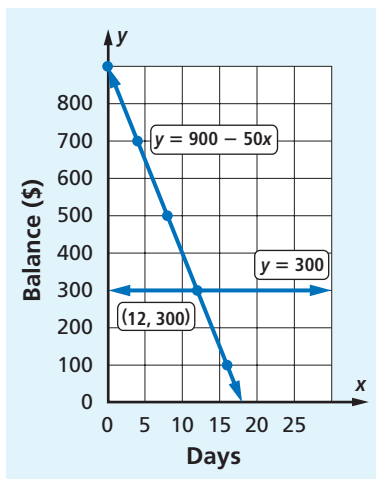## Horizontal Lines and Linear Patterns

Horizontal lines can be helpful in solving equations or inequalities.

Example 3

At the beginning of a vacation, Matt has $900 in his savings account. As long as he has at least $300 in his account, he does not have to pay a service fee. Each morning he withdraws $50 from an ATM for spending money. How many days can he withdraw $50 per day without paying a service fee?

Solution 1 Let $x =$ the number of days after Matt's vacation began. Let $y =$ the amount of money in his bank account. Then $y = 900 - 50x$.

Day (x)	Balance (y)
0	900
4	700
8	500
12	300
16	100

The first modern day ATM in the United States was introduced to consumers in 1971 by Chemical Bank.

Source: Cash Technologies, Inc.

The table of (x, y) values can be used to graph $y = 900 - 50x$. Also graphed is $y = 300$ to represent the balance that he must keep to avoid a service fee. As long as the point for Matt's balance is at or above the $y = 300$ line he does not have to pay a service fee. The lines appear to intersect at $x = 12$, **so for the first 12 days, Matt's balance is high enough to avoid paying a service fee.**

Solution 2 Translate "Matt's balance is at least $300" into an inequality and solve. Let $x =$ the number of days Matt has been on vacation. In x days, he will have withdrawn $50x$ dollars, so we want to know when $900 - 50x \geq 300$.

$$900 - 50x - 900 \geq 300 - 900 \quad \text{Subtract 900 from each side.}$$
$$-50x \geq -600 \quad \text{Collect like terms.}$$
$$\frac{-50x}{-50} \leq \frac{-600}{-50} \quad \text{Divide each side by } -50.$$
Change the sense of the
inequality.
$$x \leq 12 \quad \text{Simplify.}$$

For the first 12 days, Matt does not have to pay a
service charge.
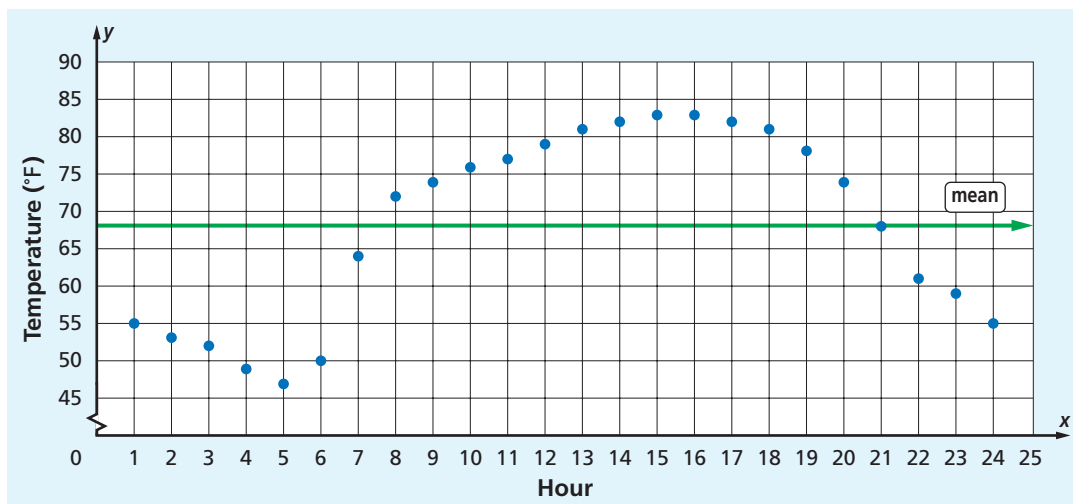
Deviation from the Mean

For statistical data in a scatterplot, a horizontal line at the mean can
help to show how the data relate to the average value. The hourly
temperatures in Flagstaff, Arizona, on a June day are shown in the
graph below.

Related to each temperature is its deviation from the mean, which
is the difference between the actual temperature and the mean
temperature.

For example, when $h = 15$ (3 P.M.), the temperature was 82.9°F,
giving a deviation of $82.9 - 68.1 = 14.8$°F from the mean. At $h = 3$
(3 A.M.), the deviation was $52 - 68.1 = -16.1$°F.

Activity

Use the table and graph of the temperatures at Flagstaff on a June day.
Notice that the values on the vertical axis begin at 45. The interval
$0 < y < 45$ is compressed on the graph since there are no data in
that interval.

1. What is the equation of the horizontal line that is graphed?

2. Give the deviation from the mean at 7 A.M. and 8 A.M.

3. For which two hours was the deviation +5.8°F?

4. The temperatures t are described by the interval __?__ $\leq t \leq$ __?__.

5. The deviations d are described by the interval __?__ $\leq d \leq$ __?__.

6. The deviation is positive when h, the number of hours since midnight, is in the interval __?__ $\leq h \leq$ __?__.

7. The deviation is negative when $h \leq$ __?__ or $h \geq$ __?__.

8. The maximum deviation and the maximum absolute deviation are __?__ and __?__ respectively. Why is the minimum deviation not equal to the minimum absolute deviation?

| Hour (h)	Temperature (°F)
1	55
2	53.1
3	52
4	48.9
5	46.9
6	50
7	64
8	72
9	73.9
10	75.9
11	77
12	79
13	81
14	82
15	82.9
16	82.9
17	82
18	81
19	78.1
20	73.9
21	68
22	61
23	59
24	55

Sometimes deviations are taken from a number other than the mean. For instance, it is common in golf to give the player's results not by the score, but by how the score deviates from par. (*Par* is the expected number of strokes a golfer should take on the hole.) For example, in the 2006 U.S. Open, par was 70 strokes for 18 holes. Some scores in the last round were; Jim Furyk 0 (his score was 70), Ryuji Imada +4 (score: 74), Jeff Sluman −1 (score: 69), and Vijay Singh +3 (score: 73). In other sports, the deviation of a team's score from its opponent's score is described with statements like "We're down by 3 points!"

Questions

COVERING THE IDEAS

1. **a.** List three ordered pairs whose *y*-coordinate is 4.

   b. Graph your points from Part a and draw a line through them.

   c. Write the equation of your line from Part b.

In 2 and 3, write an equation for each graph.

2.

3.
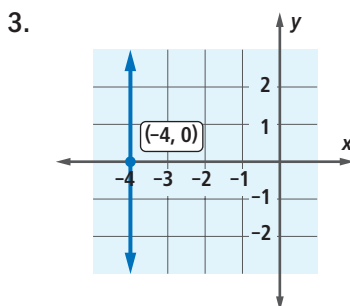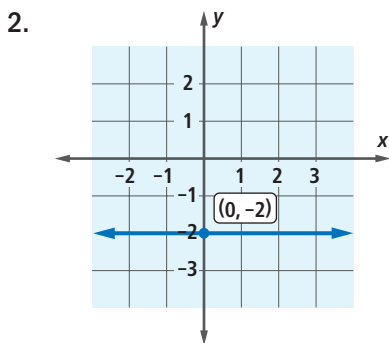
4. **Fill in the Blank** All points on a horizontal line have the same __?__-coordinate.

5. **Fill in the Blank** All points on a vertical line have the same __?__-coordinate.

In 6 and 7, an equation is given.

    a. Graph all points on a number line that satisfy the equation.

    b. Graph all points in the coordinate plane that satisfy the equation.

6. $x = 2$ 7. $y = 1.5$

8. At the beginning of the year, Kylie has $580 in her savings account. As long as she has $200 in her account she does not have to pay a service fee. How long can she withdraw $20 per week without paying a service fee?

In 9–11, the graph shows the annual snowfall on Mt. Hood near Portland, Oregon. The mean snowfall is 98.5 inches per year. This information is important for the water supply in the region, and also for people who like to ski on Mt. Hood.

9. **a.** Let S be the snowfall in year y. What is an equation of the line that is graphed?

    b. Which year had the greatest absolute deviation from the mean?

10. **a.** Which year's snowfall was closest to the mean?

    b. What does that tell about its absolute deviation from the mean?

11. **a.** In which two consecutive years was the deviation negative?

    b. Was this good or bad news for Portland residents? Explain your answer.
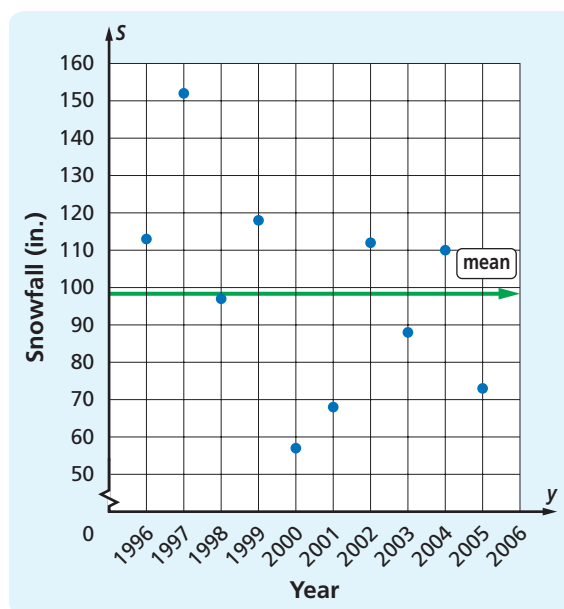
In 12 and 13, write an equation for the line containing the given points.

12. (–9, 12), (–4, 12), (0.04, 12)   13. (–6, –3), (–6, 4), (–6, 22)

14. **a.** Write an equation for the horizontal line through (7, –13).

    b. Write an equation for the vertical line through (7, –13).

15. **Fill in the Blanks** Horizontal lines are parallel to the __?__-axis and perpendicular to the __?__-axis.

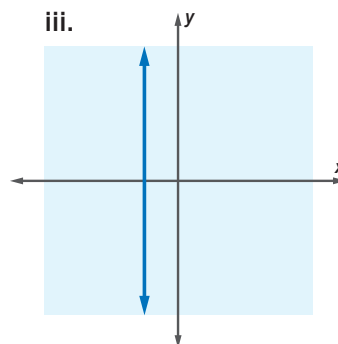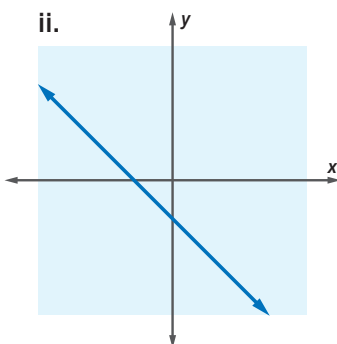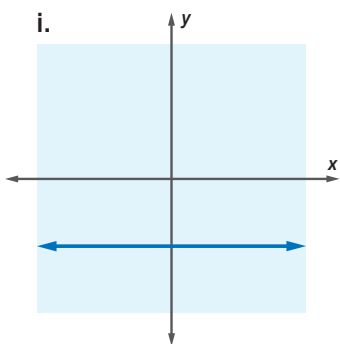16. **Matching** Match each table with the appropriate graph.

a.

x	y
−3	1
−2	0
−1	−1
0	−2
1	−3
2	−4

b.

x	y
−3	1
−3	2
−3	3
−3	4
−3	5
−3	6

c.

x	y
−4	−5
−2	−5
0	−5
2	−5
4	−5
6	−5

i.

ii.

iii.

APPLYING THE MATHEMATICS

17. Predict where the lines $y = 5$ and $x = 7$ will intersect without graphing.

18. The table at the right gives information about a set of data values that has a mean of 53. Complete the table.

Value	Deviation
45	?
42	?
?	−1
?	13
?	7

19. The temperature in Acapulco, Mexico does not vary greatly whereas in a desert location like Reno, Nevada, the temperature can change greatly over the course of a day. Suppose that at 7 A.M. the temperature in Acapulco was 80°F and in Reno it was 65°F. If the Acapulco temperature stayed steady all morning, and the Reno temperature rose 5°F per hour, when were the temperatures in the cities the same?

20. Write an equation for a line through the points $(4, m)$, $(0, m)$, $(6, m)$, and $(-8, m)$.

21. A furnace repair service charges $35 for travel time plus $70 per hour to repair furnaces. Lloyd's Smoothie Emporium needs its furnace repaired, but Lloyd is willing to spend no more than $315 on the repair.

    a. Write an equation that relates the repair cost y to the time spent x.

    b. Draw a graph of the equation that you wrote in Part a.

    c. On the same coordinate axes as in Part b, draw the line $y = 315$ to represent the money that Lloyd is willing to spend on the repair.

    d. Use your graph to determine the maximum number of hours that the furnace repairperson could work and still keep Lloyd's bill under $315.

    e. Check your answer to Part d by solving an inequality.

REVIEW

22. In 2006, the seating capacity in Fenway Park was 38,805 people. If 28,486 people show up for a baseball game, what percent of the stadium is full? Round your answer to the nearest percent. (**Lesson 4-1**)

23. After a 6% sales tax, it cost the Clarences $17,000 for a car. What was the amount before the tax was applied? (**Lesson 4-1**)

24. The Sholitons bought a farm, but they did not know the capacity of the heating-oil storage tank. At one point, the tank was $\frac{1}{8}$ full. After a delivery of 450 gallons, the tank's gauge showed that it was $\frac{7}{8}$ full. How many gallons of oil does the tank hold? (**Lesson 3-8**)

25. Last year the Guptas paid $18,550 in income taxes, more than one fourth of their earned income. (**Lessons 3-7, 3-6**)

    a. Let $I =$ the Guptas earned income last year. Write a sentence that describes the situation above.

    b. Solve the sentence.

26. Solve $2(x + 1) + 7 = 5x$. (**Lesson 3-3**)

27. Graph $y = -5x - (1 + x)$. (**Lesson 3-1**)

Major League Baseball's oldest ballpark, Fenway Park, is located in Boston, Massachusetts.

Source: Major League Baseball

EXPLORATION

28. Find a set of data with numbers that lend themselves to finding the mean. Find the mean. Then find the deviations. Describe the advantages and disadvantages of using deviations rather than the actual values.